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MHD Thermosolutal Marangoni Convection Boundary Layer Nanofluid Flow Past a Flat Plate with Radiation and Chemical Reaction

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Abstract

Objectives: To investigate the effects of Magneto Hydro Dynamic marangoni convection in nanofluids subject to radiation along with first order chemical reaction. **Method:** The analysis is taken over Copper-Water and Silver-Water nanofluids. Water with Prandtl number 6.2 is taken as base fluid. Flow equations are formed by a suitable similarity transformation. Using MATLAB, numerical solutions are obtained for Velocity, Temperature and Concentration of the nanofluid particles for different parameters. Also it is observed the heat and mass transfer effects. **Results:** The effect of magnetic parameter on fluid motion and temperature is appreciable. Further at low magnetic number ($0 \leq M \leq 2$), velocity of Cu nanofluid particles move faster than that of Ag nanofluid particles. It is observed that increase in Soret number results in decrease of mass transfer rate and this rate is more in Ag-water nanofluid. **Conclusion:** Copper nanofluid particles possess higher heat and mass transfer rates. It is also noticed that Sherwood number decreases with increase of Soret number.

Keywords: Heat and Mass Transfer, MHD, Marangoni Convection, Nanofluid, Radiation

1. Introduction

The marangoni or thermo-capillary convection is induced by the variations of the surface tension gradients. This convection has received great consideration in view of its application in the fields of welding and crystal growth. Also it is widely used in the semiconductor processing. Marangoni effect is used for drying silicon wafers after a wet processing step during the manufacture of integrated circuits. Marangoni boundary layer is first named by Napolitano[1]. Experimental work connected to marangoni convection has been discussed earlier[2]. The metallurgical applications of magnetic field include the cooling of strips or filaments in the process of drawing, annealing, and thinning of copper wires. Drawing such strips in an electrically conducting fluid subject to a magnetic field can control the rate of cooling. Tiwari and Das[3] proposed a model for nanofluid flow which is used in several papers. Chamkha[4] presented the similarity solution for magneto hydrodynamic thermosolutal marangoni convection over a flat plate. Magyari[5] has arrived analytical solutions for magneto hydrodynamic convective flow. Arifin[6] derived the governing equations of a marangoni flow and solved them using similarity transformation in a nanofluid.

Soret and Dafour effects are presented in many of the research papers recently. Kafoussias and Williams[7] presented literature on this context. Recently Hamid8 analyzed thermal diffusion and diffusion thermo effects in MHD thermosolutal marangoni convection over a porous plate. They examined the influence of Soret number on temperature and mass transfer rates. They extended their contribution towards the influence of velocity on Soret and Dafour numbers. Chamkha[9] examined the influence of chemical reaction on magneto hydrodynamic flow. These studies are focused to Newtonian fluids.

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However, due to the increasing importance of nanofluids, a tremendous amount of interest has been given to the study of convective transport of nanofluids. The word nanofluid coined by Choi[10] found a liquid suspension containing ultra-fine particles which are designed by a high-energy-pulsed process from a conducting material. The nanotechnology manufacturing materials include particles of metals such as aluminum, copper, gold, iron and titanium or their oxides. Usually water, ethylene glycol, toluene or oil will be taken as base fluids. This choice of base fluid-particle combination depends on the application for which the nanofluid is prepared. A characteristic feature of nanofluid is thermal conductivity enhancement, a phenomenon observed by Masuda[11]. Recently heat transfer on Magneto Hydro Dynamic marangoni convection in nanofluids is studied by Sastry[12]. They noticed that inclusion of the magnetic field parameter on the flow increased the temperature and decreased the velocity fields in all types of nanofluids concerned. The aim of this paper is to discuss the combined effects of a magnetic field, chemical reaction and Soret effects on nanofluid flow. The velocity, temperature and concentration profiles against magnetic field parameter and solid volume fraction are presented graphically. The heat transfer against volume fraction and mass transfer against Soret number are discussed graphically. The radiation effects on velocity, temperature profiles are also discussed.

2. Mathematical Formulation

Consider a two-dimensional thermo capillary convection boundary layer flow in a water-based nanofluid consisting Cu (Copper) and Ag (Silver). Assume that the fluid and the particles are in thermal equilibrium and no slip occurs between them. The thermo physical properties of nano particles are given by Oztop and Abu-Nada[13] (Table 1). Consider a coordinate system (x,y), where x and y are measured along the plate and normal to it. The surface tension is given by

$$\sigma = \sigma_0[1-\gamma(T-T_\infty)-\gamma^*(C-C_\infty)] \tag{1}$$

where σ_0 is surface tension at the interface and

$$\gamma = -\frac{\partial\sigma}{\partial T}, \quad \gamma^* = -\frac{\partial\sigma}{\partial C}$$

Let H_0 be uniform magnetic field in the direction normal motion are given by to the surface. The steady state governing equations of

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma^*}{\rho_{nf}}H_0{}^2u \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}-\frac{1}{(\rho C_p)_{nf}}\frac{\partial q_r}{\partial y} \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}+D_1\frac{\partial^2 T}{\partial y^2}-K_0(C-C_\infty) \tag{5}$$

along with the boundary conditions

$$\left.\begin{aligned} &v=0,\ T=T_\infty+ax^2,\ C=C_\infty+bx^2 \text{ and} \\ &\mu_{nf}\frac{\partial u}{\partial y}=\gamma\frac{\partial T}{\partial x}+\gamma^*\frac{\partial C}{\partial x} \text{ at } y=0 \end{aligned}\right\} \tag{6}$$

Here, u and v are velocity components along the x and y axes respectively. ρ_f is the fluid density, T is the temperature, C is the concentration, C_∞ is the concentration of the free fluid, C_p is the specific heat at constant pressure, D is the species diffusivity, D_1 is the coefficient that signifies the contribution to mass flux through temperature gradient and K_0 is the chemical reaction parameter. σ^* is the electric conductivity, a and b are coefficients of temperature and concentration gradients respectively.

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}} \tag{7}$$

where ϕ is the solid volume fraction of nanoparticles. The effective density,

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s \tag{8}$$

Effective thermal diffusivity

$$\alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}} \tag{9}$$

Effective heat capacitance,

$$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s \tag{10}$$

$$\frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)} \tag{11}$$

In equations (7) - (11), the subscripts *nf*, *f* and s denote the thermo physical properties of the nanofluid, base fluid and nano-solid particles respectively.

The continuity equation (2) is satisfied by introducing a stream function ψ(x, y) such that

$$u = \frac{\partial\psi}{\partial y},\ v = -\frac{\partial\psi}{\partial x} \qquad (12)$$

The following similarity variables are introduced:

$$\psi = C_1 x f(\eta),\ \eta = C_2 y \qquad (13)$$

$$\theta(\eta) = \frac{T - T_\infty}{ax^2},\ h(\eta) = \frac{C - C_\infty}{bx^2} \qquad (14)$$

where the constants C_1 and C_2 are given by

$$C_1 = \left(\frac{\left.\frac{d\sigma}{dT}\right|_C a\mu_f}{\rho_f^2}\right)^{1/3},\ C_2 = \left(\frac{\left.\frac{d\sigma}{dT}\right|_C a\rho_f}{\mu_f^2}\right)^{1/3} \qquad (15)$$

where η is the similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature and $h(\eta)$ is the dimensionless concentration. On using equations (7), (8), (9), (10), (11), (13), (14) and (15), equations (3), (4), (5), and (6) transform into the following two-point boundary value problem:

$$f''' = (1-\phi)^{2.5}[(1-\phi) + \phi\frac{\rho_s}{\rho_f}](f'^2 - ff'') + M^2\ (1-\phi)^{2.5} f' \qquad (16)$$

$$\theta'' = \frac{P_r[(1-\phi) + \phi\frac{(\rho Cp)_s}{(\rho Cp)_f}]}{(k_{nf}/k_f)(1+N_r)}(2f'\ \theta - f\theta') \qquad (17)$$

$$h'' = S_c(2hf' - fh' + K^* h) - S_r\theta'' \qquad (18)$$

$$\left.\begin{aligned} & f(0) = 0,\ \theta(0) = 1,\ h(0)=1 \\ & \frac{1}{(1-\phi)^{2.5}} f''(0) = -2(1+\varepsilon) \\ & f'(\infty) = 0, \theta(\infty) = 0, h(\infty) = 0 \end{aligned}\right\} \qquad (19)$$

The non-dimensional constants appearing in equations (16) – (19) are the magnetic parameter M, Prandtl number Pr, Radiation parameter Nr, Schmidt number Sc, Soret number Sr, scaled chemical reaction parameter K*, and thermosolutal surface tension ratio. They are defined as

$$M = \frac{\sigma_c^{1/2} H_0 \mu^{1/6}}{\rho^{1/3} \left.\frac{d\sigma}{dT}\right|_C^{1/3} a^{1/3}},\quad \Pr = \frac{\upsilon_f(\rho C_p)_f}{k_f},\quad S_C = \frac{\upsilon_f}{D}$$

$$K^* = \frac{K_0 \mu_f^{1/3} \rho_f^{1/3}}{\left.\frac{d\sigma}{dT}\right|_C^{2/3} a^{2/3}},\quad Sr = \frac{D_1(T - T_\infty)}{D(C - C_\infty)},\quad \varepsilon = \frac{\Delta C \left.\frac{d\sigma}{dC}\right|_T}{\Delta T \left.\frac{d\sigma}{dT}\right|_C}$$

$$N_r = \frac{16\sigma^* T_\infty^{\ 3}}{3k^* k_{nf}}$$

3. Skin Friction, Nusselt and Sherwood Numbers

In the problem of finding drag force, heat and mass transfer one can use the parameters like coefficient of skin friction C_f, the local Nusselt number Nu_x, and local Sherwood number Sh_x respectively.

The shearing stress at wall $\tau = -\mu_{nf}\left.\frac{\partial u}{\partial y}\right|_{y=0} = -\frac{\left.\frac{d\sigma}{dT}\right|_C axf''(0)}{(1-\phi)^{2.5}}$ (20)

The coefficient of skin friction $C_f = \frac{2\tau}{\rho_f u^2}$ (21)

where $u(x) = \sqrt[3]{\frac{\left(\left[\left.\frac{d\sigma}{dT}\right|_C a\right)\right]^2}{\rho_f \mu_f}}\ x\ f'(0)$ is surface velocity (22)

Using equations (20) and (22) in equation (21) we obtain

$$C_f Re_x^{\ 2}(1-\phi)^{2.5} = -2f''(0) \qquad (23)$$

The heat transfer rate at the surface flux at the wall is given by

$$q(x) = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} = -k_{nf}C_2 a x^2 \theta'(0) \tag{24}$$

The Nusselt number is defined as

$$Nu_x = \frac{xq(x)}{k_f[T - T_\infty]} \tag{25}$$

Using the equations (6), and (24), one can get the dimensionless wall heat transfer rate from equation (25) as

$$\frac{Nu_x\left(\frac{k_f}{k_{nf}}\right)}{C_2 x} = -\theta'(0) \tag{26}$$

The mass flux at the wall is defined as

$$m = -D\left(\frac{\partial C}{\partial y}\right)_{y=0} = -DC_2 b x^2 h'(0) \tag{27}$$

and the Sherwood number is defined as

$$Sh_x = \frac{xm}{D(C - C_\infty)} \tag{28}$$

From the equations (6),(27) and (28) one can get the dimensionless mass transfer rate as

$$\frac{Sh_x}{C_2 x} = -h'(0) \tag{29}$$

where $C_2 x$ is a dimensionless quantity.

4. Results and Discussion

Numerical solutions are obtained for the radiation and chemical reaction effects on heat and mass transfer characteristics of a MHD marangoni convective nanofluid flow over a flat plate. The non-linear ordinary differential equations (16) - (18) together with boundary conditions (19) are solved numerically using Matlab bvp4c programme. In this context two different types of nanoparticles, namely Copper and Silver, with base fluid Water (Prandtl number Pr = 6.2) are considered. In this paper the magnetic field parameter *M* is ranged from 0 to 6. The solid volume fraction ϕ is ranged from 0 to 0.8. The radiation parameter is taken from 0 to 50. The Soret number is taken from 0 to 5. The chemical reaction parameter K* is ranged from 0 to 50. Thermosolutal surface tension ratio ε is ranged from 0 to 1. The effects of magnetic field parameter M on velocity, temperature and concentration is showed in Figure 1 – Figure 3. The effect of nanoparticles volume fraction ϕ on velocity, temperature and concentration is discussed in Figure 4 – Figure 6.

The effects of radiation parameter Nr on temperature and concentration profiles are presented in Figure 7 and Figure 8 respectively. The change in concentration along with Soret number Sr is showed in Figure 9. The effects of chemical reaction parameter K* on concentration profile is displayed in Figure 10. The effects of thermosolutal surface tension ratio ε on fluid properties are showed in Figure 11 – Figure 13. The influences of the solid volume fraction ϕ on heat transfer and Soret number on mass transfer are showed in Figure 14 – Figure 15.

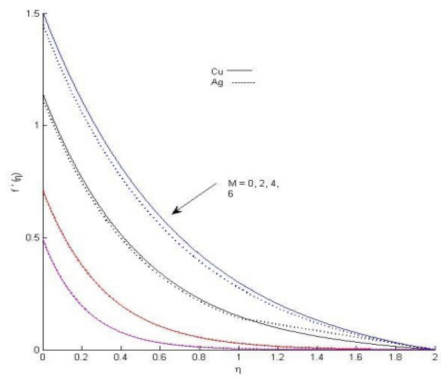

Figure 1. Velocity profile for different values of magnetic parameter M.

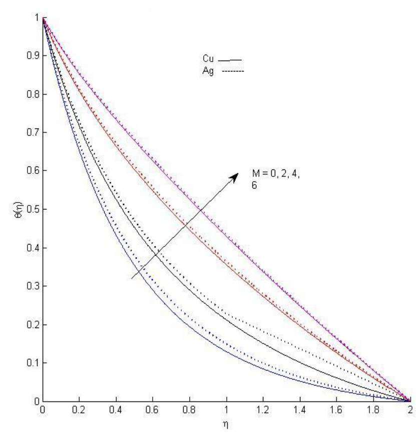

Figure 2. Temperature profile for different values of magnetic M.

When a magnetic field is applied within a boundary layer it produces a resistive type force, known as Lorentz force which acts to retard the fluid motion along the surface and simultaneously increases temperature and concentration values of the fluid within the boundary layer. The effect of magnetic parameter on fluid motion and temperature is appreciable.

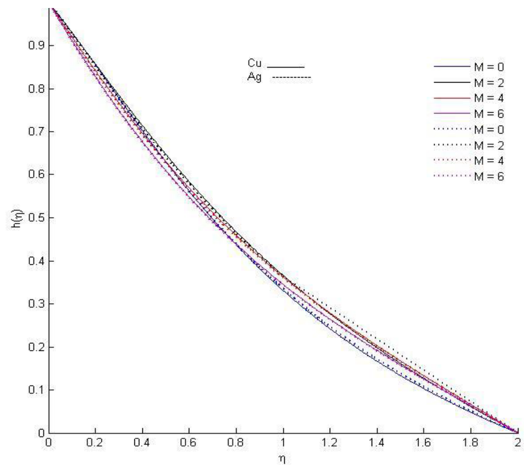

Figure 3. Concentration profiles for different values of magnetic parameter M.

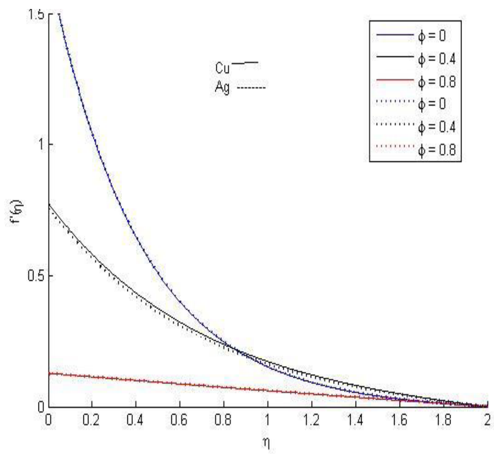

Figure 4. Velocity profile for different values of volumetric fraction ϕ .

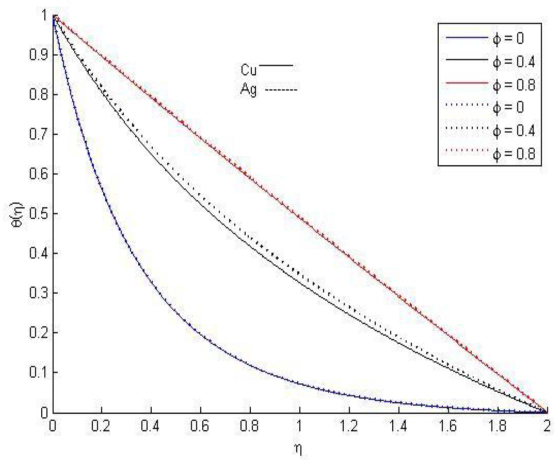

Figure 5. Temperature profiles for different values of volmetric fraction ϕ

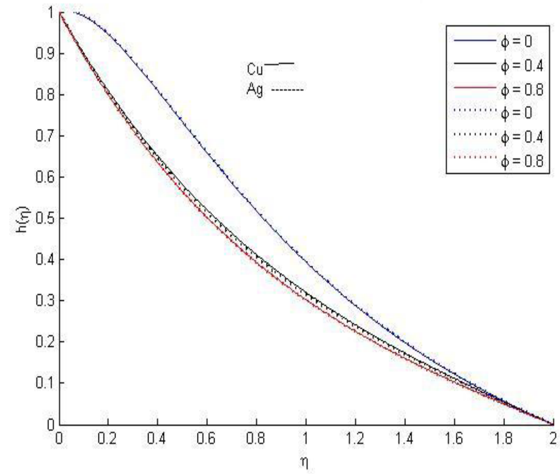

Figure 6. Concentration profiles for different values of volumetric fraction ϕ .

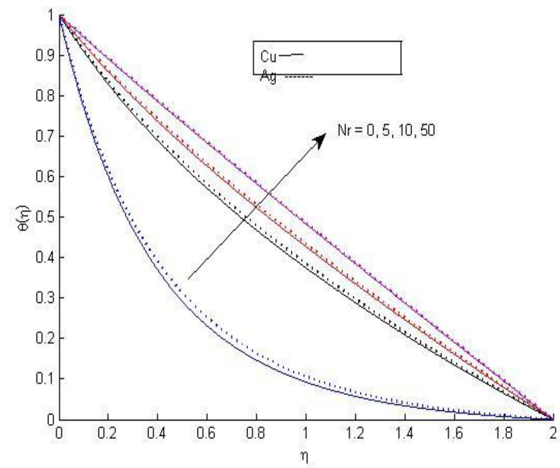

Figure 7. Temperature profiles for different values of Radiation parameter Nr.

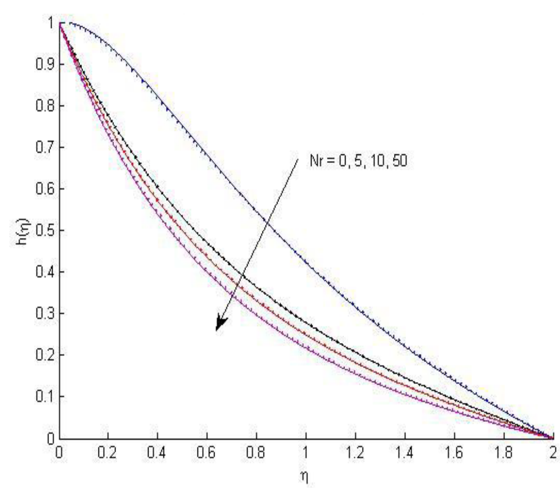

Figure 8. Concentration profiles for different values of Radiation parameter Nr.

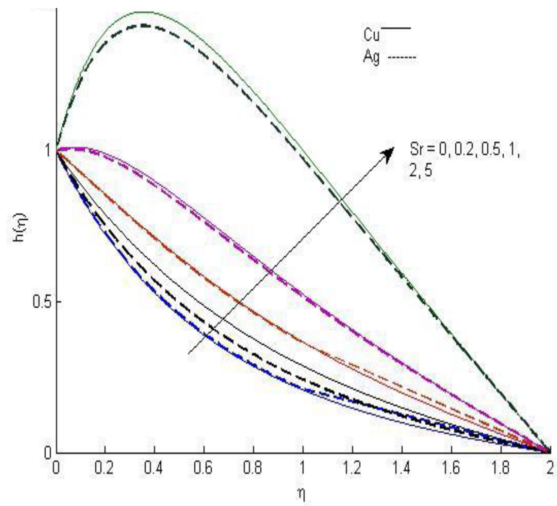

Figure 9. Concentration profiles for different values of Soret number Sr.

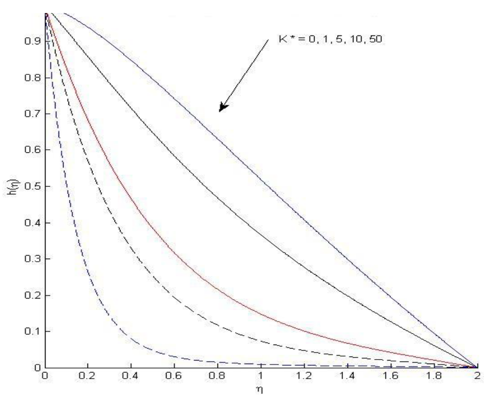

Figure 10. Concentration profiles for different values of chemical reaction parameter K*.

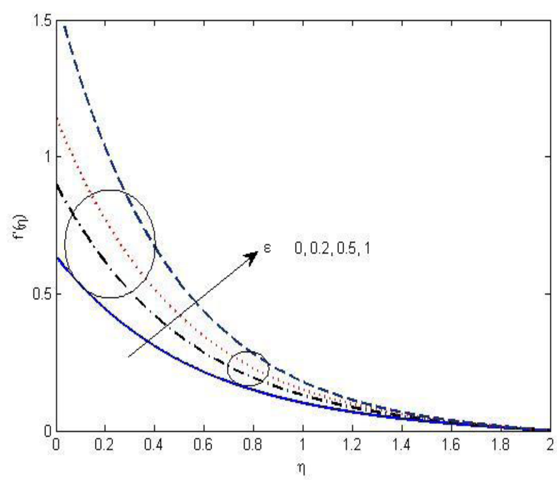

Figure 11. Velocity profiles for different values of Thermosolutal surface tension ratio ∈ .

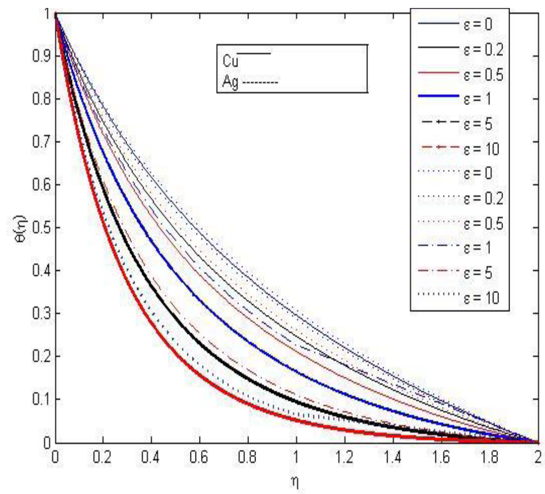

Figure 12. Temperature profiles for different values of Thermosolutal surface tension ratio ∈ .

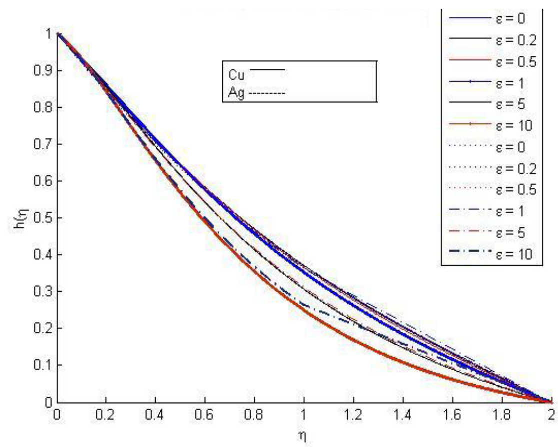

Figure 13. Concentration profile for different values of thermosolutal surface tension ratio ∈ .

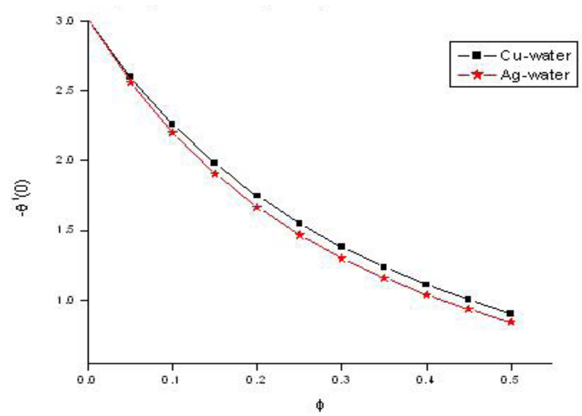

Figure 14. Rate of heat transfer for different values of volume fraction (M = 2, Sc = Sr = Nr = K*=1, Pr = 62).

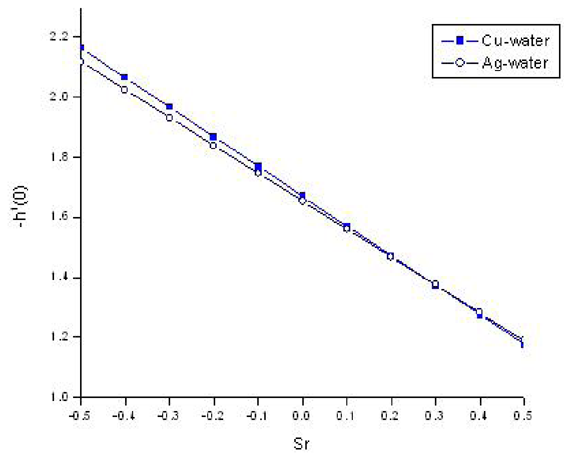

Figure 15. Rate of mass transfer for different values of Soret number Sr (M = 2, Sc = Nr = K* = 1, Pr = 6.2, = 0.2).

Further at low magnetic number ($0 \leq M \leq 2$), velocity of Cu nanofluid particles move faster than that of Ag nanofluid particles (Figure 1, Figure 2 & Figure 3). Also enhancing the volume fraction results a decrease in velocity and increase in temperature and concentration profiles (Figure 4 & Figure 5). The thermal boundary layer is larger for Cu nanofluid than that of Ag nanofluid. This shows the physical behaviour of the metals Figure 6 depicts the concentration distribution of nanofluid against volume fraction. It is observed that solutal boundary layer thickness is higher near the wall and lesser far away from the wall with the increase of volume fractionϕ (Figure 6). Figure 7 and Figure 8 depict the impact of radiation on nanofluids' temperature and concentration profiles respectively. It is observed that increase in the value of radiation parameter enhances the temperature of the nanofluid profile and reduces the species concentration. The Cu-water nanoparticles exhibit lower temperature distribution than that of Ag-water nanoparticles (Figure 7 & Figure 8). This is because of the property that Silver is more conductive metal than Copper. Figure 9 depicts the effect of Soret number on the species concentration of the nanoparticles. It is noticed that increase in Soret number enhances the concentration of the nanofluid particles. This increase is more in Cu-water nanofluid particles (Figure 9). From Figure 10 one can observe that increase in the chemical reaction parameter reduces the species concentration (Figure 10). Also from Figure 11, it is noticed that a large variation in the velocity profiles of the nanoparticles exist near the boundary layer wall and nullified far away from the wall whenever there is an increment in the thermosolutal surface tension ratio ε (Figure 11). Also from Figure 12 and Figure 13 it is noticed that thermosolutal surface tension ratio significantly decreases the fluid temperature and concentration. This finding is obtained due to the increase in the values of ε demand the increase in the marangoni convection which produces more induced flows within the boundary layer. As a consequence, the resulting flows will propagate within the boundary layers causing the maximum velocity obtained at the wall (Figure 12 & Figure 13). Figure 14 illustrate the effect of heat transfer against volume fraction. It is observed that heat transfer rate is reduced by increasing the volume fraction and this is found more in Ag-water nanofluid (Figure 14). Also from Figure 15 it is observed that increase in Soret number results in decrease of mass transfer rate and this rate is more in Ag-water nanofluid (Figure 15).

5. Conclusion

The problem of marangoni MHD convective nanofluid flow subject to radiation, chemical reaction and Soret effects past a flat porous plate is considered. Two types of nanofluids, namely Cu-water and Ag-water are considered. Governing equations of motion are solved numerically using MATLAB bvp4c programme. Results for the velocity, temperature, concentration, Nusselt number and Sherwood number for selected values of the governing parameters are showed graphically. A good agreement is found with the previous works that are presented in the literature. It is observed that Cu-water nanofluid exhibits higher wall heat and mass transfer rates as compared to Ag-water nanofluid. Also it is observed that increase in Soret number results in decrease the mass transfer rate.

Table 1. The thermo physical properties of nano particles

Physical property	Pure Water	Cu	Ag
ρ (kg/m^3)	997.1	8933	10500
C_p (J/kg K)	4179	385	235
k (W/m K)	0.613	401	429

6. References

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