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The Medium Domination Number of a Jahangir Graph $J_{m,n}$

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Abstract

In 1958 - Claude Berge introduced the domination number of a graph which is used to protect the single vertices. But in 2011 Duygu Vargor and Pinar Dundar initiated the medium domination number of a graph which is utilized to protect the pairs of vertices in a graph.

In a graph every vertices u, v Є V should be privileged and it is essential to scrutinize how many vertices are proficient of dominating both of u and v. We compute the total number of vertices that dominates all pairs of vertices and evaluate the average of this value and call it "the medium domination number" of graph. The medium domination number of G is the minimum cardinality among all the medium domination sets of G.

We prove the main result by two-dimensional induction method. First we are manipulative the medium domination number of $J_{1,3}$. Then we are calculating the medium domination number of $J_{m+1,3}$ and $J_{m,n+1}$. Finally we are getting the medium domination number of $J_{m,n}$. By using this method we can proficient to observe how many pairs of vertices are dominates in the Jahangir graph $J_{m,n}$.

In graph theory, there are many stability parameters such as the connectivity number, the edge-connectivity number, the independence number, the vertex domination number and the domination number. In this paper, we obtained the bound of the medium domination number of Jahangir graph $J_{m,n}$.

Keywords: Domination Number, Jahangir Graph, Medium Domination Number, TDV

1. Introduction

Graph theory is rapidly moving into the mainstream of mathematics mainly because of its applications in diverse fields which include biochemistry, electrical engineering, computer science and operations research. In 1958, Claude Berge has introduced the domination number of a graph. Domination theory is a special area in graph theory. In graph theory, a dominating set for a graph $G = (V, E)$ is a subset D of V such that every vertex not in D is adjacent to at least one member of D. The domination number $\gamma(G)$ is the number of vertices in the smallest dominating set for G.

In[5] defined the domination in Jahangir Graph $J_{2,m}$. In[6] studied the domination in Jahangir Graph $J_{3,m}$. Dominating set is used to protect the individual vertices in the graph but the medium dominating set is used to protect the pairs of vertices in the graph. The medium domination number of a graph was first introduced in[7]. They have defined the medium domination number of a graph.

2. Notations and Definitions

2.1 Definition

A graph G consists of a pair (V (G), E (G)) where V (G) is a non-empty finite set whose elements are called points or vertices and E (G) is a set of unordered pairs of distinct elements of V (G). The elements of E (G) are called lines or edges of the graph G.

2.2 Definition

The degree of a vertex v in a graph G is the number of edges of G incident with v and it is denoted by deg (v_i).

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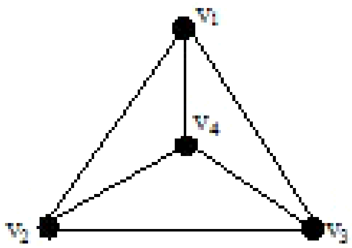

Figure 1. Jahangir Graph $J_{1,3}$.

The maximum he vertices of G is denoted by Δ (G). The minimum degree among the vertices of G is denoted by δ (G).

2.3 Definition

For G = (V, E) and ∀ u, v ∈ V; if u and v are adjacent they dominate each other then at least $dom(u, v) = 1$.

2.4 Definition

In the graph G = (V, E) and ∀ u, v ∈ V; the total number of vertices that dominate every pair of vertices is defined as

$$TDV(G) = \sum_{\forall u,v \in V(G)} dom(u,v)$$

2.5 Definition

In any connected, simple graph G of order n, the medium domination number of G is defined as

$$MD(G) = \frac{TDV(G)}{nC_2}$$

2.6 Definition

Jahangir graphs $J_{m,n}$ for n ≥ 3 is a graph on mn+1 vertices. i.e. a graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance 'n' to each other on C_{nm}.

The graph $J_{1,3}$ has a cycle C_3 and 4 vertices v_1, v_2, v_3, v_4. The fourth vertex v_4 is adjacent to remaining three vertices v_1, v_2, and v_3.

3. Preliminary Notes on Medium Domination Number

3.1 Observation

dom (u, v) ≤ m (u, v).

3.2 Observation

dom (u, v) ≤ min {deg (u), deg (v)}.

3.3 Observation

Let P_n be the path graph with n vertices; then we have $MD(P_n) = \frac{2n-3}{nC_2}$

3.4 Observation

Let C_n be the cycle graph with n vertices; then we have $MD(C_n) = \frac{2_n}{nC_2}$

3.4.1 Theorem

For G has n vertices, q edges and for deg (v_i) ≥ 2; $TVD(G) = q + \sum_{v_i \in V} (\deg v_i C_2)$

4. Main Results on Medium Domination Number of Jahangir Graph $J_{m,n}$

4.1 Lemma

The medium domination number of Jahangir graph $J_{1,n}$ is

$$MD(J_{1,n}) = \frac{\{n[n+4(1+1)+1]\}/2}{(n+1)C_2}$$

Proof: We prove this lemma by method of induction.

Step 1: The lemma is true for n = 3.

In the Jahangir graph $J_{1,3}$, there are four vertices v_1, v_2, v_3, v_4. The fourth vertex v_4 is adjacent to three vertices v_1, v_2, and v_3 with six edges v_1v_2, v_2v_3, v_3v_1, v_3v_4, v_1v_4, v_4v_2. Here all the four vertices v_1, v_2, v_3, v_4 are of odd degree. The total number of vertices that dominate every pair of vertices is the sum of number of edges and the summation of $\deg v_i C_2$.

Here the total number of vertices that dominates every pair of vertices is split-up in to four parts, first identify the number of edges in the graph $J_{1,3}$, choose the odd degree vertices (v_1, v_2, v_3) in the exterior area of the diagram, then take the even degree vertices and finally choose the last odd degree vertex v_4 from the interior diagram.

By theorem 3.1, the total number of vertices that dominate every pair of vertices is

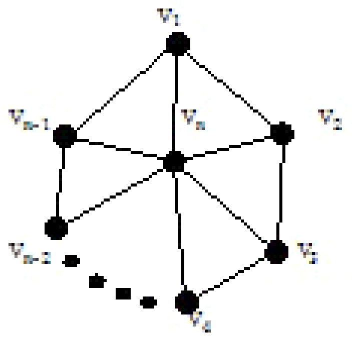

Figure 2. Jahangir Graph $J_{1,n-1}$

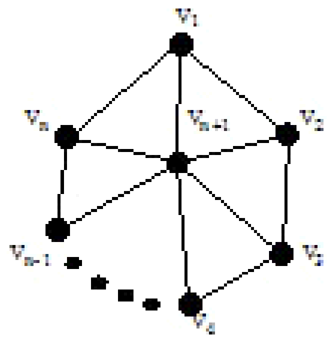

Figure 3. Jahangir Graph $J_{1,n}$

$$TVD(J_{1,3}) = q_1 + \sum_{v_i \in V} (\deg v_i C_2)$$
$$= q_1 + \sum_{oddv_i} (\deg v_i C_2) + \sum_{evenv_i} (\deg v_i C_2)$$
$$= 6 + \sum_{oddv_i} (3C_2) \left(\because \sum_{evenv_i} (\deg v_i C_2) = 0 \right)$$
$$= 6 + 3(3C_2) + 3C_2$$
$$= 18$$

For any connected, simple graph G of order n, the medium domination number of G is denoted as

$$MD(G) = \frac{TVD(G)}{nC_2}$$
$$\therefore MD(J_{1,3}) = \frac{TVD(J_{1,3})}{nC_2} = \frac{18}{4C_2}$$

Step 2: Let us consider the Jahangir graph $J_{1,n-1}$.

Here $J_{1,n-1}$ has n vertices v_1, v_2,….V_n and 2(n-1) edges v_1v_2, v_2v_3,…..,v_n v_{n-1}. In $J_{1,n-1}$ there are (n-1) vertices of odd degree with one additional vertex v_n of degree (n-1).

Assume that the result is true for $J_{1,n-1}$.

The total number of vertices that dominates every pair of vertices is

$$TDV(J_{1,n-1}) = \frac{\{(n-1)[(n-1)+4(1+1)+1]\}}{2}$$

Step 3: Now consider the Jahangir graph $J_{1,n}$.

Here $J_{1,n}$ has n+1 vertices v_1, v_2,….V_{n+1} and 2n edges v_1v_2, v_2v_3,…..,v_n v_{n+1}. In $J_{1,n}$ there are n vertices of odd degree with one additional vertex v_{n+1} of degree n. By the graphs $J_{1,n}$ and $J_{1,n-1}$ comparisons, for every (n-1) stage to n^{th} stage certain differences has been found. So each stages the additional vertices and edges will be formed. For any two consecutive stages, say ((n-1) and n), each graph was found with one additional vertex and two edges.

In each stage there are two types of vertices. There are n vertices are of odd degree and one vertex of degree (n-1). These additional vertices and edges made an impact on the total number of vertices that dominates every pair of vertices. So we come into a conclusion that each additional vertices and edges formed at each stage will be $2 + 3C_2 + \{nC_2 - (n-1)C_2\}$.

The total number of vertices that dominates every pair of vertices is

$$TDV(J_{1,n}) = TDV(J_{1,n-1}) + 2 + 3C_2 + \{nC_2 - (n-1)C_2\}$$
$$= \left\{ \frac{(n-1)[(n-1)+4(1+1)+1]\}}{2} \right\}$$
$$+ 2 + 3C_2 + \{nC_2 - (n-1)C_2\}$$
$$= \left\{ \frac{(n-1)[n+8]}{2} \right\} + 2 + 3C_2$$
$$+ \left\{ \frac{n(n-1)}{2} - \frac{(n-1)(n-2)}{2} \right\}$$
$$= \frac{(n-1)(n+8) + 4 + 6 + n^2 - n - (n^2 - 3n + 2)}{2}$$
$$= \frac{n^2 + 9n}{2} = \frac{n[n + (8+1)]}{2}$$
$$TDV(J_{1,n}) = \frac{n[n + 4(1+1) + 1]}{2}$$

$$MD(J_{1,n}) = \frac{TDV(J_{1,n})}{(n+1)C_2} = \frac{\{n[n+4(1+1)+1]\}/2}{(n+1)C_2}$$

4.2 Lemma

The medium domination number of Jahangir graph $J_{2,n}$ is

$$MD\left(J_{2,n}\right)=\frac{\left\{n\left[n+4\left(2+1\right)+1\right]\right\}/2}{\left(n+1\right)C_2}$$

Proof: We prove this lemma by method of induction.

Step 1: The lemma is true for n = 3.

In the Jahangir graph $J_{2,3}$, there are seven vertices v_1, v_2,….. v_7 with nine edges v_1v, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_7, v_7v_1, v_7v_3, v_7v_5. In these seven vertices, four vertices v_1, v_3, v_5, v_7 are of odd degree and 3 vertices v_2, v_4, v_6 are of even degree.

The total number of vertices that dominate every pair of vertices is the sum of number of edges and the summation of $\deg v_i C_2$.

Here the total number of vertices that dominates every pair of vertices is split-up in to four parts, first identify the number of edges of the graph $J_{2,3}$, choose the odd degree vertices (v_1, v_3, v_5) in the exterior area of the diagram, then take the even degree vertices (v_2, v_4, v_6) and finally choose the last odd degree vertex v_7 from the interior diagram.

By theorem 3.4.1, the total number of vertices that dominate every pair of vertices is

$$\begin{aligned}TDV\left(J_{2,3}\right)&=q_1+\sum_{v_i\in V}\left(\deg v_iC_2\right)\\&=q_1+\sum_{odd v_i}\left(\deg v_iC_2\right)+\sum_{even v_i}\left(\deg v_iC_2\right)\\&=6+3\left(3C_2\right)+3\left(2C_2\right)+3C_2\\&=9+9+3+3\\TDV\left(J_{2,3}\right)&=24\end{aligned}$$

For any connected, simple graph G of order n, the medium domination number of G is denoted as,

$$MD\left(G\right)=\frac{TDV\left(G\right)}{nC_2}$$

$$MD\left(J_{2,3}\right)=\frac{TDV\left(J_{2,3}\right)}{nC_2}=\frac{24}{7C_2}$$

Step 2: Let us consider the Jahangir graph $J_{2,n-1}$.

Here $J_{2,n-1}$ has 2n-1 vertices v_1, v_2,….V_{2n-1} and 3(n-1) edges v_1v_2, v_2v_3,….., v_{2n-3} v_{2n-1}. In $J_{2,n-1}$ there is (n-1) vertices of odd degree and (n-1) vertices of even degree with one additional vertex v_{2n-1} of degree (n-1).

Assume that the result is true for $J_{2,n-1}$.

The total number of vertices that dominates every pair of vertices is

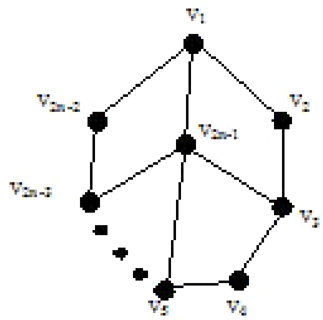

Figure 4. Jahangir Graph $J_{2,n-1}$

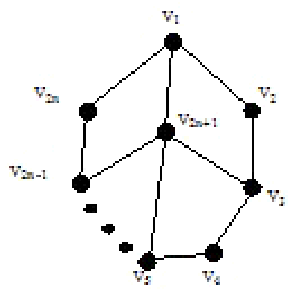

Figure 5. Jahangir Graph $J_{2,n}$

$$TDV\left(J_{2,n-1}\right)=\frac{\left\{\left(n-1\right)\left[\left(n-1\right)+4\left(2+1\right)+1\right]\right\}}{2}$$

Step 3: Now consider the Jahangir graph $J_{2,n}$

Here $J_{2,n}$ has 2n+1 vertices v_1, v_2,….V_{n+1} and (3n) edges v_1v_2, v_2v_3,…..v_{2n-1} v_{2n+1}. In $J_{2,n+1}$ there are n vertices of odd degree and n vertices of even degree with one additional vertex v_{2n+1} of degree n. By the graphs $J_{2,n}$ and $J_{2,n-1}$ comparisons, for every (n-1) stage to n^{th} stage certain differences has been found. So each stages the additional vertices and edges will be formed. For any two consecutive stages, say ((n-1) and n), each graph was found with two additional vertex and three edges.

In each stage there are two types of vertices. There are n vertices are of odd degree and one vertex of degree (n-1). These additional vertices and edges made an impact on the total number of vertices that dominates every pair of vertices. So we come in to a conclusion that each additional vertices and edges formed at each stage will be $3+3C_2+2C_2+\left\{nC_2-\left(n-1\right)C_2\right\}$.

The total number of vertices that dominates every pair of vertices is

$$TDV(J_{2,n}) = TDV(J_{2,n-1}) + 3 + 3C_2 + 2C_2$$
$$+\{nC_2 - (n-1)C_2\}$$
$$= \frac{\{(n-1)[(n-1)+4(2+1)+1]\}}{2}$$
$$+3+3+1+\{nC_2-(n-1)C_2\}$$
$$= \frac{\{(n-1)[n+12]\}}{2}+7$$
$$+\left\{\frac{n(n-1)}{2}-\frac{(n-1)(n-2)}{2}\right\}$$
$$= \frac{n^2+13n}{2} = \frac{n[n+(12+1)]}{2}$$
$$TDV(J_{2,n}) = \frac{n[n+4(2+1)+1]}{2}$$
$$MD(J_{2,n}) = \frac{TDV(J_{2,n})}{(2n+1)C_2} = \frac{\{n[n+4(2+1)+1]\}/2}{(2n+1)C_2}$$

4.2.1 Theorem

The medium domination number of Jahangir graph $J_{m,n}$ is

$$MD(J_{m,n}) = \frac{\{n[n+4(m+1)+1]\}/2}{(mn+1)C_2}, \text{ Where } n \geq 3$$

Proof: We prove this theorem by two-dimensional induction method.

Step 1: We prove this theorem is true for $J_{1,3}$.

In the Jahangir graph $J_{1,3}$, there are four vertices v_1, v_2, v_3, v_4. The fourth vertex v_4 is adjacent to three vertices v_1, v_2, and v_3 with six edges v_1v_2, v_2v_3, v_3v_1, v_3v_4, v_1v_4, v_4v_2. Here all the four vertices v_1, v_2, v_3, v_4 are of odd degree.

The total number of vertices that dominate every pair of vertices is the sum of number of edges and the summation of $\deg v_i C_2$. Here the total number of vertices that dominates every pair of vertices is split-up in to four parts, first identify the number of edges in the graph $J_{2,3}$, choose the odd degree vertices (v_1, v_2, v_3) in the exterior area of the diagram, then tame the even degree vertices and finally choose the last odd degree vertex v_4 from the interior diagram.

By theorem 3.4.1, the total number of vertices that dominate every pair of vertices is

$$TDV(J_{1,3}) = q_1 + \sum_{v_i \in V}(\deg v_i C_2)$$
$$= q_1 + \sum_{oddv_i}(\deg v_i C_2) + \sum_{evenv_i}(\deg v_i C_2)$$
$$= 6 + \sum_{oddv_i}(3C_2)\left(\because \sum_{evenv_i}(\deg v_i C_2) = 0\right)$$
$$= 6 + 3(3C_2) + 3C_2$$
$$= 18$$

For any connected, simple graph G of order n, the medium domination number of G is denoted as,

$$MD(G) = \frac{TDV(G)}{nC_2}$$
$$\therefore MD(J_{1,3}) = \frac{TDV(J_{1,3})}{nC_2} = \frac{18}{4C_2}$$

$$MD(G) = \frac{TDV(G)}{nC_2}$$
$$\therefore MD(J_{1,3}) = \frac{TDV(J_{1,3})}{nC_2} = \frac{18}{4C_2}$$

Step 2: Assume that the theorem is true for $J_{m,3.}$

Here $J_{m,3}$ has 3m+1 vertices v_1, v_2,….v_{2m-1} and 3m+3 edges. In $J_{m,3}$ there are 3 vertices of odd degree and 3(m-1) are even degree with one additional vertex v_{3m+1} of degree 3.

By theorem 3.4.1, the total number of vertices that dominate every pair of vertices is

$$TDV(J_{m,3}) = \frac{3[3+4(m+1)+1]}{2}$$

$$TDV(J_{m,3}) = \frac{3[3+4(m+1)+1]}{2}$$

Here the graph $J_{m+1,3}$ has 3m+4 vertices v_1, v_2,….v_{2m+1} and 3m+6 edges. In $J_{m+1,3}$ there are 3 vertices of odd degree and 3m are even degree with one additional vertex v_{3m+4} of degree 3.

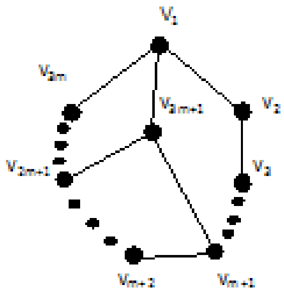

Figure 6. Jahangir Graph $J_{m,3.}$

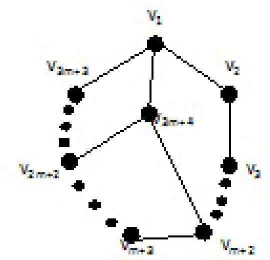

Figure 7. Jahangir Graph $J_{m+1,3.}$

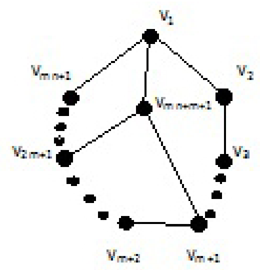

Figure 8. Jahangir Graph $J_{m,n.}$

By the graphs $J_{m,3}$ and $J_{m+1,3}$ comparisons, for every (n-1) stage to nth stage certain differences has been found. So each stages the additional vertices and edges will be formed. For any two consecutive stages, say ((n-1) and n), each graph was found with three additional vertices and 3 edges.

So we come into a conclusion that each additional vertices and edges formed at each stage will be $3+3(3$.

$$TDV\left(J_{m+1,3}\right)=TDV\left(J_{m.3}\right)+3+3\left(2C_2\right)$$

$$=\frac{3\left[3+4\left(m+1\right)+1\right]}{2}+3+3\left(2C_2\right)$$

$$=\frac{3\left[3+4m+5\right]}{2}+3+3$$

$$=\frac{9+12m+15+12}{2}=\frac{9+12m+27}{2}$$

$$=\frac{3\left[3+4m+9\right]}{2}$$

$$TDV\left(J_{m+1,3}\right)=\frac{3\left[3+4\left(m+2\right)+1\right]}{2}$$

$$MD\left(J_{m+1,3}\right)=\frac{\left\{3\left[3+4\left(m+2\right)+1\right]\right\}/2}{\left(3m+4\right)C_2}$$

$\therefore$ The theorem is true for J(m+1,3).

Step 3: Assume that the theorem is true for J(m,n), n ≥ 3.

To prove that theorem is true for J(m,n+1)

Here $J_{m,n}$ has mn+1 vertices $v_1, v_2, \ldots V_{nm+1}$ and n(m+1) edges. In $J_{m,3}$ there are n vertices of odd degree and n(m-1) are even degree with one additional vertex v_{mn+1} of degree n.

By theorem 3.4.1, the total number of vertices that dominate every pair of vertices is

$$TDV\left(J_{m,n}\right)=\frac{3\left[3+4\left(m+1\right)+1\right]}{2}$$

$$TDV\left(J_{m,n}\right)=\frac{3\left[3+4\left(m+1\right)+1\right]}{2}$$

$$\left(m+1\right)+3C_2+\left(m-1\right)\left(2C_2\right)+\left[\left(n+1\right)C_2-nC_2\right]$$

Here $J_{m,n+1}$ has mn+m+1 vertices $v_1, v_2, \ldots V_{nm+m+1}$ and (n+1)(m+1) edges . In $J_{m,n+1}$ there are (n+1) vertices of odd degree and (n+1)(m-1) are even degree with one additional vertex v_{mn+m+1} of degree (n+1). So we come into a conclusion that each additional vertices and edges formed at each stage will be $(m=1)+3C_2+(m\text{-}1)(2C_2)+[(n+1)C_2\text{-}nC_2]$

The total number of vertices that dominates every pair of vertices is

$$TDV\left(J_{m,n+1}\right)=TDV\left(J_{m,n}\right)+\left(m+1\right)+3C_2$$

$$+\left(m-1\right)\left(2C_2\right)+\left[\left(n+1\right)C_2-nC_2\right]$$

$$=\frac{n\left[n+4\left(m+1\right)+1\right]}{2}+\left(m+1\right)$$

$$+3+\left(m-1\right)+n$$

$$=\frac{n\left[n+4m+5\right]}{2}+2m+n+3$$

$$=\frac{n^2+4mn+5n+4m+2n+6}{2}$$

$$=\frac{n^2+4mn+4m+7n+6}{2}$$

$$=\frac{\left(n+1\right)\left[\left(n+1\right)+4m+4+1\right]}{2}$$

$$TDV\left(J_{m,n+1}\right)=\frac{\left(n+1\right)\left[\left(n+1\right)+4\left(m+1\right)+1\right]}{2}$$

$$MDV\left(J_{m,n+1}\right)=\frac{\left\{\left(n+1\right)\left[\left(n+1\right)+4\left(m+1\right)+1\right]\right\}/2}{\left(mn+m+1\right)C_2}$$

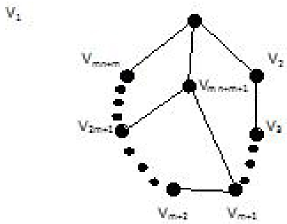

Figure 9. Jahangir Graph $J_{m,n+1.}$

∴ The theorem is true for J(m,n+1)

∴ The Theorem is true for all m,n and m ≥ 3.

∴ The medium domination number of Jahangir graph $J_{m,n}$ is

$$MD\left(J_{m,n}\right)=\frac{\left\{n\left[n+4\left(m+1\right)+1\right]\right\}/2}{\left(mn+1\right)C_2}\; where\, m\geq 3$$

$$MD\left(J_{m,n}\right)=\frac{\left\{n\left[n+4\left(m+1\right)+1\right]\right\}/2}{\left(mn+1\right)C_2}\; where\, m\geq 3$$

5. Conclusion

In graph theory, there are many stability parameters such as the connectivity number, the edge-connectivity number, the independence number, the vertex domination number and the domination number. In a graph every vertices u, v ∈ V should be privileged and it is essential to scrutinize how many vertices are proficient of dominating both of u and v. We compute the total number of vertices that dominates all pairs of vertices and evaluate the average of this value and call it "the medium domination number" of graph. In this paper, we obtained the bound of the medium domination number of Jahangir graph $J_{m,n}$.

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