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# Comparison of Fuzzy AHP and Fuzzy TOPSIS Methods for Math Teachers Selection

**M. Moayeri[1], A. Shahvarani[1*], M. H. Behzadi[2] and F. Hosseinzadeh-Lotfi[1]**

[1]Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran-1477893855, Iran; shahvarani.1393@gmail.com
[2]Department of statistics, Science and Research Branch, Islamic Azad University, Tehran, Iran

## Abstract

Math teachers' selection is a multi-criteria evaluation decision and has a strategic importance for many institutions. The conventional methods for Math teachers' selection are inadequate for dealing with the imprecise or vague nature of linguistic assessment. To overcome this difficulty, fuzzy multi criteria decision-making methods are proposed. The aim of this study is to use Fuzzy Analytic Hierarchy Process (F.AHP) and the Fuzzy Technique for Order Preference by Similarity to Ideal Solution (F.TOPSIS) methods for the selection of Math teachers' in education and institutions. The proposed methods have been applied to a Math teachers' selection problem of education in Iran. After determining the criteria that affect the Math teachers' decisions, fuzzy AHP and fuzzy TOPSIS methods are applied to the problem and results are presented. The similarities and differences of two methods are also discussed.

**Keywords:** Fuzzy AHP, Fuzzy TOPSIS, Multi-criteria Evaluation Decision, Teachers' Selection

## 1. Introduction

In real life, the evaluation data of Math teachers' selection suitability for various subjective criteria and the weights of the criteria are usually expressed in linguistic terms. Also, to efficiently resolve the ambiguity frequently arising in available information and do more justice to the essential fuzziness in human judgment and preference, the fuzzy set theory has been used to establish an ill-defined multiple criteria decision-making problems1. Thus in this paper, fuzzy AHP and fuzzy TOPSIS methods are proposed for Math teachers selection, where the ratings of various alternative under various subjective criteria and the weights of all criteria are represented by fuzzy numbers. Education system is the most important office in every country and future of every country depends on this institute. In academic institutions, teachers and students are two main pillars and without these it cannot be developed education system. Actually choose and distribution teachers is so important. Because the most important part in education is teacher, paying attention to employment status of teachers is so important. The process of evaluating and selecting eminency math teachers, although common practice in academic institution but the complexity of its own. Iranian secondary education is facing problem due to lack of quality math teachers, also for giving financial and other benefits, we need to identify good math teacher from a group of teachers. This is possible either by human expert or by a Decision Support System (DSS) developed with the help of some suitable techniques. Secondary education improves quality of life of individual. It provides knowledge to improve the quality and performance of students as well as teachers; it is very essential to evaluate math teachers. Math teachers are having many conflicting criteria among them and hence every difficult to decide their ranking, this lead to Multi Criteria Decision Making (MCDM). In this research, according to the fuzzy AHP, the best alternative is teacher that the same as fuzzy TOPSIS.

## 2. Fuzzy Sets

In order to deal with vagueness of human thought, Zadeh first introduced the fuzzy set theory[2]. A fuzzy set is a class

**Author for correspondence*

of objects with a continuum of grades of membership. Such a set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one[2]. A fuzzy set is an extension of a crisp set. Crisp sets only allow full membership or non membership at all, whereas fuzzy sets allow partial membership. In other words, an element may partially belong to a fuzzy set[3]. Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling: uncertain systems in industry, nature and humanity; and facilitators for commonsense reasoning in decision-making in the absence of complete and precise information. Their role is significant when applied to complex phenomena not easily described by traditional mathematical methods, especially when the goal is to find a good approximate solution[4]. Fuzzy sets theory providing a more widely frame than classic sets theory, has been contributing to capability of reflecting real world[5]. Modeling using fuzzy sets has proven to be an effective way for formulating decision problems where the information available is subjective and imprecise[6].

## 2.1 Linguistic Variable

A linguistic variable is a variable whose values are words or sentences in a natural or artificial language[7]. As an illustration, age is a linguistic variable if its values are assumed to be the fuzzy variables labeled young, not young, very young, not very young, etc. rather than the numbers zero, one, two, three[8]. The concept of a linguistic variable provides a means of approximate characterization of phenomena which are too complex or too ill-defined to be amenable to description in conventional quantitative terms. The main applications of the linguistic approach lie in the realm of humanistic systems-especially in the fields of artificial intelligence, linguistics, human decision processes, pattern recognition, psychology, law, medical diagnosis, information retrieval, economics and related areas[7].

## 2.2 Fuzzy Numbers

A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set $\tilde{A}$ of the real line R such that[6]:

- It exists such that one $x_0 \in R$ with $\mu_{\tilde{A}}(x_0) = 1$ ($x_0$ is called mean value of Ã)

- $\mu_{\tilde{A}}(x)$ is piecewise continuous.

It is possible to use different fuzzy numbers according to the situation. In applications it is often convenient to work with Triangular Fuzzy Numbers (TFNs) because of their computational simplicity, and they are useful in promoting representation and information processing in a fuzzy environment. In this study TFNs are adopted in the fuzzy AHP and fuzzy TOPSIS methods. Triangular fuzzy numbers can be defined as a triplet (l, m, u). The parameters l, m, and u. respectively, indicate the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event. A triangular fuzzy number Ã is shown in Figure 1[9]. There are various operations on triangular fuzzy numbers.

Definition 1.A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X a real number in the interval [0, 1]. The function value $\mu_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$ .

Definition 2. A triangular fuzzy number ã can be defined by a triplet (a1, a2, a3) shown in Figure 1. The membership function $\mu_{\tilde{a}}(x)$ is defined.

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ \dfrac{x - a_3}{a_2 - a_3} & a_2 \le x \le a_3 \\ 0 & x > a_3 \end{cases} \quad (1)$$

Let ã and $\tilde{b}$ be two triangular fuzzy numbers parameterized by the triplet (a1, a2, a3) and (b1, b2, b3), respectively, then the operational laws of these two triangular fuzzy numbers are as follows:

$$\tilde{a}(+)\tilde{b} = (a_1, a_2, a_3)(+)(b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \quad (2)$$

$$\tilde{a}(-)\tilde{b} = (a_1, a_2, a_3)(-)(b_1, b_2, b_3) = (a_1 - b_1, a_2 - b_2, a_3 - b_3) \quad (3)$$

$$\tilde{a}(\times)\tilde{b} = (a_1, a_2, a_3)(\times)(b_1, b_2, b_3) = (a_1.b_2, a_2.b_2, a_3.b_3) \quad (4)$$

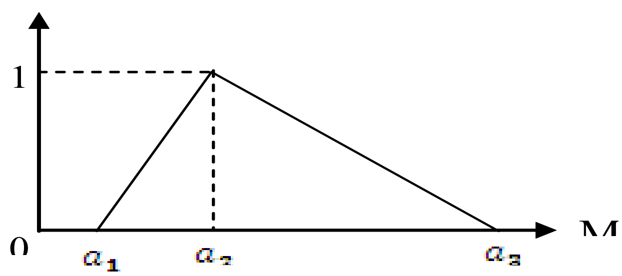


**Figure 1.** Triangular fuzzy number *a*

$$\tilde{a}\left(\begin{smallmatrix}\square\\\square\end{smallmatrix}\right)\tilde{b}=(a_1,a_2,a_3)\left(\begin{smallmatrix}\square\\\square\end{smallmatrix}\right)(b_1,b_2,b_3)=\left(\frac{a_1}{b_3},\frac{a_2}{b_2},\frac{a_3}{b_1}\right) \quad (5)$$

$$\tilde{a}=(ka_1,ka_2,ka_3) \quad (6)$$

Definition 3. A linguistic variable is a variable values of which are linguistic terms[7,10]. The concept of linguistic variable is very useful in dealing with situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions[7,10]. For example, "weight" is a linguistic variable; its values are very low, low, medium, high, very high, etc. These linguistic values can also be represented by fuzzy numbers.

Definition 4. Let $\tilde{a}_1=(a1,a2,a3)$ and $\tilde{b}_1=(b1,b2,b3)$ be two triangular fuzzy numbers, then the vertex method is defined to calculate the distance between them.

$$d(a,\tilde{b})=\sqrt{\frac{1}{3}\left[(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2\right]} \quad (7)$$

Definition 5. Considering the different importance values of each criterion, the weighted normalized fuzzy-decision matrix is constructed as

$$\tilde{V}=\left[\widetilde{v_{ij}}\right]n\times j \qquad i=1,2,\ldots n,\ j=1,2,\ldots J \quad (8)$$

Where

$$\widetilde{v_{ij}}=\widetilde{x_{ij}}\times w_i. \quad (9)$$

- A set of performance ratings of Aj (j = 1, 2,. . . , J) with respect to criteria Ci (i = 1, 2,. . . , n) called $X^{-}=\left\{(x_{\downarrow}ij)^{\sim} \quad ,i=1;2;\ldots;n;J=1;2;\ldots;J\right\}$.
- A set of importance weights of each criterion wi (i = 1, 2,. . . ,n).

## 3. Fuzzy Analytic Hierarchy Process

First proposed by Thomas L. Saaty, the Analytic Hierarchy Process (AHP) is a widely used multiple criteria decision-making tool[11]. The analytic hierarchy process, since its invention, has been a tool at the hands of decision-makers and researchers, becoming one of the most widely used multiple criteria decision making tools[12]. Although the purpose of AHP is to capture the expert's knowledge, the traditional AHP still cannot really reflect the human thinking style[13]. The traditional AHP method is problematic in that it uses an exact value to express the decision-makers opinion in a comparison of alternatives[14]. And AHP method is often criticized, due to its use of unbalanced scale of judgments and its inability to adequately handle the inherent uncertainty and imprecision in the pair-wise comparison process[9]. To overcome all these shortcomings, fuzzy analytical hierarchy process was developed for solving the hierarchical problems. Decision-makers usually find that it is more accurate to give interval judgments than fixed value judgments. This is because usually he/she is unable to make his/her preference explicitly about the fuzzy nature of the comparison process[13]. The first study of fuzzy AHP is proposed by[15], which compared fuzzy ratios described by triangular fuzzy numbers. Buckley initiated trapezoidal fuzzy numbers to express the decision-makers evaluation on alternatives with respect to each criterion[16]. Chang introduced a new approach for handling fuzzy AHP, with the use of triangular fuzzy numbers for pair-wise comparison scale of fuzzy AHP, and the use of the extent analysis method for the synthetic extent values of the pair-wise comparisons[17].

1) Each index in system should be resolved into several levels. Every index at the same level is subject to the upper index and governs the lower index. Then a hierarchical structure model about the problem can be constructed.
2) The relationship of indexes in system should be analyzed. With a rule, one index should be compared with another index at the same level about the importance to the upper index. Then a comparison matrix about the comparison process can be got.
3) The weight of every index can be got with the comparison matrix based on the rule and the consistency of comparison matrix should be tested. Then with weight of indexes, the total arrangement weight of the level to system can be got.

Based on the importance of the index to the upper level's index about a rule, we can get the weight of this index. When the weights of all indexes at the same level, the AHP is used to get the weight of index with hierarchical structure model.

(1) The comparison matrix

In order to get the importance to the upper level's index, we can compare the index i and j at the same level.

And the importance should be assigned by deciders and experts. By the method of AHP, the estimate about importance of an index should be quantified with some ratio scale. The 1-9 scale method is employed by this paper. If there are n indexes at this level, the comparison matrix is C= (Cij) and Cij is the assignment about importance of the index i to j.

(2) Weight calculation

The weight computing problem is how to get the maximized eigenvalue and eigenvector of comparison matrix. Calculating each row comparison matrix elements' product Mi.

$$\mathbf{M_i} = \prod_{j}^{n} \mathbf{a_{ij}} \qquad i = 1,2,\dots n \tag{5}$$

Calculating Nth root Wi of product Mi

$$\bar{\mathbf{W}}_\mathbf{i} = \sqrt[n]{\mathbf{M}_1} \tag{6}$$

Normalized vector $\mathbf{W_i} = \left[\bar{\mathbf{W}}_1, \bar{\mathbf{W}}_2, \dots \bar{\mathbf{W}}_\mathbf{n}\right]^\mathbf{T}$

$$\mathbf{W_i} = \frac{\bar{\mathbf{W}}_\mathbf{i}}{\sum_{j=1}^{n} \bar{\mathbf{w}}_\mathbf{j}} \tag{7}$$

W= $[W_{1,} W_2,\dots, W_n]^T$ is the eigenvector.

Calculating the maximized eigenvalue of comparison matrix.

$$\sum_{\lambda\max=i=1}^{n} \frac{(AW)_i}{nw_i} \tag{8}$$

(3) The consistency of comparison matrix

Although comparison matrix makes critical thinking mathematical, the consistency of comparison matrix and critical thinking should be tested. When the importance of indexes is estimated by experts, the result of estimate by different experts must be consistency. So when the AHP is employed, the consistency of comparison matrix should be tested to ensure the consistency of critical thinking provided by different experts. We can use consistency ratio C.R. to test the consistency of comparison matrix.

$$\mathrm{CR} = \mathrm{CI}/\ \mathrm{RI} \tag{9}$$

C.I. is consistency index and $C.I = \dfrac{\lambda\max - n}{n-1}$ (10)

R.I. is random index and the value of R.I. can be got with Table 1.

If C.R. <0.1, the comparison matrix is accepted[18].

## 4. Fuzzy TOPSIS Method

The TOPSIS is widely used for tackling ranking problems in real situations. Despite its popularity and simplicity in concept, this method is often criticized for its inability to adequately handle the inherent uncertainty and imprecision associated with the mapping of the decision-maker's perception to crisp values. In the traditional formulation of the TOPSIS, personal judgments are represented with crisp values. However, in many practical cases the human preference model is uncertain and decision-makers might be reluctant or unable to assign crisp values to the comparison judgments[19]. Having to use crisp values is one of the problematic points in the crisp evaluation process. One reason is that decision-makers usually feel more confident to give interval judgments rather than expressing their judgments in the form of single numeric values. As some criteria are difficult to measure by crisp values, they are usually neglected during the evaluation. Another reason is mathematical models that are based on crisp value. These methods cannot deal with decision-makers' ambiguities, uncertainties and vagueness which cannot be handled by crisp values. The use of fuzzy set theory[2] allows the decision-makers to incorporate unquantifiable information, incomplete information, non-obtainable information and partially ignorant facts into decision model[20]. As a result, fuzzy TOPSIS and its extensions are developed to solve ranking and justification problems[21–27].

Fuzzy TOPSIS method tries to estimate as far is a particular alternative near the ideal solution. Distance of alternatives can be in positive or negative direction. The method calculates two values: the Fuzzy Positive Ideal Solution (FPIS), which represents a project benefit and the Fuzzy Negative Ideal Solution (FNIS), a cost of project. The method selects the alternative which has the smallest distance from the positive-ideal solution and the greatest distance from the negative-ideal solution[28–31].

The mathematics concept of Fuzzy TOPSIS can be described as follows[32].

**Table 1.** Values of *R.I*

| N | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| R.I | 0 | 0 | 0.52 | 0.88 | 1.1 | 1.24 | 1.34 | 1.4 | 1.44 | 1.48 | 1.51 | 1.53 | 1.55 | 1.57 | 1.58 |

Step 1: Determination of Linguistic Terms, Membership Functions and the weighting of evaluation criteria

Determine the linguistic variables for all criteria. Each linguistic variable is assigned a set of membership functions; determine weights of evaluation criteria and the ratings of alternatives are considered as linguistic terms.

**Step 2:** Construct the fuzzy decision matrix

Decision matrix is directly associated with linguistic variables and the criteria alternatives. If assumed that the number of criteria is n and the count of projects is m, fuzzy decision matrix will be obtained with m rows and n columns as in the following matrix:

$$\begin{array}{cc} & \begin{array}{ccc} c1 & c2 \ldots & \ldots\ cn \end{array} \\ \tilde{D} = & \begin{bmatrix} \widetilde{x_{11}} & \widetilde{x_{12}} \ldots & \widetilde{x_{1n}} \\ \widetilde{x_{21}} & \widetilde{x_{22}} \ldots & \widetilde{x_{2n}} \\ \widetilde{x_{m1}} & \widetilde{x_{m2}} \ldots & \widetilde{x_{mn}} \end{bmatrix} \end{array} \tag{10}$$

$$\tilde{\mathbf{x}}_{ij} = \left(a_{ij}, b_{ij}, c_{ij}\right) \tag{11}$$

$$a_{ij} = \min\left\{a_{ij}^{k}\right\}, b_{ij} = \frac{1}{k}\sum_{k=1}^{k} b_{ij}^{k}, c_{ij} = \max\left\{c_{ij}^{k}\right\} \tag{12}$$

where A1,A2,...,Am alternatives, quality of teachers which must be ranked according to established criteria C1,C2,...,Cn, xij is the rating of alternative, Ai with respect to criterion Cj. Also, it is necessary to aggregate weighted values of criteria, their importance in the evaluation of projects.

$$\tilde{w} = \left(\tilde{w}_1, \tilde{w}_2, \ldots \tilde{w}_n\right) \tag{13}$$

where W is the weight vector with the values of criteria.

**Step 3:** Normalize the fuzzy decision matrix Normalization of fuzzy decision matrix is accomplished using linear scale transformation. The calculations are done using formulas (14),(15).

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+}\right) \text{and } c_j^+ = \max c_{ij}\left(\text{benefit}\right) \tag{14}$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{a_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{c_{ij}}\right) \text{and } a_j^- = \min a_{ij}\left(\text{cos t}\right) \tag{15}$$

If in the teacher quality assessment we use the criteria whose value indicates the benefit, we use formula (14). Otherwise, for the criteria which represent the cost in normalization of matrix formula (15) is used. In the quality evaluation the cost benefit criteria will be used. The normalized fuzzy decision matrix can be represented by Equation (16):

$$\tilde{R} = \left[\widetilde{r_{ij}}\right]_{m\times n}, i = 1, 2, \ldots m; j = 1, 2, \ldots J \tag{16}$$

where rij is the normalized value of xij=(aij, bij, cij).

**Step 4:**Calculate the weighted fuzzy decision matrix. The weighted normalized fuzzy decision matrix V is computed by multiplying the weights (wj) of evaluation criteria with the normalized value rij from fuzzy decision matrix. The weighted normalized decision matrix can be represented by Eq. (17):

$$\tilde{V} = \left[\widetilde{v_{ij}}\right]_{n\times j}, i = 1, 2, \ldots n; j = 1, 2, \ldots J \tag{17}$$

Where:

$$\tilde{v}_{ij} = \tilde{r}_{ij}(\bullet)\tilde{w}_i \tag{18}$$

**Step 5:** Determine the fuzzy positive-ideal solution (FPIS A+) and fuzzy negative-ideal solution (FNIS A-)

According to the weighted normalized fuzzy decision matrix, in this step we determine the positive and negative displacement from the ideal solution. Their ranges belong to the closed interval [0,1]. FPIS and FNIS are defined as triplet (1,1,1) or (0,0,0), otherwise the values determined by using the following formula:

$$A^{\uparrow}+ = (\left(v^{\uparrow}+\right)_{\downarrow}1, \left(v^{\uparrow}+\right)_{\downarrow}2, \ldots, \left(v^{\uparrow}+\right)_{\downarrow}n) \tag{19}$$

$$A^{\uparrow}{}_{\downarrow} = (\left(v^{\uparrow}-\right)_{\downarrow}1, \left(v^{\uparrow}-\right)_{\downarrow}2, \ldots, \left(v^{\uparrow}-\right)_{\downarrow}n) \tag{20}$$

Where $\tilde{v}_j^+ = (1,1,1)$ *and* $\tilde{v}_j^- = (0,0,0)$, ,j=1,2,…n.

**Step 6:** Calculate the distance of each alternative from FPIS and FNISThe distance ( $d_j^+$ and $d_j^-$ ) of each alternative $A^+$ from and $A^-$ can be calculated as:

$$d_i^+ = \sum_{j=1}^{n} d\left(\tilde{v}_{ij}, \widetilde{v^+}_{ij}\right) i = 1, 2, \ldots, m \tag{21}$$

$$d_i^- = \sum_{j=1}^{n} d\left(\tilde{v}_{ij}, \widetilde{v^-_{ij}}\right) \; i = 1, 2, \ldots, m \tag{22}$$

The distance between two fuzzy numbers $\tilde{\alpha} = (a_1, a_2, a_3)$ and, $\tilde{b} = (b_1, b_2, b_3)$, can be calculated as:

$$d_v(a,b) = \sqrt{\frac{1}{3}\left[(a_1+b_1)^2 + (a_2+b_2)^2 + (a_3-b_3)^2\right]} \quad (23)$$

Step 7: Calculate the closeness coefficientThe closeness coefficient CCi is defined to determine the ranking order of all alternatives. The index CCi indicates that the alternative is close to the FPIS( $d_j^+$ ) and far from the FNIS( $d_j^-$ ). The closeness coefficient of each evaluated teacher quality can be calculated as:

$$CC_i = \frac{d_i^+}{d_i^+ + d_i^-} \quad (24)$$

**Step 8:** The ranging order of all alternatives, the ranking of alternatives is carried out based on the calculated closeness coefficients. The alternative with the highest coefficient represents the best alternative.

## 5. The Evaluation Framework

### 5.1 Criteria Selection

In the first phase, criteria and sub-criteria to be used in ranking determined. One of the important steps of the proposed model is to determine all the important criteria and their relationship with the decision variables. This step is crucial because the selected criteria and sub criteria can influence the final choice. Here in this project the criteria and sub-criteria are selected based on the Existing literature & expert's opinion. These are six alternatives (T1,T2,T3) taken are eminency math teachers in Kerman. The criteria and sub-criteria selected are described in Table 2.

#### *5.1.1 Application with Fuzzy AHP Method*

In this section, fuzzy AHP method is proposed for the same problem of the math teachers' selection. Here proposed a group decision based on fuzzy AHP. Firstly decision-makers prepared questionnaires forms and then with division against other importance carry out pair-wise comparison. Decision-makers use the linguistic variables, to evaluate the ratings of alternatives with respect to each criterion and they converted into triangular fuzzy numbers. A pair wise comparison is performed by using Fuzzy linguistic terms in the scale

**Table 2.** Criteria and Sub-Criteria

| Criteria | Sub-criteria |
|---|---|
| Creativity (C1) | C11: non Cognitive, C12: cognitive, C13: motivational |
| Personality (C2) | C21:Extraversion,C22: Neuroticism, C23:Agreeableness-C24:compatibility,C25: Responsibility |
| Attitude (C3) | C31:Job dependency,C32:matching job with business, C33:design job,C34:Associates, C35:Environmental stress and complexity of work, C36: Balancing in career and family life, C37:training, evaluation and promotion,C38: physical conditions work |

of 1-9 described by the Triangular Fuzzy Numbers in the Table 3.

Weights of the criteria and sub-criteria are calculated and Further the sub-criteria overall weights are multiplied by the corresponding main criteria weighs to obtain final weight of the sub-criteria as results are described below in Tables 4.

The results of the overall sub-criteria weights indicate that the priorities are highest in Responsibility followed by personality criteria. Teachers feedbacks of three alternative math teachers are collected with respect to each of the sub-criteria using fuzzy linguistic preference scale and the corresponding weights are generated as described in Table 5.

Fuzzy Score of alternative eminence math teachers, namely teacher1, teacher2 and teacher3 of Kerman along with the final score are expressed in Table 6.

| | Teacher 1 | Teacher 2 | Teacher 3 |
|---|---|---|---|
| Sum of Weights | 0.31182 | 0.39977 | 0.30968 |

Alternative T2 which has the highest priority weight is selected as a best math teacher selection for education. The ranking order of the alternatives with fuzzy AHP method is T2 > T1 > T3.

#### *5.1.2 Application with Fuzzy TOPSIS Method*

In this section fuzzy TOPSIS method is proposed for the math teachers' selection problem of the education complex. Table 7 defines the linguistic terms and shows the membership functions of these linguistic terms.

The algorithm of this method can be described as follows:

**Table 3.** The linguistic scale and corresponding triangular fuzzy numbers15

| Linguistic Scale | Explanation | TFN | Inverse TFN |
|---|---|---|---|
| Equal Importance | Two activities contribute equally to the objective | (1,1,1) | (1,1,1) |
| Moderate Importance | Experience and judgment slightly favor one activity over another | (1,3,5) | (1/5,1/3,1) |
| Strong Importance | Experience and judgment strongly favor one activity over another | (3,5,7) | (1/7,1/5,1/3) |
| Very Strong Importance | An activity is favored very strongly over another, its dominance | (5,7,9) | (1/9,1/7,1/5) |
| Demonstrated Importance | The evidence favoring one activity over another is highest possible order of affirmation | (7,9,11) | (1/11,1/9,1/7) |

**Table 4.** Ranking of criteria and sub-criteria

| Criteria | Final weigh | Ranking | Sub criteria | Local weigh | Overall weigh | Ranking |
|---|---|---|---|---|---|---|
| Creativity (C1) | 0.373842 | 2 | C11 | 0.39587 | 0.147993 | 3 |
| | | | C12 | 0.409596 | 0.153124 | 2 |
| | | | C13 | 0.194533 | 0.072725 | 5 |
| Personality (C2) | 0.38977 | 1 | C21 | 0.095863 | 0.037364 | 11 |
| | | | C22 | 0.039888 | 0.015547 | 14 |
| | | | C23 | 0.142896 | 0.055696 | 6 |
| | | | C24 | 0.283143 | 0.11036 | 4 |
| | | | C25 | 0.43821 | 0.17080 | 1 |
| Attitude (C3) | 0.23639 | 3 | C31 | 0.186415 | 0.044066 | 9 |
| | | | C32 | 0.219809 | 0.051961 | 7 |
| | | | C33 | 0.187821 | 0.0443991 | 8 |
| | | | C34 | 0.160332 | 0.037901 | 10 |
| | | | C35 | 0.098438 | 0.023269 | 12 |
| | | | C36 | 0.074138 | 0.017525 | 13 |
| | | | C37 | 0.039484 | 0.009333 | 15 |
| | | | C38 | 0.033562 | 0.007934 | 16 |

1. Construct the fuzzy decision matrix:

Linguistic variables converted into triangular fuzzy numbers to form fuzzy decision matrix as shown in Table 8.

2. Normalize the fuzzy decision matrix

The normalized decision matrix can be calculated by applying Equation (14) as shown in Table 9.

3. Construct weighted normalized fuzzy decision matrix

**Table 5.** Weights of alternatives

| Sub criteria | Weights of the Alternatives | | |
|---|---|---|---|
| | Teacher1 | Teacher2 | Teacher3 |
| Non-cognitive | 0.237 | 0.355 | 0.408 |
| cognitive | 0.439 | 0.291 | 0.269 |
| motivational | 0.267 | 0.384 | 0.348 |
| Extraversion | 0.209 | 0.543 | 0.247 |
| Neuroticism | 0.461 | 0.346 | 0.194 |
| Agreeableness | 0.33 | 0.341 | 0.33 |
| compatibility | 0.468 | 0.379 | 0.153 |
| Responsibility | 0.277 | 0.444 | 0.277 |
| Job dependency | 0.153 | 0.552 | 0.268 |
| matching jobs and career | 0.371 | 0.391 | 0.716 |
| design job | 0.209 | 0.513 | 0.280 |
| Associates | 0.216 | 0.499 | 0.285 |
| Environmental stress and complexity of work | 0.274 | 0.396 | 0.257 |
| Balancing career and family | 0.220 | 0.477 | 0.303 |
| training, evaluation and promotion | 0.211 | 0.565 | 0.250 |
| physical conditions work | 0.266 | | 0.266 |

**Table 6.** Final weights of alternatives

| Sub criteria | Final score of the Alternatives | | |
|---|---|---|---|
| | Teacher1 | Teacher2 | Teacher3 |
| Non-cognitive | 0.03507 | 0.05254 | 0.06038 |
| cognitive | 0.06722 | 0.04456 | 0.04119 |
| motivational | 0.01941 | 0.02792 | 0.02531 |
| Extraversion | 0.00781 | 0.02029 | 0.00922 |
| Neuroticism | 0.00717 | 0.00538 | 0.00302 |
| Agreeableness | 0.01838 | 0.01899 | 0.01838 |
| compatibility | 0.05165 | 0.04183 | 0.01689 |
| Responsibility | 0.04731 | 0.07584 | 0.04731 |
| Job dependency | 0.00674 | 0.02432 | 0.01181 |
| matching jobs and career | 0.01928 | 0.02032 | 0.03720 |
| design job | 0.00928 | 0.02278 | 0.01243 |
| Associates | 0.00819 | 0.01891 | 0.01081 |
| Environmental stress and complexity of work | 0.00638 | 0.00921 | 0.00598 |
| Balancing career and family | 0.00385 | 0.00836 | 0.00531 |
| training, evaluation and promotion | 0.00197 | 0.00527 | 0.00233 |
| physical conditions work | 0.00211 | 0.00370 | 0.00211 |

**Table 7.** Linguistic terms and membership function

| Linguistic terms | Very Poor (VP) | Poor (P) | Medium (M) | Good (G) | Very Good (VG) |
|---|---|---|---|---|---|
| Membership function | (1,1,3) | (1,3,5) | (3,5,7) | (5,7,9) | (7,9,11) |

**Table 8.** Aggregation fuzzy decision matrix

| Criteria & sub Criteria | T1 | T2 | T3 |
|---|---|---|---|
| C1 | (7,9,11) | (5,7,9) | (7,9,11) |
| C2 | (5,7,9) | (5,7,9) | (7,9,11) |
| C3 | (1,3,5) | (3,5,7) | (3,5,7) |
| C11 | (3,5,7) | (3,5,7) | (1,3,5) |
| C12 | (5,7,9) | (5,7,9) | (5,7,9) |
| C13 | (1,3,5) | (5,7,9) | (1,3,5) |
| C21 | (3,5,7) | (3,5,7) | (1,3,5) |
| C22 | (1,1,3) | (1,1,3) | (1,1,3) |
| C23 | (3,5,7) | (5,7,9) | (5,7,9) |
| C24 | (5,7,9) | (5,7,9) | (7,9,11) |
| C25 | (5,7,9) | (7,9,11) | (7,9,11) |
| C31 | (5,7,9) | (7,9,11) | (3,5,7) |
| C32 | (5,7,9) | (5,7,9) | (3,5,7) |
| C33 | (5,7,9) | (5,7,9) | (3,5,7) |
| C34 | (3,5,7) | (3,5,7) | (5,7,9) |
| C35 | (5,7,9) | (5,7,9) | (3,5,7) |
| C36 | (7,9,11) | (7,9,11) | (7,9,11) |
| C37 | (5,7,9) | (7,9,11) | (7,9,11) |
| C38 | (5,7,9) | (1,1,3) | (1,3,5) |

After decision matrix normalization, the next step is to calculate the weighted Fuzzy decision matrix. The results of this operation are shown in Table 10.

4. Determine FPIS and FNIS: the fuzzy positive ideal solution (FPIS, A+) and fuzzy negative ideal solution (FNIS, A−) are determined.

The ranking order of all alternatives can be obtained once the closeness coefficient is determined. This method allows the decision makers to select the most feasible alternative. The closeness coefficient of each alternative is calculated by applying Equation (18) as follows. Table 11 shows the final result and candidates rating. According to these closeness coefficients, the ranking order of the three candidates will be **T3, T1 and T2**.

According to the closeness coefficient of three alternatives, the ranking order of three alternatives is

**Table 9.** Normalized aggregation fuzzy decision matrix

| Criteria & sub Criteria | T1 | T2 | T3 |
|---|---|---|---|
| C1 | (0.64,0.82,1) | (0.56,0.78,1) | (0.64,0.82,1) |
| C2 | (0.54,0.64,0.82) | (0.56,0.78,1) | (0.64,0.82,1) |
| C3 | (0.09,0.27,0.45) | (0.33,0.56,0.78) | (0.27,0.45,0.64) |
| C11 | (0.33,0.56,0.78) | (0.33,0.56,0.78) | (0.11,0.33,0.56) |
| C12 | (0.56,0.78,1) | (0.56,0.78,1) | (0.56,0.78,1) |
| C13 | (0.11,0.33,0.56) | (0.56,0.78,1) | (0.11,0.33,0.56) |
| C21 | (0.33,0.56,0.78) | (0.27,0.45,0.64) | (0.09,0.27,0.33) |
| C22 | (0.11,0.11,0.33) | (0.09,0.09,0.27) | (0.09,0.09,0.11) |
| C23 | (0.33,0.56,0.78) | (0.45,0.64,0.82) | (0.45,0.64,0.33) |
| C24 | (0.56,0.78,1) | (0.45,0.64,0.82) | (0.64,0.82,0.56) |
| C25 | (0.56,0.78,1) | (0.64,0.82,1) | (0.64,0.82,0.56) |
| C31 | (0.45,0.64,0.82) | (0.64,0.82,1) | (0.27,0.45,0.64) |
| C32 | (0.45,0.64,0.82) | (0.45,0.64,0.82) | (0.27,0.45,0.64) |
| C33 | (0.45,0.64,0.82) | (0.45,0.64,0.82) | (0.27,0.45,0.64) |
| C34 | (0.27,0.45,0.64) | (0.27,0.45,0.64) | (0.45,0.64,0.82) |
| C35 | (0.45,0.64,0.82) | (0.45,0.64,0.82) | (0.27,0.45,0.64) |
| C36 | (0.64,0.82,1) | (0.64,0.82,1) | (0.64,0.82,1) |
| C37 | (0.45,0.64,0.82) | (0.64,0.82,1) | (0.64,0.82,1) |
| C38 | (0.45,0.64,0.82) | (0.09,0.09,0.27) | (0.09,0.27,0.45) |

**Table 10.** Weighted normalized aggregation fuzzy decision matrix

| Criteria & sub Criteria | T1 | T2 | T3 |
|---|---|---|---|
| C1 | (0.21,0.27,0.33) | (0.19,0.26,0.33) | (0.21,0.27,0.33) |
| C2 | (0.15,0.21,0.27) | (0.19,0.26,0.33) | (0.21,0.27,0.33) |
| C3 | (0.03,0.09,0.15) | (0.11,0.19,0.26) | (0.09,0.15,0.21) |
| C11 | (0.11,0.19,0.26) | (0.11,0.19,0.26) | (0.04,0.11,0.19) |
| C12 | (0.19,0.26,0.33) | (0.19,0.26,0.33) | (0.19,0.26,0.33) |
| C13 | (0.04,0.11,0.19) | (0.19,0.26,0.33) | (0.04,0.11,0.19) |
| C21 | (0.07,0.11,0.16) | (0.05,0.09,0.13) | (0.02,0.05,0.07) |
| C22 | (0.02,0.02,0.07) | (0.02,0.02,0.05) | (0.02,0.02,0.02) |
| C23 | (0.07,0.11,0.16) | (0.09,0.13,0.16) | (0.09,0.13,0.07) |
| C24 | (0.11,0.16,0.20) | (0.09,0.13,0.16) | (0.13,0.16,0.11) |
| C25 | (0.11,0.16,0.20) | (0.13,0.16,0.20) | (0.13,0.16,0.11) |
| C31 | (0.06,0.08,0.1) | (0.08,0.1,0.13) | (0.03,0.06,0.08) |
| C32 | (0.06,0.08,0.1) | (0.06,0.08,0.1) | (0.03,0.06,0.08) |
| C33 | (0.06,0.08,0.1) | (0.06,0.08,0.1) | (0.03,0.06,0.08) |
| C34 | (0.03,0.06,0.08) | (0.03,0.06,0.08) | (0.06,0.08,0.1) |
| C35 | (0.06,0.08,0.1) | (0.06,0.08,0.1) | (0.03,0.06,0.08) |
| C36 | (0.08,0.1,0.13) | (0.08,0.1,0.13) | (0.08,0.1,0.13) |
| C37 | (0.06,0.08,0.1) | (0.08,0.1,0.13) | (0.08,0.1,0.13) |
| C38 | (0.06,0.08,0.1) | (0.01,0.01,0.03) | (0.01,0.03,0.06) |

**Table 11.** Fuzzy topsis result

| Teacher | $d_j^+$ | $d_j^-$ | CCi | Ranking |
|---|---|---|---|---|
| T1 | 16.68 | 2.43 | 0.8729 | 3 |
| T2 | 16.46 | 2.02 | 0.8907 | 1 |
| T3 | 16.75 | 2.37 | 0.8761 | 2 |

determined as T2>T3>T1. The first alternative is closer to the FPIS and farther from the FNIS.

There have reached the same result with fuzzy AHP approximately and Teacher1 is selected as first alternative. Fuzzy AHP and fuzzy TOPSIS methods are both appropriate for the selection math teachers or other multi-criteria decision-making problems of the education. But these two methods have some limitations and advantages. According to the problem the most appropriate method should be chosen. In this research can summarize the differences and similarities between fuzzy AHP and fuzzy TOPSIS methods as follows:

- When these two methods are compared with respect to the amount of computations, fuzzy AHP requires more complex computations than fuzzy TOPSIS.
- Pair-wise comparisons for criteria, sub criteria and alternatives are made in fuzzy AHP, while there is no pair-wise comparison in fuzzy TOPSIS and are based on their relative distances to positive ideal solution and negative ideal solutions.
- TOPSIS has been proved to be one of the best methods addressing rank reversal issue that is the change in the ranking of the alternatives when a non-optimal alternative is introduced.
- In the extent analysis of fuzzy AHP, the priority weights of criterion or alternative can be equal to zero. In this situation, we do not take this criterion or alternative into consideration. This is the one of the disadvantages of this method.
- Both in fuzzy AHP and fuzzy TOPSIS can adopt linguistic variables.
- In this research, the ranking results of the fuzzy AHP and fuzzy TOPSIS are the same approximately. This shows that when the decision-makers are consistent with him/her in determining the data, two methods independently, the ranking results will be same.

## 6. Conclusion

Decision-makers face up to the uncertainty and vagueness from subjective perceptions and experiences in the decision-making process. By using fuzzy AHP and fuzzy TOPSIS, uncertainty and vagueness from subjective perception and the experiences of decision-maker can be effectively represented and reached to a more effective decision. In this study math teachers selection with fuzzy AHP and fuzzy TOPSIS method has been proposed. Although two methods have the same objective of selecting the best math teachers for the education, they have differences. In fuzzy TOPSIS decision-makers used the linguistic variables to assess the importance of the criteria, sub criteria and to evaluate the each alternative with respect to each criteria and sub criteria. These linguistic variables converted into triangular fuzzy numbers and fuzzy decision matrix was formed. Then normalized fuzzy decision matrix and weighted normalized fuzzy decision matrix were formed. After FPIS and FNIS were defined, distance of each alternative to FPIS and FNIS were calculated. And then the closeness coefficient of each alternative was calculated separately. According to the closeness coefficient of three alternatives, the best alternatives have been determined as Teacher2. In fuzzy AHP, decision-makers made pair-wise comparisons for the criteria, sub criteria and alternatives under each criteria and sub criteria. Then these comparisons integrated and decision-makers' pair wise comparison values are transformed into triangular fuzzy numbers. The priority weights of criteria, sub criteria and alternatives are determined by Saaty extent analysis. According to the combination of the priority weights of criteria, sub criteria and alternatives, the best alternative is determined. According to the fuzzy AHP, the best alternative is T1 that the same as fuzzy TOPSIS. Educations and institution should choose the appropriate method for their problem according to the situation and the structure of the problem they have. In future studies, other multi-criteria methods like fuzzy PROMETHEE and ELECTRE can be used to handle math teachers selection problems.

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