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Metamodel for Mathematical Modelling Surfaces of Celestial Bodies on the Base of Radiolocation Data

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Abstract

The paper proposes for the mathematical modelling surfaces of celestial bodies by the given radar data use the models, produced from the geometrical metamodel G. The metamodel G consists of the corresponding to the dimensions of a space geometrical objects (points, lines, surfaces), the mathematical methods of interpolation, interlinations, interflatation, and the set of rules for producing mathematical models. Using the metamodelling approach allows us to consider from a unique point of view the different methods for the modelling surfaces of bodies and also to develop a computer tool which accelerates and simplify the modelling process, starting from the problem specification and finishing visualization and interpretation of the obtained solution. The correctness of the derived from the metamodel G the set of the models M_1, M_2 ... M_N results from the correspondence of the structure of the model objects of G to the structure of experimental data, recorded in the process of radiolocation as values of the functions in the points, and traces of the functions on the given lines and surfaces.

Widely used in cartography Digital Elevation Model (DEM), can be also produced from G by using for the description of the bodies the surfaces in the form of triangles. Using the metamodel G allows us to integrate the DEM method for the data specification with setting data in points and on lines (which are the basic objects of the metamodel G) and so apply more precise (comparatively with the classical interpolation) methods of interlination and interflatation of functions.

Another advantage of the proposed metamodelling approach is a possibility of development of complex geometrical models by composition of the basic elements of the metamodel. As an example, the paper proposes a new method for the reconstruction of the surface of a celestial body by the data, given on the system of strips – interstripation (form the inter – in between – of the strips).

Keywords: Interflatation, Interlination, Interpolation, Interstripation, Metamodel, Model of a Surface, Software Tools

1. Introduction

The most used on practice for description of the surfaces of Earth and other celestial bodies is the Digital Elevation Model (DEM)[1], sometimes referred as the Digital Terrain Model (DTM)[2]. The idea of DEM is a replacement of a surface of a celestial body by multifaceted surface, where each face is a triangle. The coordinates of the vertices of these triangles are set by the researcher in the process of radiolocation sensing. Within each of the triangle, the structure of the investigated part of the surface of the Earth or other body is considered as homogeneous. Note, that practice often needs a more accurate description of a surface within such the triangle, which size can be rather big.

In addition, there is certain inconsistency of DEM to the experimental data, derived from a radar (or sonar). Inclusion in the DEM of experimental data, having a different geometrical structure, generally speaking, is not a trivial task. For example, this concerns the use in the description of such the typical elements of the Earth's surface, as the banks of rivers or seas. The reason is that their inclusion in the description of the surface requires processing of each triangle (a face of the multifaceted surface) in the DEM separately.

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Figure 1 shows an example of mapping an ocean floor using sonar.

The lines $x = x_k \left(k = \overline{1, M_1}\right)$ and $y = y_i \left(i = \overline{1, M_2}\right)$ here are the courses of the ship with sonar, z = f(x,y) is the equation of the surface of the ocean floor that needs approximate reconstruction. From this example follows the natural use of geometrical lines, as carriers of the experimental sonar data, and, actually, the neediness of inclusion of the line model object in the metamodel *G*.

Another important example is the mapping a celestial body surface using a radar (see Figure 2). In this case, the radiolocation data are given on the system of strips, located on the surface of such the body. That is why the development of the method for reconstruction of a body surface accordingly to the data, specified on the system of strips (the interstripation) is a challenging task.

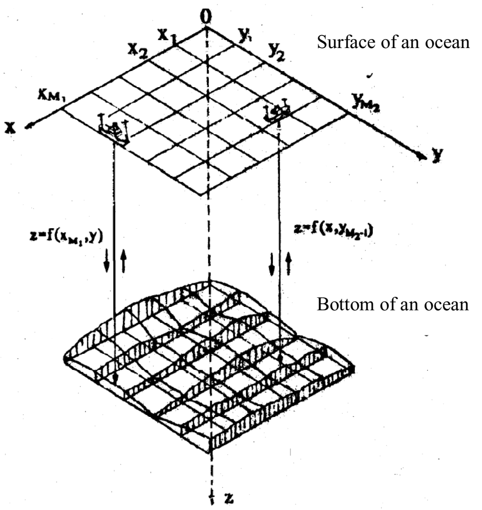

Figure 1. Mapping the ocean floor using sonar data.

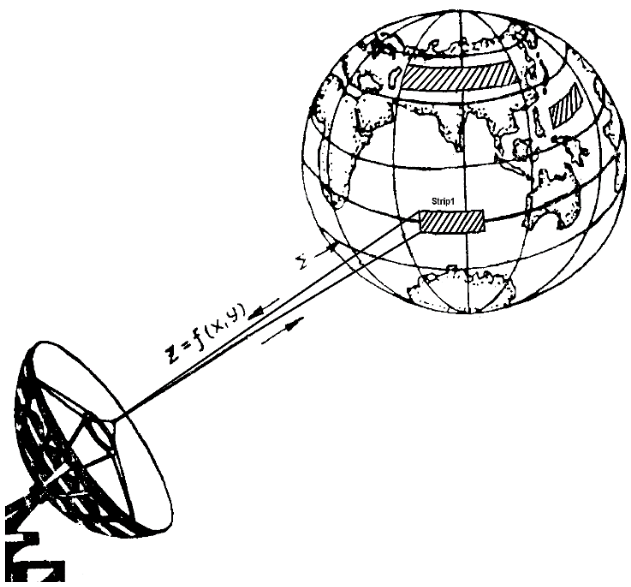

Figure 2. Mapping the surface of a celestial body using radar data.

In general, these strips may overlap, be located at different angles, as also be received at different moments of time. For example, when "Magellan" spacecraft mapped Venus[3], its radar scanned the same part of the surface at different turns with a time interval of several tens of months; during this time Venus was heavily destroyed by an earthquake, or better say, "venusquake". Thus, the state of a specific part of the surface can have a large deviation from the state, which was recorded on the same strip during the movement of the spacecraft via one of the previous turns.

Given examples shows that use of the metamodel *G*, we discuss in details in the next sections of the paper, is entirely natural for specification and processing radiolocation data. This is primarily due to the structure of the experimental data used in computer cartography. Consequently, the problem of generalization of approaches to modelling surfaces of bodies in the framework of the metamodel *G*, that includes DTM as a special case, is relevant.It gives the opportunity for more accurate description of surfaces of planets and other celestial bodies, both through the use of geometrical objects, which more closely match the available experimental data, and the application of modern mathematical methods for approximation of functions of many variables - interlination and interflatation of functions[4].

2. Formulation of the Problem

The paper proposes for processing radiolocation and sonar data use the mathematical models of the surface, derived from the geometrical metamodel *G*. Metamodel *G* consists of the set of the model objects {*P, L, S*}, where *P* is the point, *L* is the line, *S* is the surface; the mathematical methods $\{O_p, O_l, O_f\}$ applicable to these model objects, where O_p is the operator of interpolation, O_l - the operator of interlination, O_f - the operator of interflatation, and the rules for generating models M_1, M_2 ... M_N from *G*.

The feature of the proposed metamodelling approach is defining the metamodel *G* at two levels - formal (mathematical) and visual (geometrical), i.e. as a set of graphical objects, which are used for manipulation and visualization of the metamodel elements. These levels of the metamodel definition are given to the user through GUI of a corresponding software tool[5]. For example, changing a location of a visual object on a screen will change its radius-vector, defined at a formal level.

Note, that use for the modelling of such the basic geometrical objects like point, line and surface, has been the subject of multiple discussions in the scientific literature, starting from the work "Discourse on Method" of René Descartes, first published in 1637[6]. The different levels of the definition of these objects, i.e. the geometrical and the analytical, exactly the basis of the method of Descartes' coordinates. On the base of the René Descartes approach, Isaac Newton and Gottfried Leibniz later created the differential and integral calculus.

Let's note here works of Ukrainian academician Vladimir Rvachev(see e.g.[7]), which also are based on the Descartes' approach. In his works, V. Rvachov expanded the method of Descartes with functions of many variables, properties of Boolean functions, and k-ary logic. It gave the possibility to develop the common method for building equations of shapes with complex boundaries.

In our case we combine the theories of mathematical modelling with metamodelling approach of computer science, used for software systems development.

In[8] we propose the metamodelling architecture for modelling domains having different mathematical structure. It allow us to develop metamodel for modelling multi-dimensional domains[9]. Another application is development and application of metamodels as integrated logical and algebraic systems[10].

In this paper we expand the metamodelling approach of software engineering and show, that it can be used not only for generating program code, but also for development of solutions of complex mathematical problems. Here, we consider metamodel as a model of mathematical models. The approach allows us to find a general and more effective solutions by definition of a new level of abstraction lying behind existing model abstractions. Such the metamodel produces a set of domain models (in the context of this paper, the models of surfaces of bodies).

New models are developed by putting constraints on the geometrical structure of the metamodel elements (for example, from the line, we produce the line segment, from the surface - the triangle, the strip, etc.) and by their composition, i.e. by development of more complex models from the basic elements of the metamodel.

We write the relationship between metamodel and derived models as $G \Rightarrow M$, where the models M is the set of $\{\{P_1, P_2 ... P_A\}, \{L_1, L_2,...L_B\}, \{S_1, S_2,...S_C\}\}$, where

$\{P_1, P_2...P_A\}$–the set of points,
$\{L_1, L_2,...L_B\}$–the set of lines,
$\{S_1, S_2,...S_C\}$–the set of surfaces,
A + B + C = N–the total number of objects.

Each object of the set $\{\{P_1, P_2...P_A\}, \{L_1, L_2,...L_B\}, \{S_1, S_2,...S_C\}\}$ is a carrier of the mathematical properties, here, the distribution of radar or sonar data. Important that these specific models use the operators $\{O_p, O_l, O_f\}$, which are the significant part of the metamodel G.

3. Materials and Methods

The problems of building maps of surfaces of celestial bodies on the base of the given radar data is relevant both from a practical and a theoretical point of view.

In the proposed metamodel G, the information about the surface Σ, the map of which we want to create, is specified by the values of the unknown function of two $z = f(x, y)$ or three $\Phi(x, y, z) = 0$ variables in the specified system of points $\{P_1, P_2...P_A\}$; its traces on the specified system of lines $\{L_1, L_2,...L_B\}$; and its traces on some planes or surfaces $\{S_1, S_2,...S_C\}$ (generally speaking, curvilinear).

For the more precise recovery of the function, we need also take into account an existing additional information about studied surface (the class of smoothness of the surface, its closeness to the known surfaces - planes, spheres, cylinders, etc.). Use of the metamodel allows us to build the computer tools that automate and simplify application of many mathematical methods (for example, automatically switch from the Cartesian to the spherical or the cylindrical system of coordinates). The surface can also be nothing else than a digital snapshot of a body. In this case, the computer tool allows us to consider the surface, as a function of two variables.

3.1 Characteristic of the Metamodel Operators

Operators of the metamodel G are the formulas of spline-interpolation, spline-interlination and spline-interflatation[4]. Interlination (interflatation) of functions of many variables we call the reconstruction (possibly, approximate) of this function by using their tracks and traces of their derivatives up to the given order on the system of lines (or surfaces, respectively). Interlination and interflatation of here functions is a natural generalization of interpolation, which is reconstruction of (possibly, approximate) the function by using its values and its derivatives up to some order on the specified system of points.

Interflatation of the function $f(x_1,...,x_n)$ of n variables using the function traces (or traces of its derivatives up to the given order $\le N$) on M surfaces of the dimension m is a reconstruction (possibly approximate) of the function f at the arbitrary points of the domain.

If $m = 0$ then this approximation is a generally known *interpolation* of a function by its values in M points (for $n \ge 1$).

If $m = 1$ (for $n \ge 2$), then this approach is called an interlination (sometimes, *blending function interpolation*) on M lines.

Here are the key statements about restoration of functions of many variables using operators of interlination and of interflatation of functions. Interflatation of functions can be used:

- In the methods for solving Linear or Nonlinear Integro-Differential Equations (LIDE or NIDE);
- For solutions of boundary value problems for differential equations with partial derivatives, which bring a boundary problem for the areas of complex form to the systems of ordinary LIDR or NIDR;
- In the theory of approximation of functions;
- In the digital processing multidimensional signals;
- In computer tomography;
- At describing the surfaces of automobiles, ships, aircrafts, space bodies, etc.;
- In cartography.

Let's give existing and introduce some new definitions using common mathematical symbols.

Let's $n, M \in \mathrm{N}, m, N \in \overset{0}{\mathrm{N}}-$ are the given numbers, $\prod_k, k=\overline{1,M}$ are the given m – directional surfaces in R^n ($0 \le m < n$). We will assume for convenience that the *point* is also the surface of the dimension $m=0$, and the *line* is the surface of the dimension $m=1$. In addition, we assume the given functions $\varphi_{k,p}(x)$, $k=\overline{1,M}$, $p=\overline{0,N}$, which are the traces of operators $L_{k,p}$ $f(x)$ of the function $f(x)$ (generally speaking, unknown), i.e. $\varphi_{k,p}(x)|_{\Pi_k} = L_{k,p} f(x)|_{\Pi_k}, k=\overline{1,M}, p=\overline{0,N}$. Operators $L_{k,p}$ $f(x)$ may be partial derivatives or the normal derivatives $L_{k,p} f(x)|_{\Pi_k} = \partial^p f(x)/\partial \nu_k^p|_{\Pi_k}, p=\overline{0,N}$ for the case of $m = n-1$ etc. (ν_k is the normal vector to $\prod_k$).

Definition 1. Operators $O\left(\left\{\varphi_{k,p}\right\},x\right) := O\left(\left\{L_{k,p}\right\},\{\Pi_k\},\left\{\varphi_{k,p}\right\},x\right)$ we call operators of interflatation if $L_{l,q}O(\{\varphi_{k,p}\}, x)|_{\Pi_l} = \varphi_{l,q}(x)|_{\Pi_l}, l=\overline{1,M}, q=\overline{0,N}$.

If $m = 0$, then $\prod_k \in R^n$ are the points in R^n traces can be values of the function $f(x)$ and its partial derivatives at these points. Then $O(\{\varphi_{k,p}\}, x)$ are operators of interpolation in M points. If $m = 1$, $n \ge 2$, then $\prod_k$ are the lines in R^n and operators $O(\{\varphi_{k,p}\}, x)$ are the operators of interlination on these lines.

Definition 2. Let $x = (x_1,...,x_n)$

$$O(\{\varphi_{k,p}\},x) = \sum_{l=1}^{M}\sum_{q=0}^{N} \gamma_{l,q}(\{\varphi_{k,p}\},x) h_{l,q}(x),$$

Where $h_{l,q}(x) = h_{l,q}(\{L_{k,p}\}, \{\prod_k\}, \{x)$- are some system of auxiliary functions that do not depend on the approximating function $f(x)$ and $\Upsilon_{l,q}(\{\varphi_{k,p}\}, x) = \Upsilon_{l,q}(\{L_{k,p}, \{\prod_k\}, \{\varphi_{k,p}\}, x)$ are the linear operators from the functions $\varphi_{k,p}$, $k=\overline{1,M}, p=\overline{0,N}$. Then we will call operators $O(\{\varphi_{k,p}\}, x)$

Table 1. The formal definition of the metamodel G

The concept of the metamodel	The type of information that allows to specify the concept of the metamodel	Approximating method of the metamodel
Point	The value of the function $f(x_1, \ldots, x_n)$ and its derivatives (up to the given order) at the specified points	*Interpolation* of functions of one or more variables $n(n\ge1)$
Line	Traces of the function $f(x_1, \ldots, x_n)$ and its derivatives (up to the given order) on the specified lines	*Interlination* of functions of two or more variables $n(n\ge2)$
Surface	Traces of the function $f(x_1, \ldots, x_n)$ and its derivatives (up to the given order) on the specified surfaces of dimensionality $m(0\le m\le n-1)$	*Interflatation* of functions of three or more variables $n(n\ge3)$

the linear operators of interflatation (interpolation, interlination). Otherwise, these operators we will call a non-linear operators of interflatation (interpolation, interlination).

Definition 3. Let the auxiliary functions $h_{l,q}(x) = h_{l,q}(\{L_{k,p}\}, \{\Pi_k\}, x)$ are rational, polynomial, trigonometric functions or a spline-functions, or functions that are built using R-functions[7], etc. Then we will call operators $O(\{\varphi_{k,p}\}, x)$ the operators of rational, polynomial, trigonometric, spline-interflatation (interpolation, interlination).

Definition 4. If $f(x) \in C^r(R^n)$, $r \geq N \geq 1$ and $O(\{\varphi_{k,p}\}, x) \in C^r(R^n)$, then operators $O(\{\varphi_{k,p}\}, x)$ are called operators, which preserve the class of differentiation $C^r(R^n)$, to which the approximation function $f(x)$ belongs. Else operators $O(\{\varphi_{k,p}\}, x)$ are called operators, that do not preserve the class of differentiation $C^r(R^n)$, to which the approximation function $f(x)$ belongs.

Definition 5. If $\exists l,q : L_{l,q}\ O(\{\varphi_{k,q}\},x)|_{\Pi_l} \neq \varphi_{l,q}(x)|_{\Pi_l}$, then the operators $O(\{\varphi_{k,q}\}, x)$ are the operators of rational, polynomial, trigonometric, spline-approximation.

Here are some formulas that used to build the operators of the metamodel (interlination and of interflatation of functions).

3.2 Operators of Interlination Without Saving Class of Differentiation $C^r(R^2)$, $r \geq 1$

3.2.1 Operators of Rational Interlination on M Lines

Let $n = 2$ and $\Pi_k : \omega_k(x) := a_k x_1 + b_k x_2 - \gamma_k = 0, k = \overline{1,M}$, $a_k^a + b_k^2 = 1$,

$\varphi_{k,s}(x) = \partial^s f / \partial v_k^s(x)|_{\Pi_k} = \partial^s f / \partial v_k^s(x_1(\gamma_k - a_k x_1)/b_k)$ if $b_k \neq 0; v_k = \nabla\omega_k(x) = (a_k, b_k)$

or $\varphi_{k,s}(x) = \partial^s f / \partial v_k^s(x)|_{\Pi_k} = \partial^s f / \partial v_k^s((\gamma_k - b_k x_2)/a_k, x_2)$ if $a_k \neq 0$,

$$O_{k,N} f(x) = \sum_{s=0}^{N} \varphi_{k,s}(x - \omega_k(x)\nabla\omega_k(x))\frac{\omega_k^s(x)}{s!},$$

$$H_k(x) = \prod_{\substack{i=1\\ i\neq k}}^{M} \omega_i^{N^*}(x) / \sum_{l=1}^{M}\prod_{\substack{i=1\\ i\neq l}}^{M} \omega_i^{N^*}(x)\ N^* = N+1, if\ N = 2q+1,$$

$$q \in \overset{0}{\mathrm{N}}; N^* = N+2,\ if\ N = 2q, q \in \overset{0}{\mathrm{N}};$$

Theorem 1. If at any point intersect no more than two lines, Π_k and Π_1 then the operator

$$O_{M,N}(\{\varphi_{k,s}\},\{\Pi_k\},x) = \sum_{k=1}^{M} O_{k,N} f(x) H_{k,N}(x)$$

has the property

$\partial^s O_{M,N}(\{\varphi_{k,s}\},\{\Pi_k\},x)/\partial v_k^s(x)|_{\Pi_k} = \varphi_{k,s}(x)|_{\Pi_k}, k = \overline{1,M}, s = \overline{0,N}$

Comment 1. If $\Pi_k : \omega_k(x) = 0, k = \overline{1,M}$ is an arbitrary set of lines or surfaces in the R^n, $n \geq 2$ and $\partial^p \omega_k(x)/\partial v_k^p|_{\Pi_k} = \delta_{0,p}, p = \overline{0,N}$, the assertion of the theorem 1 remains in force, in condition, that at one point intersects no more, then n lines or surfaces (with $n > 2$).

3.2.2 Polynomial, Trigonometric and Spline-interlination on a Set of Mutually Perpendicular Straight Lines

Let $G = I^2, I = [0,1], 0 = x_{k,0} < \ldots < x_{k,M_k} = 1, k = 1,2;$

$\partial^{sk} f(x)/\partial x_k^{sk}|_{x_k = x_{k,i_k}} = \varphi_{k,i_k,s_k}(x_{3-k}),$

$$B_k f(x) = \sum_{i_k=0}^{M_k}\sum_{s_k=0}^{N} \varphi_{i_k,s_k}(x_{3-k}) h_{M_k,N,s_k}(x_k), h^{(q)}_{M_k,i_k,s_k}(x_{k,j}) =$$

$\delta_{q,i_k}\delta_{i_k,j}, q, s_k = \overline{0,N}; i_k, j = \overline{1,M_k}$ $h_{M_k,}N, s_k(x_k)$ are the basis system of functions of one variable for polynomial, trigonometric or spline interpolation.

Theorem 2. Operators $Of(x)–(B_1 + B_2 – B_1B_2)f(x)$ have the following properties:

$$\partial^p Of(x)/\partial x_k^p = \partial^p Of(x)/\partial x_k^p, x_k = x_{k,i_k}, p = \overline{0,N},$$
$$i_k = \overline{0,M_k}, k = 1,2,$$

In addition, if $R_{12}f(x) := (I – O)f(x)$ — the final member of the approximation function $f(x)$ operators $Of(x)$, then

$$R_{12}f(x) := (I – O)f(x) = (I – B_1)(I – B_2)f(x),$$

It follows that the $R_{12}f(x) = O(\varepsilon)^2$, if $(I – B_k)f(x) = O(\varepsilon)$, $\varepsilon \to 0$, $k = 1, 2$.

3.2.3 The Cost Saving Schemes for Calculations of Operators of Polynomial, Trigonometric and Spline Interpolation, Obtained by using the Appropriate Operators of Interlination

In general, these operators have the form $\bar{O}f(x) = (\bar{B}_1 + \bar{B}_2 - B_1B_2)$. Operators $\bar{B}_k f(x)$ are obtained from operators $B_k f(x)$ by using the following replacement $\varphi_{i_k,s_k}(x_{3-k}) \approx \Phi_{i_k,s_k}(x_{3-k})$ in $B_k f(x)$, where $\Phi_{i_k,s_k}(x_{3-k})$ are polynomial, trigonometric, or spline-interpoliants having the properties $\left\|\varphi_{i_k,s_k}(x_{3-k}) - \Phi_{i_k,s_k}(x_{3-k})\right\| = O(\varepsilon^2), \|.\| = \|.\|_C$

Theorem 3. Operators of interpolation $\bar{O}f(x)$ use fewer amount of the values of function $f(x)$, than the classical operators $B_1B_2 f(x)$ (at condition that they approximate $f(x)$ with an error $O(\varepsilon^2)$).

3.3 Operators of Interlination and Interflatation of Functions with Saving Class $C^r(R^n)$, $r \geq 1$

If at restoring the surface of the body not only values of the approximating function are used, but also values of its derivatives up to the order $N \geq 1$, then it is recommended to use the interlination operators for functions, preserving the class of differentiation $C^r(R^n)$, $r \geq 1$, to which the approximating function belongs. The theory of constructing such the operators can be found in[4].

Note, that operators for spline interpolation of functions of three variables, built using operators of spline interflatation of function on the system of mutually perpendicular planes, have very high accuracy, in comparison with the classical spline interpolation operators. Therefore, we will focus more on the formulas for their construction.

4. Operators for 3D Interpolation, are Built with the Operators of 3D Interflatation

Let

$$f(x) \in C^{r,r,r}(I^3), r = 1,2, u_{k,i_k}(x) = f(x)|_{x_k = i_k/M},$$
$$0 \leq i_k \leq M, k = \overline{1,3}$$

$$L_{k,M} f(x) = \sum_{i_{k=0}}^{M} u_{k,i_k}(x) h(Mx_k - i_k), h(t) = (|t-1| - 2|t| + |t+1|)/2$$

Theorem 4. Operators $Of(x) = (L_{1,M} + L_{2,M} + L_{3,M} - L_{1,M}L_{2,M} - L_{1,M}L_{3,M} - L_{2,M}L_{3,M} + L_{1,M}L_{2,M}L_{3,M})f(x)$ have the properties $Of(x)|_{x_k = j_k/M} = f(x)|_{x_k=j_k/M}, j_k = \overline{0,M}, k = \overline{1,3}$,

$$\|f - Of\| = O(M^{-3r}) \forall u \in C^{r,r,r}(I^3), r = 1,2$$

Theorem 5. We make replacement u_1, $i_1(x) = f(i_1/M, x_2, x_3) \approx$

$$u_{1,i_1}(x) = \sum_{j_2=0}^{M^{3/2}} \sum_{j_3=0}^{M^3} f(i_1/M, j_2/M^{3/2}, j_3/M^3) h(M^{3/2}x_2 - j_2) h(M^3 x_3 - j_3)$$
$$+ \sum_{j_2=0}^{M^3} \sum_{j_3=0}^{M^{3/2}} f(i_1/M, j_2/M^3, j_3/M^{3/2}) h(M^3 x_2 - j_2) h(M^{3/2} x_3 - j_3)$$
$$- \sum_{j_2=0}^{M^{3/2}} \sum_{j_3=0}^{M^{3/2}} f(i_1/M, j_2/M^{3/2}, j_3/M^{3/2}) h(M^{3/2}x_2 - j_2) h(M^{3/2} x_3 - j_3)$$

Similar replacements will do to for other functions of one and two variables in Of. We then get operator $\bar{O}f(x)$, which has the properties:

1) $\|f - \bar{O}f\| = 0(M^{-3r})$;
2) $\bar{O}f(x)$ Uses $Q = 6(M+1)(M^{3/2}+1)(M^3+1) = O(M^{5.5})$ values of the function f. Note that the classic 3D spline interpolation operators $L_{1,M^3}L_{2,M^3}L_{3,M^3}f(x)$ piecewise-linear on each of the three variables have the same error and use $Q_{classic} = (n^3+1)^3 = O(n^9)$ values of the function f.

Similar assertions also hold for the approximation operators of blended approximation using experimental data on the system of mutually perpendicular lines.

5. Results and Discussion

Let's consider the mathematical methods of recovery of the body surface based on radar data given on systems of crossed strips. The proposed method significantly uses operators of spline-interlination and spline-interflatation of functions of two variables.

Suppose, we have a system of strips $S_i : \alpha_i \leq \omega_i(x,y) \leq \beta_i, i = \overline{1,n}, \quad \omega_i := a_i x + b_i y - c_i, \quad a_i^2 + b_i^2 = 1$. Consider also known the surface reliefs $S : z = f(x, y) \in C(R^2)$ above each strip: $f_i(x,y) = \begin{cases} f|_{S_i}, & \text{if } (x,y) \in S_i \\ 0, & \text{if } (x,y) \notin S_i \end{cases}, \ i = \overline{1,n}.$

By this information we need restore (possibly, approximately) the function $f(x, y)$. This problem occurs, in particular, in mapping the surface according to the radar data, received from the satellite, that moves over the territory S by fixed paths (obviously, that radiolocation data covers the strips along the path). Below is one of the possible approaches to the solution of the problem.

First of all, when all the strips are parallel and have only shared borders (i.e. there is no overlay)

$$S_i = \alpha_i \le \omega_i(x,y) \le \alpha_{i+1}, \quad i=\overline{1,n}, \quad -\infty < \alpha_1 \ldots, \alpha_{n+1} < \infty$$

this problem has a trivial solution, its operator is developed as

$$O(\{f_i\}: x,y) = f_k(x,y), \quad (x,y)\in S_k, \quad i=\overline{1,n}.$$

So let's consider the general case. We introduce the following notation:

$$S_{k,p} = S_k \cap S_p,$$

$$f_{k,p}(x,y) = f(x,y)\big|_{S_{k,p}} = f_k(x,y)\big|_{S_p} = f_p(x,y)\big|_{S_k},$$

$$\Omega_i(x,y) = \begin{cases} \omega_i(x,y) - \alpha_i, & \omega_i(x,y) < \alpha_i \\ 0, & \alpha_i \le \omega_i(x,y) \le \beta_i, \\ \omega_i(x,y) - \beta_i & \omega_i(x,y) > \beta_i \end{cases}$$

$$G_i(x,y) = \prod_{j=1, j\ne i}^{n} \Omega_j^2(x,y) \Big/ \sum_{k=1}^{n} \prod_{j=1, j\ne k}^{n} \Omega_j^2(x,y), \quad i=\overline{1,n}.$$

Obviously,

$$G_i(x,y)\big|_{S_p} = \begin{cases} 1, & p=i, \\ 0, & p\ne i, \end{cases} \qquad \sum_{i=1}^{M} G_i(x,y) = 1.$$

These properties of the functions $G_i(x, y)$ provide the ability to prove the following theorem.

Theorem 6. Operator $O(\{f_i\}; x, y) =$

$$= \sum_{i=1}^{n} G_i(x,y) f_i(x,y) - \sum_{S_{k,p}\ne 0} G_k(x,y) G_p(x,y) f_{k,p}(x,y)$$

has the properties:

$$f_i(x,y) \in C(R^2), \ i=\overline{1,n} \ \Rightarrow 0(\{f_i\}; x,y \in C(R^2));$$

$$O(\{f_i\}; x,y)\big|_{S_q} = f_q(x,y)\big|_{S_q}, \quad q=\overline{1,n}.$$

Proof. We write for $q=\overline{1,n}$

$$\begin{aligned} O(\{f_i\}; x,y)\big|_{S_q} &= \sum_{i=1}^{n} G_i(x,y) f_i(x,y)\Big|_{S_q} \\ &\quad - \sum_{S_{k,p}\ne 0} G_k(x,y) G_p(x,y) f_{k,p}(x,y)\Big|_{S_q} \\ &= f_q(x,y)\big|_{S_q} + \sum_{i=1, i\ne q}^{n} f_{i,q}(x,y)\Big|_{S_q} \\ &\quad - \sum_{k=1, k\ne q}^{n} f_{k,q}(x,y)\Big|_{S_q} = f_q(x,y)\big|_{S_q}. \end{aligned}$$

Theorem is proven.

So, the operators $O(\{f_i\}; x, y)$ gives the possibility to recover an unknown surface in the points between the strips by the information, given on these strips.

For the better approximation, we should take in mind that the functions $f_i(x,y)$, $i=\overline{1,n}$ can be defined as a set of photos along the strip, and these pictures may overlap, i.e. have the common subdomains, not only common borders.

This means that to build $f_i(x,y)$, $i=\overline{1,n}$, in the points R^2, we might want to use anti-aliasing algorithms, not only the algorithms that exactly restore a surface of the specified subdomain on the strip S_i, $i=\overline{1,n}$. In addition, it is necessary to be able to continue the function $f_i(x,y)$, $i=\overline{1,n}$ outside the border of strips.

Here is one possible algorithm for such the continuation. Let the strip S_i has the local coordinate system $\omega_i := a_i x + b_i y - c_i$, $\tau_i := -b_i x + a_i y$. Then the function, here $\omega_i = \omega_i(x, y)$

$$\tilde{f}_i(x,y) = \begin{cases} f_i(x,y), & (x,y)\in S_i, \\ f_i\big(x-(\omega_i-\alpha_i)a_i, y-(\omega_i-\alpha_i)b_i\big), & \omega_i < \alpha_i, \\ f_i\big(x-(\omega_i-\beta_i)a_i, y-(\omega_i-\beta_i)b_i\big), & \omega_i > \beta_i \end{cases}$$

is continued on R^2 and $\tilde{f}_i(x,y) = f_i(x,y), (x,y)\in S_i$.

6. Conclusion

This paper discussed the metamodel G, which allows us to consider from a unique point of view the different approaches to development models of a surface of a body. Producing such the models from the metamodel *G* is appropriate due to the correspondence of the structure of the model objects of *G* to the structure of experimental data, obtained from a radar or a sonar.

The proposed metamodel *G* uses for the mathematical modelling the operators that restore the function $f(x, y)$ by given traces of the function and their derivatives on a system of points, lines and surfaces. Using operators of interlination, interflatation and blended approximation leads to more accurate results than gave us the classical operators of polynomial, trigonometric, spline interpolation and approximation. Thus, to achieve the same accuracy of approximation we can use less, than in classical methods, number of experimental data.

The new mathematical method for restoring the surface of the body is proposed, which is based on radar data, given on a system of intersecting strips. This method significantly uses operators of spline-interlination and spline-interflatation of the functions of two variables.

7. References

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