Measurement Techniques and Instruments for Measurement of Dynamic Characteristics of Gear Pump

C. Raghunathan^{*} and C. Manoharan

King College of Technology, Namakkal, 637 020, Tamil Nadu, India; raghusalem@yahoo.co.in, c_m_66@yahoo.co.in

Abstract

Overall dynamic analysis of gear pump performance is mandatory in industrial scenario due to various operational, design and physical parameters, for which the main impact is being caused by friction, torque, vibration assembly, surface smoothness, load and speed for which, we are adopting various techniques/methods for analyzing the friction, vibrations in axial and radial directions with respect to high accuracy and sensitivity with the help of mechanical, pneumatic and optical setup, in which the following methods are included: direct reading along with precision supports and storage capacitor. Based on the above, we obtained solutions/findings for frictional torque/force, load which comparatively experiential & theoretical and lost dynamic characteristics of precision gear assemblies. Most of the research was carried out for damping, squeeze oil film thickness and sliding velocity so the present work is centered on the above said measurement techniques for analyzing the dynamic performance of gear pump.

Keywords: Assembling loads and Gear Pump, Dynamic Characteristics, Frictional Torque, Surface Smoothness, Vibrations

1. Introduction

The different parameters of gear are checked by special setups and instruments. Depending on the nature of the requirements of instrument gear verification of all the parameters of the gear can often be replaced by determination of the friction torque and, when necessary, vibrations of the gear rings.

In this case the friction torque and vibrations of the gear to be measured can be divided into the following groups: Friction torque of gear contact (contact torque); Friction torque in the dynamic condition consisting of constant and variable components of friction torque; spatial vibrations of gear consisting of vibrations of gear rolling elements in the axial and radial directions¹. These methods and instruments for the measurement of friction torque and vibrations can be classified into groups with the following general requirements: High accuracy of measurements exceeding the accuracy of

the instruments in which the gears are mounted. High sensitivity that allows measurement of the parameters of imprecision gear. Conditions of measurement that must fully initiate the operational range of the gear. Minimum inertia of the sensing element of the measuring setup. Minimum non-linearity. The working conditions of many gears differs so much that friction torque and vibrations during operation sometimes change by more than two orders of magnitude. Hence the range of measuring instruments should be large and linear. Universality and simplicity of maintenance for different types of gear instruments should be large and linear.

The possibility of measurements with simultaneous recording of readings, high productivity of measurements, Such high demands sometimes involve finding different variations of the solution of the problem for each individual case so that the principle and methods vary for apparently similar measurements².

*Author for correspondence

2. Measurement of Friction Torque and Vibrations of Gear

The friction torque of a gear is a function of many variables. Analytical determination is almost impossible where it appreciably affects the working of the complete assembly or machine. The study and determination of the components of friction torque in gear started simultaneously with the appearance of precision instruments, in the first place, navigation instruments. In the last couple of decades this parameter has become the decisive factor in evaluating the quality and suitability of a gear for application in instrument rolling contact gear.

The working principles of the majority of instruments and stands for the measurement of friction torque is based on the fact that the forces acting on the moving part of the torque meter-transducers assembly deflect it from the initial equilibrium position. The moving part of the torque meter is maintained in equilibrium in this new position under the action of two opposing forces; the force due to the object being measured and the reacting force due to the elastic number of the torque meter that records the quantity to be measured.

In the majority of cases the measurement of vibrations is based on the contact less method of measuring displacements. First of all, the methods for measuring friction torque can be divided into two groups: direct and indirect. The following methods belong to the first type is shown in Figure 1.



Figure 1. Setup for measurement and recording of friction torque in gear at low speeds.

- Mechanical, in which the following methods are included: 1) Direct reading, where the torque to be measured is read and recorded directly from the instrument scale without additional effort or transformation. Instruments of the pendulum type, setups for rundown of the rotor, mechanical friction balances and others belong to this group. As a rule, direct reading instruments are meant for measurement of the mean value of the constant friction torque component and contact torque static torque. 2) Pneumatic, based on the use of pneumatic systems for the creation of a force balancing the friction torque, for example, membranes and others.
- **Optical**, in which optical systems are used to increase the accuracy of reading and recording the torque.
- Vibration of Gear: Optical instruments and setups form a small group because optical systems have only the role of recording element and the friction torque is balanced by pneumatic, elastic rods and other systems³. Hence optical setups can be included in other groups. The principle scheme for the setup with an optical system for reading and recording friction torque is balanced by pneumatic, elastic rods and other systems. Let us describe this scheme is shown in Figure 1.

The gear under test is placed in the chuck 4 and set in motion by an electric motor with reduction gear. The friction torque is balanced by the spiral spring/connected at one end with the stationary part of the casing and at other with the axis, on which mirror 2 and damper (dash-pot) 3 are fixed. Friction torques in the gear move the measuring system by twisting the spring through an angle proportional to the friction torque being measured. Due to this, the ray of light falling on the mirror from the source of light 5 through the optical system 6 is deflected through the same angle and is recorded by the optical system 9 and 10 on the moving paper tape of the camera 7 in the form of a curve. A scale service the purpose of visual reading.

Nowadays the strain gauge technique finds wide application in the measurement of friction torque in gear. The basic principle of strain gauge transducers is the charge in the resistance of the metal or semiconductor under the action of an applied load. The resistance strain gauge finds very wide application because of its small size and the possibility of measuring both static and dynamic torques. Most popular are wire, foil and semiconductor resistance strain gauges.

3. Measurements of Friction Torque in Gear

The pneumatic instrument MDSL–1, Figure 2 which is the best developed among them is meant for measurement of contact torque in radial gear of an outer diameter up to 20mm. The gear under test 1 is placed in the replaceable arbor 2 in the moving cup. The cup can rotate on the gear 3 mounted on the stationary axis 4. To ensure concentricity a rocker arm 6 is placed in the inner ring of the gear with the help of conical pad 5 with replaceable weights 7 create axial loading of the gear. A hemisphere and balancing weights 8 are fixed onto the rocker arm to receive the pressure of the air jet.

The torque needed to overcome the contact torque in the gear is determined by the angle of turn of the rocker arm and is read from the graduated scale 9 on the flange of the cup. The torque necessary to turn the rocker arm is created by the jet of air impinging form the calibrated nozzle. The air pressure is regulated by a pneumatic valve and is measured using a manometer graduated in units of friction torque.

Before starting measurements the mark 1 is lined up with the indicator notch on the base of the instrument and the end face of the hemisphere 9 with the stationary indicator. During measurement the air jet pressure on the hemisphere increases until the rocker arm is displaced. By adjusting all the points simultaneously with the indicator notch the contact torque at various points in the raceway is measured for various locations of the gear⁴.

A more universal pneumatic instrument is shown in Figure 1. The instrument allows measurement of the

(10) 6 (2)(8) (1)9 (4)1 Moving cup 6 Rocker arm 7 Replaceable weight 2 Arbor 3 Contact bearing 8 Balencing weight 9 Graduated scale 4 Stationary axis 5 Conical pad 10 Stationary indicator

Figure 2. MD Sh-1 instrument.

constant and variable components of friction torque. Corrugated diaphragms connected by an air duct with a projection micro manometer have the role of the sensitive measuring element in this setup. When the inner ring of the gear under Test I is rotated with the help of an electric motor the outer ring with the arbor 2 fixed on it and connected with the diaphragm 3 turns through a certain angle proportion al to the constant component of the friction torque. The closed volume of air tract limited by the corrugated diaphragm of the manometer and measuring diaphragm 3 changes in proportion to the load applied to the diaphragm. As a result the air pressure changes, causing deflections of the diaphragm of the sensitive manometer. The mirror 5 which is fixed to the manometer turns under the action of the load in the direction of the diaphragm. The variable component is recorded as the mirror turns one way on the other from the equilibrium position, corresponding to the constant component of friction torque.

4. Apparatus for the Analysis of Friction Torque

In principle the setups for the measurement of friction torque in gear differ very little from setups measuring friction torque in rolling contact, the basic kinematic scheme for the measurement of friction torque components in precision gear is shown in Figure 3. It works as follows; when the rotor rotates in the support of instrument 2 the friction torque is disturbed. Under the action of the resistance the support 2 turns along with the rigidly connected casing 3 which are held on elastic crossed, for example, flat metal plates 4. When the suspended casing 3



Figure 3. Instrument for measuring friction torque in precision supports.

rotates with the support the load due to friction torque is transmitted through the lever 5, rigid needle 6 and semiconductor strain gauge bonded to the elastic plate 7, to the amplifying - recording apparatus⁵. The value of the constant component M_{cons} observed on the oscilloscope screen can be determined on the basis of the reading shown on the scale of the micrometer mounted in the strain gauge amplifier and recorded with the help of an automatic relay, etc. The variable component Mv can, however, be observed and recorded by an indicator in the shape of anscillogram or photograph.

The needle 6 is suspended in a magnetic field in which the lines of force between poles coincide with its longitudinal geometric axis. Due to the permanent magnet 8 the fixture of the needle does not require additional mechanical tightening of the plate with the strain gauge 7 relative to the elastic suspension and this appreciably enhances the sensitivity of the turn of suspension around its geometric axis, i.e the suspension in the static position has practically zero restoring torque. This design of suspensions works practically without friction torque for small angles (0-1°) of turn because the value of the friction of elasticity of plane elements is very small. The value of the torque Mi to be measured can be represented as a sum.

$$Mi = M_s + M_r + M_r + M_p,$$
 (1)

- Where, Ms is the friction torque of the rotating rotor in the support;
- M_r is the restoring torque of the crossed elastic elements of suspension
- M'_r is the restoring torque of the suspension caused by the change in temperature of the surrounding medium
- Mp is the restoring torque of the elastic plate with the bonded strain gauge

To change the sensitivity of turn of the suspension, the magnitude and direction of the restoring torque of the suspension can be regulated by using the regulating compression (not shown in the figure), effectively changing the deforming length of the crossed elastic elements. By regulating the tensile or compressive loads acting on the elastic element the value of M_T will be reduced to zero. Under the action of the action of the tensile P_T and compressive Pc loads on the elastic elements for small angles of turn of the suspension ($\varphi_2 = 0 - 1^{\circ}$ C) the restoring torque can be computer using the formulas.

$$M_r T = n_e P_T \phi^2 \frac{\cos i\beta}{\cos ix - \cos i\beta} (l_{el} - \eta i \tan i\beta)$$
(2)

. .

Where n is the number of crossed elastic elements

$$\alpha = \frac{\tanh - 1 \, le_1 + le_2 \cosh \chi - \eta \sinh \chi}{\eta (\cosh \chi - 1) - le_2 \sinh \chi}$$
(3)

Hence
$$\gamma = \alpha + \frac{Le}{\eta}; \eta = \sqrt{\frac{Ele}{PT,c}}$$

Ie =
$$\frac{b_e h_e^3}{12}$$
; $\chi = \frac{Le}{\eta}$; $i = \sqrt{-1}$ (5)

(4)

where $le = le_1 + le_2$ is the total length of the elastic flat element between the fixed points; Le_2 is the length of the flat elastic plate equal to the inner radius of the cylindrical suspension; E is young's modulus of elasticity of the flat plate; b_e and h_e are the thickness and width of the elastic element ($h_e > b_e$). The restoring torque is equal to zero when $le_1 = \eta I \tan i\beta$ for formula (4.44) and when $le_1 = \eta \tan \beta$ for formula (4).

A change in temperature of the surrounding medium by Δt° causes a compressive load and presses the elastic elements. Then the additional restoring torque of the suspension will be

$$M'_{r} = 1.68.10^{-1} \cdot n_{e} \cdot \phi^{2} \sqrt{\frac{E}{Pc}} b_{e}^{3} \cdot b_{e}^{5} \cdot \alpha_{1} \cdot \Delta t^{\circ}$$
(6)

Where a_t is the coefficient of linear expansion of elastic elements.

A simplified kinematic scheme of the mechanical part of the instrument for the measurement of friction torque components of precision gear with and without axial vibrations when the rotor is in the vertical, horizontal and inclined positions is shown in Figure 4.

The instrument has high sensitivity of measurement, is universal and is meant basically for the measurement of friction torque in precision gear of different sizes in air and in some special medium at high temperatures, different loads and velocity ranges of the rotor operation^{4,6}.

Let us analyze its working principle. The rotor 4 with the lower and upper test bearings is fixed to the lower 3 and upper 6 vibration elements which could be, for example, piezoelements of linear vibration. Piezo crystals of inverse effect are electrically connected to the block for excitation and control of frequencies. The lower vibration element 3 is rigidly connected to a chuck in the same axis as the axis of rotation. The upper vibration element 6 is axially connected to the chuck holder 5. The reciprocating-rotating motion of the chuck along with the lower 3



Figure 4. Instruments for measuring friction torque in rolling contact.

and upper 6 vibration elements and gear is derived from the drive 14, which through pulley 15, belt transmission 13, pulley 11, worm gears 9 and hob 10, transmits the rotation to the worm gear 2 on the shaft of which the chuck is rigidly fixed. In order to achieve accurate coaxially and normality and to eliminate radial wobbling of the axis of ration of the rotor 4 in the direction of the working plane of the chuck the upper vibration element 6 can be shifted with the help of micro screws 17 and 18 in two perpendicular planes and roughly move along the guides in the vertical plane. This makes it possible to carry out the test with rotor systems of different sizes. The lower and upper vibration elements work independently of each other and allow axial vibrations which differ in amplitude and frequency.

For the measurement of M const and M var in a special medium, for example, in a special liquid and at high temperatures a photographic hermetically sealed chamber is used in which are installed the rotor 4, vibroelement 3 and semiconductor strain gauge 7 movable in the radial direction. The temperature of the surrounding medium is controlled by a thermistor connected to a control block for temperature.

The angle of inclination of the rotor 4 with bearings to the vertical position is regulated by the turn of the whole upper part around the axis 16 along the dial. When the upper part turns, the electric drive 14 is rotated by hand. In the case instead of the pully 11a handle is installed which can be fixed with the help of a bolt 12. To change the drive ratio to the chuck there is another worm gear 9, 10 on which instead of handle 8 a follower pulley 11 is fitted.

5. Friction Torque Measuring Instrument (FTM) with Correction for Additive Error

As mentioned earlier, among the well-known methods of friction torque measurement the best one is based on the conversion of force into electrical signal with the help of strain gauges. It is necessary to reduce the considerable additive error observed during prolonged measurement using this method⁷. Correction is an effective means that we will use stepped extrapolation with capacitance memory for the purpose of correction. Recording of the shift signal can be done either in a closed or an open circuit. Let us consider first of all the case where recording of the drift amplifier with correction is carried out in a closed circuit as shown in Figure 5.

Here a_1 , K_1 and a_2 , K_2 are the drift and gain of the first and the second amplifier (a_1 takes into account the drift of the transducer IMSP also); r_c and r_o are the resistance of the switch K_2 in closed and open positions; U_{ak} is the voltage of the shift of switch. To obtain a more general analysis the input switch and transducer of the measuring device are not shon in the block diagram and only the drift a_1 due to them is considered. The switch K_2 will be in the off position for a period of time ΔT . During this time the torque being measured should be disconnected from the transducer. Having determined the equilibrium value of the voltage on the condenser:

$$U_{c} = a_{1}K_{1} + \frac{U_{d} + a_{2}K_{2}}{1 + K_{2}},$$
(7)

It finds the drift voltage that is brought over to the input of the first amplifier after shifting the switch to the off position:

$$E_1 = \frac{a_1 + U_{dk}}{K_1 (1 + K2)},$$
(8)



Figure 5. Block diagram of instrument measuring friction torque in gear with storage capacitor in closed loop circuit.

The drift voltage at the output of the first amplifier is equal to $U_{ot 1} = a_1 K 1$.

If the dynamic domain of the first amplifier allows, the residual drift of the second block - diagram will not exceed the residual drift of the first. From the point of view of registering the second block-diagram it is preferable since the storage capacitor has more charge resistance. Besides, in the case of the second block-diagram (open circuit) the danger of self – excitation is absent. Hence for further analysis of measuring devices we use the second block-diagram. The block-diagram as shown in Figure 6 designed on this basis consists of the device for rotation, strain gauge SG, signal amplifier of the strain gauge A₁, output amplifier A₂, control device AA, electromagnet EM and switch S.

When the inner ring of the gear B rotates the outer ring transmits friction torque through the beam. Deflection of the beam of the strain gauge leads to unbalance in the bridge where there are two semiconductor resistance strain gauges fixed to the beam^{8,9}. The unbalance signal is amplified by the amplifier A₁ and then by A₂. When the electromagnet responds the switch closes and the condenser C stores the voltage at the output of the amplifier A₁. This is made up of the total instrument drift of the amplifier and the gauge. The closing time of the switch S should be sufficient to stabilize the gauge after removing the torque from it. After switching on and releasing the electromagnet friction torque is again applied to the gauge. The time taken by the correction (important parameter) depends basically on transition processes of computation of friction torque to the gauge. It can be carried to a value not exceeding 0.1 sec.



Figure 6. Block diagram of measuring unit of friction torque in gears.

Let us consider the cumulative error of bearing. Let the
following be given: K _d is the gauge factor of the strain gauge
(certain data on the gauge factor of the strain gauges are
given in Table 1); K_1 is gain in amplifier 1; U_n is noise voltage
applied to the input of the first amplifier; $\Omega_{_{\rm u}}$ is the upper fre-
quency of noise; $\mathbf{i}_{_{\mathrm{in}}}$ is input current of the second amplifier.
Then from expression, taking into account the nonrandom
current component of the drift of the second amplifier and
possible coordination between the drifts of the two ampli-
fiers, we get an expression for the cumulative error of IMSP
when measuring the instantaneous value of friction torque.

$$\sigma_{a} = \left\{ \left[\left(\sqrt{D_{\epsilon n 1}} + \frac{\sqrt{D_{\epsilon n 2}}}{K_{1}} \right)^{2} + U_{n}^{2} + \frac{U_{n}^{2}}{1 + \Omega_{u} T_{c}^{*}} \right]^{1/2} + \frac{i_{in} T}{K_{1} C} \left\{ \frac{1}{K_{d}} \right.$$
(9)

Where $T_c^* = R_{olt}C$;

 $D_{en1} = D_{end} + D_{ena}$ is the dispersion of the residual drift introduced by the gauge and the first amplifier determined by equation (6.18); D_{en2} is the dispersion of the drift of the second amplifier. The increase in the value of K, and reduction in the value of T lead to a reduction in the cumulative error. The limiting minimum value is given by the expression.

$$T_{a\min} = \frac{U_n}{K_d} \tag{10}$$

If we assume that the amplifiers do not introduce noise and that all the noise is due to the strain gauge,

$$U_{\rm ns} = 4 \times 10^{-3} \sqrt{\rm R\Delta F}, \qquad (11)$$

Where U_{ns} is the noise voltage, μ V;

R is the bridge resistance of strain gauges, Ohm;

 Δ F is the equivalent band width of amplifier noise, kHz.

Then for R = 10^2 Ohm, $\Delta F = 0.1$ kHz and $K_d = 1$ W/N we get minimum measurable load $\Delta P = 1.26 \times 10^{-8}$ N, which with a lever arm of 1 = 2 cm is the limiting sensitivity of IMSP given by $\Delta M = 2.52 \times 10^{-10}$ nm. In actual practice a larger value is obtained.

Type of transducer	Dimension of plate mm	Natural resonance frequency Hz	Base of transducer mm	Supply voltage for bridge V	Sensitivity V/N
Semiconductor	$40 \times 10 \times 0.2$	120	10	4	0.5
Wire type	$32 \times 8 \times 0.08$	90	10	8	0.06
Foil type	$32 \times 8 \times 0.08$	90	10	8	0.05

Table 1. Characteristics Data

6. Experimental Study of the Dynamic Characteristics of Precision Gear and Assemblies

The continuous progress in technology poses fundamentally new demands on the working of the rolling contact gear used in flight vehicles, ships, instruments, etc.

In many units the support is provided basically by precision and rolling contact gear elements and sliding gear which have a minimum loss to friction torque and a low level of disturbance due to vibrations. One could list a whole series of devices of precision instrumentation in which friction torque causes appreciable deterioration of the dynamic characteristics. They include gyroscopic instruments, information input and output systems, computers, audio and video recording apparatus and counters. For example, in tape recorders oscillations in the speed of tape movement evaluated by the coefficient of detonation are caused by defects in the geometric sizes, accuracy of rotation and variation in friction torque in the ball gears used as rolling contact support of all the rotating components (tone channel, guides, etc.). In the gyroscopic instruments used in modern flight vehicles the gyroscope rotor should have very large kinetic moment and low level of vibration in the working condition and the gimbal suspension should have high sensitivity and rotational accuracy.

To ensure the required accuracy, reliability and longevity the support for rotation and gear of the devices should be of high quality. They are complex vibrating systems that excite vibrations in the shafts and rotors mounted on them with parameters which depend on the actual working conditions. Theoretical study of the dynamics of gear supports is often complicated by the impossibility of taking into account all the factors affecting the level of vibration that are met with in practice¹⁰. Hence experimental study of the dynamic and vibrational characteristics of gear supports under near-real working conditions is the only way to obtain accurate data on the behavior of the system.

7. Study of Friction Torque in Gear and Gear Assemblies

Friction torque in gears is a complex, physical process caused by contact deformations of rubbing elements, macro-and micro surface roughness of raceways, properties of the lubricant, resistance to the flow of lubricant or the surrounding medium, loads, the number of revolutions and the physical properties of the materials of contact elements⁸. Since it is difficult to compute friction torque it is necessary to use the results of tests showing the effect of various operational parameters on the components of friction torque. This is very important for selection of the optimum working conditions of gears and its assembly. Axial load has an appreciable effect on the friction torque components. Dependence of Friction Torque on loading will be seen from the curves Figure 7 radial load affects friction torque to a lesser degree.

The curves shown here were obtained on the basis of a large number of measurements of the constant friction torque component of a gear train of bulk class 809 loaded in the axial direction. Experiments were carried out with gear without lubrication. From an analysis of the curves it follows that for a load up to 20 N the friction torque is nearly proportional to the load

$$M_n = AQ$$
,

Where A is the coefficient of proportionality Q is the load

Up to a certain value, if the load is gradually increased friction torque increases negligibly. However, with a load around 5 N its value goes up appreciably¹¹. With an increase in load up to 20 N and more and also with any deterioration in the cleanliness of raceway surfaces the friction torque considerably increases.

8. Discussion and Conclusion

With an increase in axial loading the friction torque increases irrespective of lubrication and speeds of rotation. In the majority of cases of very nearly linear dependence of friction torque on loading is obtained. Only at high speeds does the friction torque change more intensively with the use of viscous lubricants. This is because, firstly, all manufacturing defects in gear and defects in the



Figure 7. Variation in constant component of friction torque with load and varying surface smoothness of mass-produced gear.

mounting in the assembly are more apparent at higher speeds and, secondly, with an increase in speed viscous lubricants cause a greater increase in friction torque.

9. References

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