# Stability Analysis of the Variable Speed Wind Turbine using Sliding Mode Control

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#### Abstract

The sliding mode control is one among the best techniques for analysing the dynamics as well as maintaining the stability of a system. The problem of lack of stability and robustness in a variable speed wind turbine can be resolved by using this sliding mode control technique. The desired sliding mode dynamics can be attained by a suitable design of the switching function and on the switching surface, the sliding occurs; following the system achieving its desired system dynamics. This paper presents the sliding mode control scheme to improve the overall stability of the variable speed wind turbine system. The proposed strategy is applied and the results are obtained using MATLAB. The results obtained demonstrate its effectiveness in improving the overall system stability.

Keywords: Doubly Fed Induction Generator, Sliding Mode Control, State Space, Variable Speed Wind Turbine

## 1. Introduction

The increased worldwide demand for the energy requires alternatives for the depleting fossil fuels, so renewable resources, particularly wind energy, are called to play a vital role in the near future. This kind of energy harness from the wind requires particular type of turbines and generators. Variable Speed Wind Turbines (VSWT) using Doubly Fed Induction Generators (DFIG) are unceasingly growing their market share now a day, since it is likely to track the changes in wind speed by adjusting the shaft speed and thus upholding the ideal power generation. Lack of stability and robustness in spite of model qualms and external instabilities are the main problems of the variable speed wind turbines. It can be resolved by using the sliding mode control technique.

In section II, the sliding mode control technique is elaborated. The steps involved in the design of a variable structure control are also included. The modelling of a small scale variable speed wind turbine is shown in section III. The recognized model of the plant is necessary for the reaching law to synthesise the variable control law. In section IV, the variable structure controller is designed and a sliding mode control law is implemented to make the state variables to reach the desired steady state value. A convenient control law is proposed instead of applying classical sliding mode approaches such as the twisting or super twisting algorithms. The objective of the sliding mode controller design is the stabilisation of the variable speed wind turbine. The simulation results obtained using MATLAB software package are shown in section V. The results show the convergence of the sliding variables to zero in a specific time, ensuring the system to reach the steady state values.

## 2. Sliding Mode Control

Sliding Mode Control (SMC) strategy also termed as variable structure control (VSC), is a dynamic system whose structure vicissitudes with the current value of its state<sup>1,4</sup>. The SMC system is comprised of independent structures, together with a switching logic between each

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of the structures. With the proper switching logic, a variable structure system can achieve the desired properties of each of the independent structures the system is comprised of. Also the system switches infinitely many times in a single time instant<sup>2</sup>.

Consider a plant described as,

$$X = A.x + B.u \tag{1}$$

$$y = C.x + D.u \tag{2}$$

Where x is an n-vector, u is a scalar, and A and B are of appropriate dimensions. The design of the variable structure control is recognised to have the following steps:

- Determination of the switching function s(x) such that the sliding mode on the switching plane is constantly stable. ie., s(x) = 0.
- Determination of a control law such that a reaching condition must be pleased. ie.,

$$u(\mathbf{x}) = \begin{cases} u^{+}(\mathbf{x}), \text{ when} \\ u^{-}(\mathbf{x}), \text{ when} \end{cases}$$
(3)

This infers that any state, starting from any initial state, will move toward the switching plane on the switching surface and attain the steady state in finite time<sup>1</sup>.

Instead of first establishing an analytical expression of a reaching condition and then designing a control law to meet the situation, here a different but much more convenient method is adopted, called the reaching law approach<sup>3,5</sup>. This reaching law unswervingly dictates the dynamics of the switching function s(x) = 0 and then, a VSC control law is created from the reaching law with recognized model of the plant and known constraints of perturbations.

A convenient reaching law for the VSC of a plant is given by,

$$\dot{s}(t) = -qs(t) - \epsilon \operatorname{sign}(t), \epsilon > 0, \ q > 0$$

$$\tag{4}$$

# 3. Modelling of a Small Scale Variable Speed Wind Turbine

The system modelling is stimulated from the study in<sup>6</sup>. The mechanical equation for the generator is given by,

$$J_{tot.S.}\Omega_{G} = N.T_{r} + K_{G.}I - B.\Omega_{G}(s)$$
<sup>(5)</sup>

where,

 $J_{tot}$  is the total inertia of the system [Kg/m<sup>2</sup>].

 $\Omega_{\rm _G}$  is the angular velocity of the generator [rad/s]. N is the gear ratio.

Tr is the force provided by the rotor [Nm].

 $K_{c}$  is the generator's torque constant [Nm/A].

I is the current through the generator coil [A].

B is the viscous friction coefficient for the system [Nm/ (rad/s)].

The electrical equation for the generator is given by,

$$U_L(\mathbf{s}) = \mathbf{I}(\mathbf{s})\mathbf{R}_G + K_G \Omega_G(\mathbf{s})$$
(6)

where,

 $U_{L}$  is the voltage provided by the load [V].

 $R_{G}$  is the terminal resistance of the generator [ $\Omega$ ].

The servomechanism can be modelled by a first order linear differential equation as,

$$\dot{\beta} = \frac{1}{\pi} \cdot \beta + \frac{1}{\pi} \cdot \beta_{ref} \tag{7}$$

where,

 $\beta$  is the pitch duty cycle.

The state, which is wanted to control, together with the input and output of the system, has to be recognized, in order to derive a state model. They are listed below:

States: The angular velocity,  $\Omega_G$  and the pitch duty cycle,  $\beta$ . Input: Voltage on the generator terminals,  $U_L$  and pitch duty cycle,  $\beta_{ref.}$ 

Output: The angular velocity,  $\Omega_{G}$ 

The state space model of the system is derived using (5) and (6) combined with (7).

The equation can be solved with respect to I and is given by,

$$I = \frac{U_L - K_G \cdot \Omega_G}{R_G} \tag{8}$$

Inserting the above equation into the mechanical differential equation for the generator, it becomes

$$\dot{\Omega}_{G} = \frac{K_{G}}{J_{tot}} \cdot I - \frac{B}{J_{tot}} \cdot \Omega_{G} + \frac{N \cdot T_{r}}{J_{tot}}$$
(9)

ie.,

$$\dot{\Omega}_{G} = \frac{K_{G}}{R_{G}J_{tot}} U_{L} - \frac{K_{G}^{2} + B.R_{G}}{R_{G}J_{tot}} \Omega_{G} + \frac{N.T_{r}}{J_{tot}}$$
(10)

 $T_r$  is linearized around a specific operating point, as it is a nonlinear function of together wind speed, rotor speed and pitch angle. The linearization is done with respect to  $\Omega_G$  and  $\beta$ , thus resulting in the gradient at the operating point as,

$$\widehat{T}_{r}\left|\left(\Omega_{G},\beta\right)=\frac{\partial T_{r}}{\partial\Omega_{G}}\right|\left(\Omega_{G},\beta\right).\widehat{\Omega}_{G}+\frac{\partial T_{r}}{\partial\beta}\left|\left(\Omega_{G},\beta\right)\widehat{\beta}\right|$$
(11)

ie.,

$$\widehat{T}_r = B_r . \widehat{\Omega}_G + K_\beta . \widehat{\beta} \tag{12}$$

Replacing  $T_r$  with (10), we get

$$\dot{\Omega}_{G} = \frac{K_{G}}{R_{G}.J_{tot}}.U_{L} - \frac{K_{G}^{2} + (B - N.B_{r}).R_{G}}{R_{G}.J_{tot}}.\Omega_{G} + \frac{N.K_{\beta}}{J_{tot}}.\beta$$
(13)

Using the following matrix notation,

$$\dot{X} = A.x + B.u \tag{14}$$

$$y = C.x + D.u \tag{15}$$

The system can now be represented by the set of matrices as,

$$\dot{x} = \begin{bmatrix} \frac{K_G^2 + (B - N \cdot B_r) \cdot R_G}{R_G \cdot J_{tot}} \frac{N \cdot K_\beta}{J_{tot}} \\ 0 & -\frac{1}{\pi} \end{bmatrix} \cdot \begin{bmatrix} \Omega_G \\ \beta \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{K_G}{R_G \cdot J_{tot}} & 0 \\ 0 & \frac{1}{\pi} \end{bmatrix} \cdot \begin{bmatrix} U_L \\ \beta_{ref} \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \Omega_G \\ \beta \end{bmatrix} + 0 \cdot \begin{bmatrix} U_L \\ \beta_{ref} \end{bmatrix}$$
(17)

In order to verify whether this model is realistic, the poles are calculated and both are seem to be negative. The motor and gear parameters have been determined from the datasheet as follows:

Table 1. Motor and gear parameters

S.NO	Parameters	Value
1.	Generator torque constant, K <sub>G</sub>	38.2*10 <sup>-3</sup> Nm/A
2.	Viscous force coefficient, B	42.4*10 <sup>-6</sup> Nm/(rad/s)
3.	Gear ratio, N	1/11
4.	Speed coefficient, B <sub>r</sub>	0.773*10-3
5.	Terminal resistance, R <sub>G</sub>	7.19Ω
6.	Total inertia of system, J <sub>tot</sub>	199*10 <sup>-6</sup> Kgm <sup>2</sup>
7.	Pitch coefficient, K <sub>g</sub>	38.4
8.	Time constant, $\tau$	0.02s

# 4. Design of Variable Structure Controller

Consider a plant,

$$\dot{x}(t) = A.x(t) + b.u(t)$$
 (18)

Where x is an n-vector, u is a scalar, and A and B are of appropriate dimensions<sup>7</sup>.

Consider a linear switching function as,

$$s(\mathbf{t}) = \mathbf{c}^T \, \mathbf{x} \tag{19}$$

Then the linear switching plane is given by,

$$\mathbf{s}(\mathbf{t}) = \mathbf{C}^T \, \mathbf{x}(\mathbf{t}) = \mathbf{0} \tag{20}$$

On substituting equation in equation, we get

$$s(t) = c^{T} . A.x(t) + c^{T} . b.u(t)$$
 (21)

Solving u(t), an appropriate control is obtained as

$$u(t) = (c^T . b)^{-1} . c^T . A.x(t)$$
 (22)

#### 5. Simulation Results

To prove the effectiveness of the SMC technique in VSWT, a small scale variable speed wind turbine system in [6] is used and the parameters of that system are,

$$A = \begin{bmatrix} -353.0614 & 0 \\ 0 & -0.02 \end{bmatrix};$$
  

$$b = \begin{bmatrix} 26.6982 & 0 \\ 0 & 0.02 \end{bmatrix};$$
  

$$c = \begin{bmatrix} 1 & 0 \end{bmatrix};$$
  

$$d = \begin{bmatrix} 0 \end{bmatrix};$$
  

$$c^{T} = \begin{bmatrix} -13.1867 & 0.0375 \\ -50.0000 & 49.0000 \end{bmatrix}$$

The initial condition assumed is  $x(0) = \begin{bmatrix} 8 & 9 \end{bmatrix}^T$ . With the overhead values, the system is simulated and the results attained are depicted in Figure 1, Figure 2 and Figure 3.

In Figure 1, the sliding surface is shown. As can be noticed, s(t) reach steady state irrespective of the initial perturbations.

In Figures 2 and 3, the state variables, viz., the speed and pitch angle of wind turbine are shown. It is clear from the



**Figure 1.** Evolution of the sliding surface, s(t).



**Figure 2.** Sliding variable,  $\Omega$  vs. Time.



**Figure 3.** Sliding variable,  $\beta$  vs. Time.

figures that the two variables converge smoothly to steady state without any overshoot.

# 6. Conclusion

This paper dealt with the problem of lack of stability of the variable speed wind turbines using sliding mode control approach which ensures system stability. The results obtained clearly shows that the system reach the steady state irrespective of the initial perturbations.

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