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Space Debris Removal Strategies and their Feasibility

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Abstract

The need for debris mitigation is illustrated in the context of historic launch activates and operational practices. This has led to the existing space debris environment, with consequent collision flux levels that are based on detailed population and evolution models. Therefore mitigation of space debris has become a major concern for us humans lately. National space agencies have proposed many space debris mitigation measures to reduce and stabilize the predicted long term growth of space object population. Through this article we take a closer look at the mathematical analysis of three main strategies adapted to reduce and stabilize the growth of space debris.

Keywords: Debris Mitigation, Debris Removal Strategies, Space Debris, Space Environment, Orbital Mechanics

1. Proposed Space Derbies Mitigation Strategies

The aim of this study is to calculate the minimum delta-v required to remove multiple target objects that lie within the lower earth orbit altitudes. Three such strategies have been analyzed below showing their pros and cons to find the optimum solution that gives the best out puts of delta-v, time, specific-impulse and the fuel mass of the missions.

1.1 Strategy One

As per this strategy illustrated in Figure 1, initially, the spacecraft will directly go to the first selected debris from earth and collect it to deorbit the debris object back to 100km disposal altitude above earth. After disposing the first debris object the spacecraft will continue to ascend from the point of disposal back to the second selected debris object. Recollect the object to deorbit the debris back to the disposal region as of the first[1]. This collection and disposal mechanism will proceed back and forth until all the selected debris have been cleared to complete the mission.

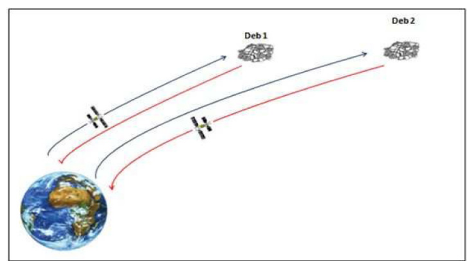

Figure 1. Schematic of Strategy one.

1.2 Strategy Two

This strategy which is demonstrated in Figure 2 is based on the notion that the spacecraft will directly go to the first selected object and attach a separate iron engine to the debris object that will direct the debris object separately back to the disposal region, which is 100 km above the earth. The spacecraft will then move to the next selected object from the current orbit and it will engage in a similar mechanism as the latter; which is to attach another iron engine that will direct the second debris object back to the disposal region. This process will be on repeat until all the selected objects have been linked to the multiple irons engines and the debris clearance procedure commence.

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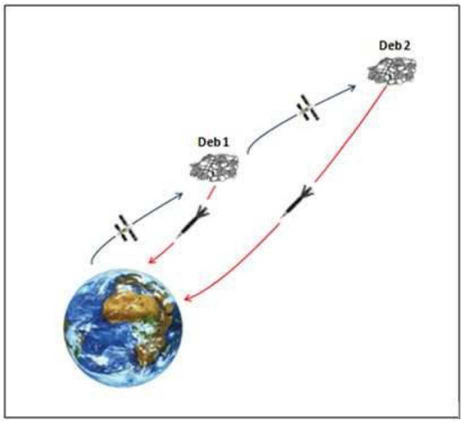

Figure 2. Schematic Strategy 2.

1.3 Strategy Three

This is a mechanism whereby the spacecraft will initially go up to an altitude of 382 km[2]. and that will be allocated as a collecting orbit which is illustrated in Figures 3 and 4; this orbit will consist of all the debris along with the space-craft prior to the pacing maneuver mechanism. The spacecraft will then maneuver itself to the first selected object and collect it to deorbit the debris objects to the collection orbit. The spacecraft will release the object on the collection orbit and proceed on to the second selected object to capture it and deorbit it back to the collection orbit, where the spacecraft will release the second debris object with a gap of an angle of 13 degrees to the first debris object from the center of the orbit. The spacecraft will then move to the next selected object and continue to put the next selected debris objects back to the collecting orbit with a gap of equal angles of 13 degrees. Finally, the spacecraft will maneuver itself to the collecting orbit keeping an angle of 13 degrees to the final object added to the collection orbit. The spacecraft will perform pacing maneuver to deorbit each object to the disposal region approximately 100 km above Earth.

1.4 Deciding the Collection Orbit for Strategy Three

When selecting the collection orbit there are multiple factors that have to be pre analyzed prior to making the final decision, as the collection orbit is the focal point of disposal of the chosen debris. It must be certified that the chosen collection orbit is free from clutter and is not congested with space particles as the omnipresence of space debris will result in accidental collisions of the existing debris, current operational satellites and the disposed debris. It should also be noted that an altitude of 800km to 1000km will not be suitable for this mission due to its high congestion of debris within the region[3].Also it should be noted those regions below 300kms will also not befitting due to the high resistive forces the spacecraft will have to succumb to when performing the pacing maneuver mechanism.

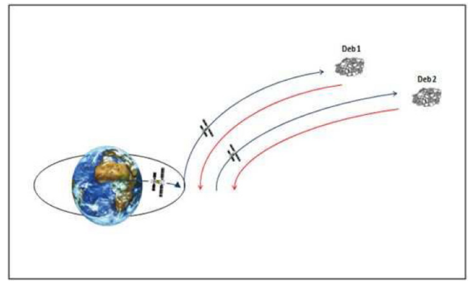

Figure 3. Schematic of Strategy three.

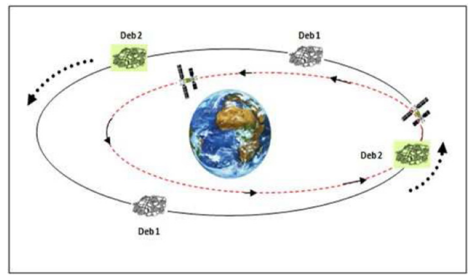

Figure 4. Schematic of strategy three with the pacing maneuver.

The collection orbit requires an area that has no other tractable object, which is large enough to cause future collisions in debris and space-crafts. Also due to the pacing maneuver its best to choose an orbit where the disposal region will also be within the pacing maneuver orbit so that the spacecraft will be saving on impulse, delta-v and fuel mass on the disposing process for the debris. Most of the LEO operational satellites are in the region of between 400km to 600km of altitude[4]. After a few calculations[5] it was deduced that to be within criteria and to have the safest orbit for the collection - 382km altitude is the best orbit. When performing the pacing maneuver at this altitude there is a region that goes below the level of 100km but above 90 km altitude that gives the required time and perfect place to dispose the debris objects with minimum external resistance (atmospheric drag, solar activity, etc.) acting on the spacecraft.

1.5 Concluding Remarks on Strategy One, Two and Three

Henceforth, from the three strategies noted above, only the first and the third strategies have been analyzed in mathematical detail. The second strategy has been omitted from here due to political and financial constraints. It can be justly stated that strategy two is complex, costly, and unsafe and invites unwanted political interferences. A thorough mathematical analysis will be undertaken to determine which of the two chosen strategies, i.e. one or two, will give out the optimum delta-v, specific impulse, fuel mass and time of the mission to understand a sequence of missions to carry forward, for disposing the space junk in the future.

2. Plane Change Maneuver Methodologies and Option

2.1 Orbit Altitude Changes

In-plane maneuver changes energy and size of an orbit generally from parking orbit (lower altitude) to a higher altitude which is the mission altitude in this case the orbit that has the selected debris object such could be geosynchronous. Since the objective orbit and the current orbit have no intersection point, the maneuvering requires a transfer orbit. Figure 5 represent a Hohmann transfer orbit. This situation, the ellipse is tangent to both final and initial orbits at the apogee and the perigee of the transferring orbit. Most fuel efficient transfer mechanism would be the Hohmann transfer and here the orbits are tangential where it gives collinear velocity vectors. Hohmann transfer is best for the transfers between two coplanar and circular orbits.

The changing velocity delta-v is applied directly in the direction of motion when transferring from the smaller to a larger orbit, and when larger orbit to a smaller orbit transformation, the change of velocity delta-v is in the opposite direction of motion.

Note that the final total delta-v for the transferring process is the sum of the velocity changes at the apogee and the perigee of the transfer ellipse. Difference between the magnitudes of the velocities in each orbit is the velocity change (delta-v) since the collinear property of the velocity vectors. R_A and R_B is known in this case since we collect the information from the satellite situation report to find the following values to compute the total delta-v

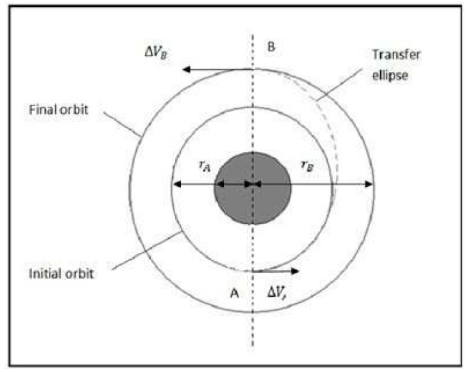

Figure 5. Vary of velocities with orbital altitude changes.

Equation (a) $$a_{tx} = \frac{r_A + r_B}{2}$$

Equation (b) $$V_{iA} = \sqrt{\frac{GM}{r_A}}$$

Equation (c) $$V_{fB} = \sqrt{\frac{GM}{r_B}}$$

Equation (d) $$V_{txA} = \sqrt{GM\left(\frac{2}{r_A} - \frac{1}{a_{tx}}\right)}$$

Equation (e) $$V_{txB} = \sqrt{GM\left(\frac{2}{r_B} - \frac{1}{a_{tx}}\right)}$$

Equation (f) $$\Delta V = V_{txA} - V_{iA}$$

Equation (g) $$\Delta V_B = V_{fB} - V_{txB}$$

Equation (h) $$\Delta V_T = \Delta V_A + \Delta V_B$$

Note that equations b and c are from the same equation, and equations d and e are derived from the same equation[6].

Using electrical propulsion to produce a constant low-thrust burn will result in spiral transfer where it is an option for changing the size of an orbit by simply taking the change of velocities of the circular velocities of the two orbits.

$$\Delta V = V_2 - V_1$$

2.2 Orbit Plane Changes

Inclination changes such to changing the orientation of a satellite's orbital plane the direction of the velocity vector must be changed. Delta-v component must be perpendicular to the orbital plane for this maneuver which can is displayed in Figure 6. This is so that it's perpendicular to the initial velocity vector. "Simple plane change" is

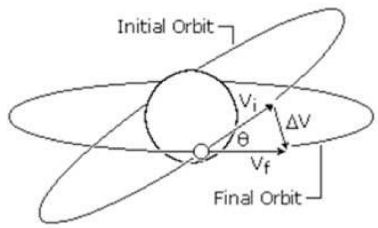

Figure 6. Orbital Plane changes.

the name given for the maneuver if the size of the orbit remains constant. Using the cosine laws the required change in velocities can be found. For a situation where the final and the initial (before and after burn) velocities are equal the delta-v expression can be simplified as below

$$\Delta V = 2V_i \sin\left(\frac{\theta}{2}\right)$$

where, V_i is the velocity before and after the burn, and θ is the angle change required.

From above if we make the angel at 60 the result will be such that the change in required velocity is equal to the current velocity. Resulting propellant consumptions are very high for a small change in velocity for a plane change. To keep this at minimum levels the plane change should be carried out at the point where the velocity of the satellite is minimum, for an elliptical orbit which will be the apogee point. In some scenario's it could be beneficial to boost the satellite in to a higher orbit and then at the apogee change the orbit plane and return the satellite to its initial orbit.

Both plane and the size of the orbit require changes in typical orbital transfer. Such could be at low altitude zero-inclination orbit at geosynchronous altitude transfer from an inclined parking orbit. A Hohmann transfer could be done to change the size of the orbit and to make the orbit equatorial a simple plane change could be carried out which is a two-step transferring mechanism.

Combining the plane change at the apogee of the transfer orbit with the tangential burn is a more efficient mechanism in term of delta-v, both directions and magnitude of the velocity vectors must be changed, using cosine rule the change in velocity can be found.

$$\Delta V = \sqrt{V_i^2 + V_f^2 - 2V_i V_f \cos\theta}$$

where, V_i, V_f and θ theta are initial velocity, final velocity and angle change required respectively.

From the above equation we can note that with the minimum as almost no effect of delta-v or propellant a small altitude change can be combined with a small plane change[7]. In general situations for a geosynchronous transfer higher plane changes are done at apogee and small plane changes are done at the perigee.

There is also another maneuver using three burns. 1st burn is to place the s/c in to a transfer orbit with a high apogee point that of the final orbit with a coplanar maneuver. Once the s/c reaches the transfer orbit apogee point combined plan change maneuver is kicked in to action. This action will cause the s.c to place itself in a 2nd coplanar transfer orbit with the final orbit. In this situation the perigee altitudes of both the final and the transfer orbit will be the same. At the point the space craft reaches the perigee of the 2nd transfer orbit to place the space craft on to the final orbit, another coplanar maneuver is taken place. This is a very time consuming maneuvering method but it could save a significant amount of propellant when it is viewed on delta-v perspective. meanwhile the time window must be taken in to account when analyzing due to complications due to atmospheric occurrence in space.

The magnitude of the angel change (difference), between the final and initial inclination is directly the value of a plane change used to modify the inclination only. Same descending and ascending nodes are shared for the final and the initial orbits. At one of these two nodes the S/C can do the plane change maneuver for the required mission aim.

2.3 First Maneuver Option

Once the debris object has been selected, its altitude and plane inclination will be allocated accordingly. The spacecraft could lie on a different orbit or could be on Earth. Space craft will ascend its current location to that of the debris altitude. Upon reaching the desired altitude of the chosen debris object, the spacecraft will perform an inclination changing maneuver to the inclination of the debris object and ensure that the spacecraft and the debris will be on the same altitude, right ascension and inclination. For most of the high congested orbits that a spacecraft such as the late space shuttle, altitude to the lower earth orbits are small in terms of the earth radii.beyond geosynchronous distance or even out side the best transferring mechanism for circular coplanar orbits is the Hohmann transfer method. The total velocity increment required for the transfer, The semi-major axis of the transfer, the velocity increments at the ends of the transfer orbit and the semi-major axis of the transfer orbit are the information what we need to do the calculations for the circular coplanar orbital transfers.

Example: noted below is debris in a circular orbit, which is in an altitude of 700km with a 28 degree inclination and a spacecraft that lies in an initial orbit with an altitude of 382km.

The Hohmann transfer[8] is used with the following assumptions:

1. $R_E = 6378Km$ (radius of Earth)
2. $\mu_E = 398600 km^3/s^2$ (gravitational constant of Earth)
3. Assume that all of the orbits are circular and coplanar
4. Assume that there is no decay in orbital height on debris due to atmospheric drag and level of solar activity during mission operations

Please note all calculations have been adopted from: Howard Curtis - Orbital Mechanics for Engineering Students (2005).

Maneuver Option Calculations

Referring to Figure 7, circular orbit speed for orbit at 700Km [7].

equation (1)

$$V_{DEB1} = \sqrt{\frac{\mu}{R_E + 700}} \tag{1}$$

where,

$\mu = 39800$ and $R_E = 6378$Km

Therefore,

$$V_{DEB1} = \sqrt{\frac{398600}{7078}}$$

$V_{DEB1} = 7.504354957$Km/s

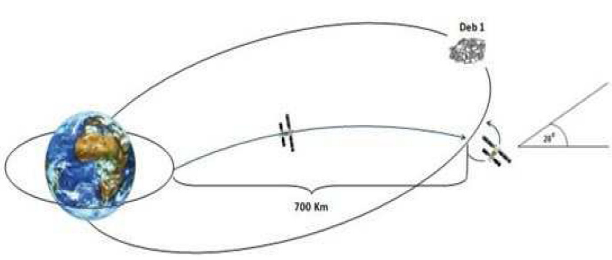

Figure 7. Schematic of option one calculations.

Rotational velocity

When the space craft is at $\theta = 28^\circ$ and 700 Km altitude[7].

equation (2)

$$\Delta V_{D\theta} = 2V_{DEB1}\sin\left(\frac{\theta}{2}\right) \tag{2}$$

$$\Delta V_{D\theta} = 2\times 7.504354957\sin\left(\frac{\theta}{2}\right)$$

$$\Delta V_{D\theta} = 3.630935553 Km/s$$

Delta-v to an altitude of 700km [7]:

where,

$R_{col} = 6760$Km and $R_{DEB1} = 7078$Km

$$\Delta V_{DEB1} = \frac{\mu}{R_{DEB1}}\left(1 - \sqrt{\frac{2R_{col}}{R_{col} + R_{DEB1}}}\right) \tag{3}$$

$$\Delta V_{DEB1} = \sqrt{\frac{398600}{7078}}\left(1 - \sqrt{\frac{2\times 6760}{6760 + 7078}}\right)$$

$$\Delta V_{DEB1} = 0.08672693323\ Km/s$$

$$Total\,\Delta V = altitude\,\Delta V + rotational\,\Delta V$$

$$\Delta V_{TO1} = \Delta V_{DEB1} + \Delta V_{D\theta} \tag{4}$$

$$\Delta V_{TO1} = 3.717662486\ Km/s$$

2.4 Second Maneuver Option

Once the debris object has been selected, its altitude and plane inclination will be allocated accordingly and in this scenario unlike in the previous option, the spacecraft will have to lie in a different orbit before the plane change maneuver. In this option we explore a situation whereby the spacecraft will lie below of the altitude of the debris. The inclination change will be done on that lower altitude and thereafter the spacecrafts orbital plane and the debris orbital plane will lie parallel to each other. After the inclination the spacecraft will be increased to the same altitude as the debris altitude and thus it will be ensured that the spacecraft and debris lie side by side[9].

Example: consider debris in a circular orbit, which is in an altitude of 700km with a 28 degree inclination and a spacecraft that lies in an initial orbit with an altitude of 382km.

Referring to Figure 8, circular orbit speed at altitude 382Km which is a parking orbit.

From equation (1) $V_{col} = \sqrt{\frac{\mu}{R_E + 382}}$ and

where, $R_{col} = 6760$ Km

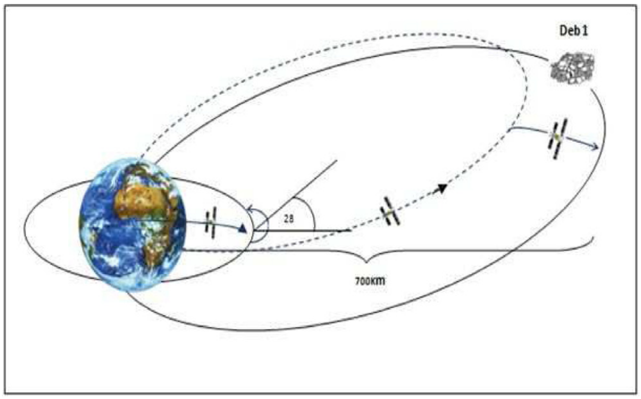

Figure 8. Schematic of option two calculations.

V_{col} = 7.678834354 Km/s

Rotational velocity at the parking orbit for an angle 28°

From equation (2)

$$\Delta V_{col\theta} = 2V_{col}\sin\left(\frac{\theta}{2}\right)$$

$$\Delta V_{col\theta} = 2\times 7.678834354\ \sin\left(\frac{28}{2}\right)$$

$$\Delta V_{col\theta} = 3.715356326\ Km/s$$

ΔV at 700 Km altitude, from equation (3)

ΔV_{DEB1} = 0, 08672693323Km/s

Total ΔV for second maneuver option, from equation (4)

$$\Delta V_{TO2} = \Delta V_{DEB1} + \Delta V_{COL\theta}$$
$$\Delta V_{TO2} = 3.802083259$$

Comparison of maneuver options

$$\Delta T_{TO1} = 3.717662486\ Km/s$$
$$\Delta T_{TO2} = 3.802083259\ Km/s$$

First maneuver option is 8.4420773 % less than the second maneuver option.

2.5 Concluding Remarks on Maneuver One and Two

From the above two maneuver options it is deduced that the first maneuver option has the least total delta-v. This option will be taken forward for further calculations for strategies one and three mathematical analysis.

3. Strategy One Evaluation

Consider the following example mission for removal of two debris objects with the assumptions that the two debris objects have the same right ascension, inclination and mass. No other external forces will be taken into calculations on the spacecraft.

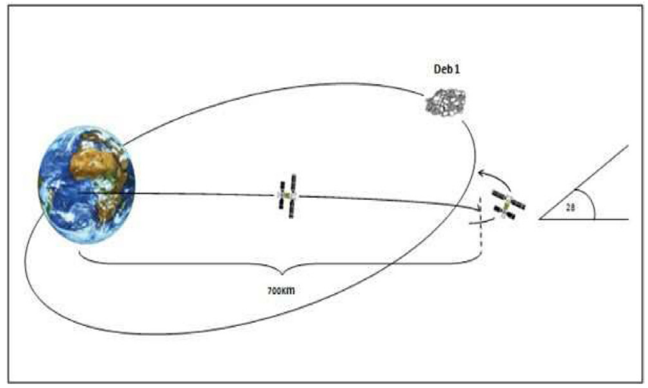

Figure 9. Schematic of strategy one calculations.

Hereby we are considering debris in a circular orbit, which is in an altitude of 700km with a 28 degree inclination and a spacecraft that launches from earth as shown in the Figure 9.

ΔV from earth to initial selected debris object

From equation (3)

$$\Delta V_{DEB1} = \frac{\mu}{R_{DEB1}}\left(1-\sqrt{\frac{2R_E}{R_{DEB1}+R_E}}\right)$$

where,

RDEB1 = 7078Km and RE = 6376Km

Therefore

$$\Delta V_{DEB1} = \sqrt{\frac{398600}{7078}}\left(1-\sqrt{\frac{2\times 6378}{7078+6376}}\right)$$

$$\Delta V_{DEB1} = 0.1978003554\ Km/s$$

Time taken for the spacecraft to reach the initial debris orbit DEB1:

$$T_{DEB1} = \frac{\pi}{\sqrt{\mu}}(a)^{\frac{3}{2}} \tag{5}$$

where,

a = 7078Km

$$T_{DEB1} = \frac{\pi}{\sqrt{398600}}(7078)^{\frac{3}{2}}$$

$$T_{DEB1} = 2963.105148\ s$$

From equation (1) circular speed of orbit at 700Km altitude.

$$V_{DEB1} = \sqrt{\frac{\mu}{R_E+700}}$$

$$V_{DEB1} = 7.50435497\ Km/s$$

From equation (2) rotational velocity of the space craft at 700Km at $\theta = 28^{\circ}$

$$\Delta V_{D1\theta} = 2V_{DEB1}\sin\left(\frac{\theta}{2}\right)$$

$$\Delta V_{D1\theta} = 3.630935553\ Km/s$$

Consider debris (Rocket Body) approximately 3000kg and a spacecraft of approximately 1000kg [7].

Specific impulse of the spacecraft from the earth to the first selected debris DEB1:

$$I_{SPD1} = m\Delta V \tag{6}$$

$$I_{SPD1} = m\ (\Delta V_{DEB1} + \Delta V_{D1\theta})$$

$$I_{SPD1} = 1000\ (0.1978003554 + 3.630935553)$$

$$I_{SPD1} = 3828.735908\ N\ s$$

De-orbiting the DEB1 to 100Km altitude (natural burning disposal region)

$$\Delta V_{DIS1} = \sqrt{\frac{\mu}{R_{DIS}}}\left(\frac{2R_{DEB1}}{R_{DIS}+R_{DEB1}} - 1\right) \tag{7}$$

where,

$R_{DIS} = 6478$Km

$$\Delta V_{DIS1} = \sqrt{\frac{398600}{6478}}\left(\frac{2\times 7078}{6478+7078} - 1\right)$$

$$\Delta V_{DIS1} = 0.1717157636\ Km/s$$

Time taken by space craft from orbit DEB1 to disposal region

$$T_{DIS1} = \frac{\pi}{\sqrt{\mu}}(a)^{\frac{3}{2}}$$

From equation (5)

where,

$$a = \frac{7078+6478}{2}$$

Therefore

$T_{DIS1} = 2776.729487\ s$

Specific impulse of the space craft for the de-orbiting process

From equation (6)

$$I_{SPDIS1} = m_{(S.C+DEB)}\Delta V_{DIS}$$

$$I_{SPDIS1} = 4000 \times 0.1717152630$$

$$I_{SPDIS1} = 686.8630544\ N\ s$$

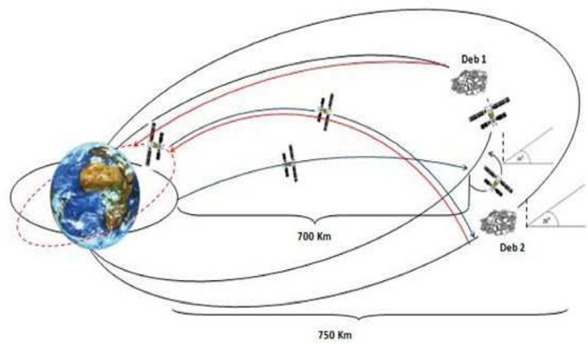

Figure 10. Schematic of strategy one calculations.

We hereby consider the second debris object at an altitude of 750 km above earth which lies on the same plane inclination as of the deorbited debris.

Referring to Figure 10, consider a debris object at 750Km altitude as the second retrieval selection.

From equation (3)

$$\Delta V_{DEB2} = \frac{\mu}{R_{DEB2}}\left(1 - \sqrt{\frac{2R_{DIS}}{R_{DIS}+R_{DEB2}}}\right)$$

where,

$R_{DEB2} = 7178$ Km, R_{DIS} = radius of disposal altitude = 6478 Km

Therefore,

$$\Delta V_{DEB2} = \sqrt{\frac{398600}{7178}}\left(1 - \sqrt{\frac{2\times 6478}{6478+7178}}\right)$$

$$\Delta V_{DEB2} = 0.1808090053\ Km/s$$

Time taken for space craft to reach DEB2 from disposal altitude (R_{DIS})

From equation (5)

$$T_{DEB2} = \frac{\pi}{\sqrt{\mu}}\left(\frac{R_{DIS}+R_{DEB2}}{2}\right)^{\frac{3}{2}}$$

$$T_{DEB2} = \frac{\pi}{\sqrt{398600}}\left(\frac{6478+7178}{2}\right)^{\frac{3}{2}}$$

$$T_{DEB2} = 2792.106193\ s$$

ΔV for the angle 28° should not be reconsidered since the space craft is already in the inclination from the previous maneuver.

From equation (6)

Specific impulse to the next debris object at 750 Km altitude.

$I_{SPD2} = m\Delta_{VDEB2}$

Total mission impulse for the selected two objects to be deorbited

$$I_{TM1} = I_{SPD1} + I_{SPDIS1} + I_{SPD2} + I_{SPDIS2} \tag{8}$$

Assuming that the debris object m =1000kg

$I_{SPD2} = 1000 \times 0.1808090053$

$I_{SPD2} = 180.8090053\ N\,s$

De-orbiting delta-v to an altitude of 100km (the disposal altitude) from the second debris object DEB2:

$$\Delta_{VDIS2} = \sqrt{\frac{\mu}{R_{DIS}}}\left(\sqrt{\frac{2R_{DEB2}}{R_{DIS} + R_{DEB2}}} - 1\right) \tag{9}$$

$$\Delta_{VDIS2} = \sqrt{\frac{398600}{6478}}\left(\sqrt{\frac{2 \times 7178}{6418 + 7178}} - 1\right)$$

$$\Delta_{VDIS2} = 0.1851845322\ Km/s$$

Time to DEB2 from disposal region is similar to the time from DEB to the disposal altitude

$T_{DEB2} = T_{DIS2} = 2792.106193\ s$

Specific impulse for the de-orbiting process from DEB2 to disposal region.

From, equation (6)

$I_{SPDIS1} = m_{S.C+DEA}\Delta V_{DIS2}$

$I_{SPDIS1} = 4000 \times 0.1851845322$

$I_{SPDIS1} = 240.7381329\ Ns$

Total mission impulse for the selected two objects to be deorbited

From, equation (8)

$I_{TM1} = I_{SPD1} + I_{SPDIS1} + I_{SPD2} + I_{SPDIS2}$

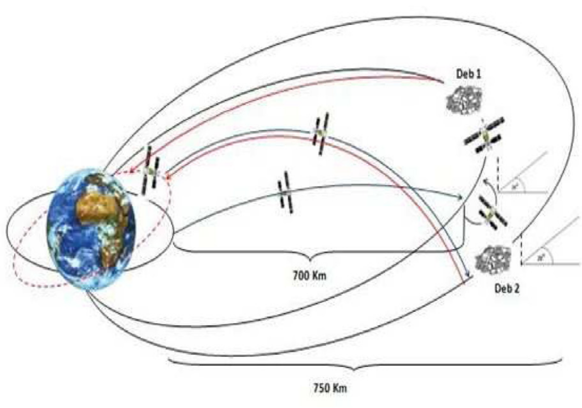

Figure 11. Schematic of strategy three calculations.

I_{TM1} = 3828.735908 + 686.8630544 + 180.8090053 + 740.7381329

$I_{TM1} = 5437.146101\ N\,s$

Total time for mission completion

$T_{TM1} = T_{DEB1} + T_{DIS1} + T_{DEB2} + T_{DIS2}$

T_{TM1} = 2963.105148 + 2776.729482 + 2792.106193 + 2791.106193

T_{TM1} = 11324.04202 s

T_{TM1} = 3.145568617 hrs

Total ΔV for the first strategy

$\Delta V_{T1} = \Delta V_{DEB1} + \Delta V_{D1\theta} + \Delta V_{DIS1} + \Delta V_{DEB2} + \Delta V_{DIS2}$

ΔV_{T1} = 0.1978003554 + 3.630935553 + 0.1717157636 + 0.1851845322 + 0.1808090053

ΔV_{T1} = 4.36644521 Km/s

4. Strategy Three Evaluation

For this calculation we consider debris in a circular orbit, which is in an altitude of 700km with a 28 degree inclination and a spacecraft that lies in an initial orbit with an altitude of 382km as shown in Figure 11.

ΔV from earth launch side to initial selected collection orbit

$$\Delta V_{col} = \sqrt{\frac{\mu}{R_{col}}}\left(1 - \sqrt{\frac{2R_E}{R_E + R_{col}}}\right)$$

where, R_E = 6378Km , R_{col} = 6760Km , μ = 398600Km

$$\Delta V_{col} = \sqrt{\frac{398600}{6760}}\left(1 - \sqrt{\frac{2 \times 6378}{6378 + 6760}}\right)$$

$$\Delta V_{col} = 0.1124582392\ Km/s$$

Time taken by space craft to reach the collection orbit from earth.

$$T_{col} = \frac{\pi}{\sqrt{\mu}}(a)^{\frac{3}{2}}$$

$$T_{col} = \frac{\pi}{\sqrt{398600}}(6760)^{\frac{3}{2}}$$

$$T_{col} = 2765.675799\ S$$

ΔV from collection orbit to the initial debris object selected at 700 Km.

Assuming $R_{DEB1} = 7078Km$

$$\Delta V_{DEB1} = \sqrt{\frac{\mu}{R_{DEB1}}}\left(1 - \sqrt{\frac{2R_{col}}{R_{col} + R_{DEB1}}}\right)$$

$$\Delta V_{DEB1} = \sqrt{\frac{398600}{7078}}\left(1 - \sqrt{\frac{2 \times 6760}{6760 + 7078}}\right)$$

$$\Delta V_{DEB1} = 0.08672693323\ Km/s$$

Time taken for the space craft to reach the first selected DEB1 orbit from the collection orbit.

$$T_{DEB1} = \frac{\pi}{\sqrt{\mu}}\left(\frac{R_{col} + R_{DEB1}}{2}\right)^{\frac{3}{2}}$$

$$T_{DEB1} = \frac{\pi}{\sqrt{398600}}\left(\frac{6760 + 7078}{2}\right)^{\frac{3}{2}}$$

$$T_{DEB1} = 2863.823321\ s$$

Velocity of the DEB1 object which is the same as strategy one circular speed at 700 Km altitude

$$V_{DEB1} = 7.50435497\ Km/s$$

Rotational velocity of the space craft at 700 Km altitude is the same as strategy 1

$$V_{DI\theta} = 3.630935553$$

Taking the mass of the debris object as 3000Kg and space craft as 1000Kg

Specific impulse of the space craft from earth to DEB1 orbit

From equation (6)

$$I_{SPD1} = m\Delta V$$

$$I_{SPD1} = m\ (\Delta V_{col} + \Delta V_{DEB1} + \Delta V_{DI\theta})$$

$$I_{SPD1} = 1000\ (0.1124582392 + 0.08677693323 + 3.630935553)$$

$$I_{SPD1} = 3830.120725\ N\ s$$

De-orbiting delta v back to the collection orbit:

From equation (9)

$$\Delta V_{DE1} = \sqrt{\frac{\mu}{R_{col}}}\left(\sqrt{\frac{2R_{DEB1}}{R_{col} + R_{DEB1}}} - 1\right)$$

$$\Delta V_{DE1} = \sqrt{\frac{398600}{6760}}\left(\sqrt{\frac{2 \times 7078}{6760 + 7078}} - 1\right)$$

$$\Delta V_{DE1} = 0.08772942535\ Km/s$$

The time taken from collection orbit to the debris object is the same for the returning process

$$T_{DEB1} = T_{r1} = 2863.823321\ s$$

Specific impulse of the space craft for the de-orbiting process

$$I_{SPC1} = m_{D+S.C}\ \Delta V_{DEB1}$$

$$I_{SPC1} = 4000\ (0.08772942535)$$

$$I_{SPC1} = 350.9177014\ N\ s$$

We hereby consider the second debris object at an altitude of 750km above earth, which lies on the same plane inclination as of the deorbited debris as shown in Figure 12.

Delta-v to the second debris object

Similar to the strategy one example we take an object that is in the altitude 750 Km above earth.

$$\Delta V_{DEB2} = \sqrt{\frac{\mu}{R_{DEB2}}}\left(1 - \sqrt{\frac{2R_{col}}{R_{col} + R_{DEB2}}}\right)$$

Assuming $R_{DEB2} = 7128 Km$

$$\Delta V_{DEB2} = \sqrt{\frac{398600}{7128}}\left(1 - \sqrt{\frac{2 \times 6760}{6760 + 7128}}\right)$$

$$\Delta V_{DEB2} = 0.09973988959\ Km/s$$

The time taken to reach the second debris object DEB2 from collection orbit:

$$T_{DEB2} = \frac{\pi}{\sqrt{\mu}}\left(\frac{R_{col} + R_{DEB2}}{2}\right)^{\frac{3}{2}}$$

$$T_{DEB2} = \frac{\pi}{\sqrt{398600}}\left(\frac{6760 + 7128}{2}\right)^{\frac{3}{2}}$$

$$T_{DEB2} = 2879.35885\ s$$

Specific impulse to the second debris object:

$$I_{SPD2} = m\Delta V_{DEB2}$$

$$I_{SPD2} = 1000 \times 0.09973988959$$

$$I_{SPD2} = 99.73988959\ N\ s$$

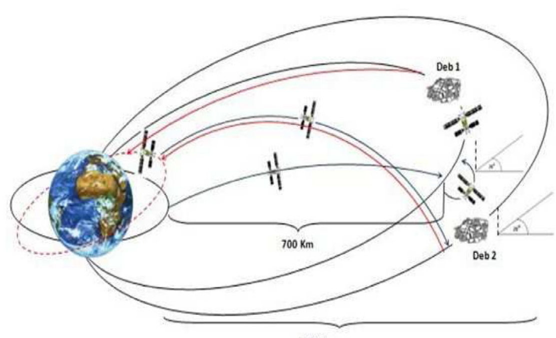

Figure 12. Schematic of strategy three calculations.

De-orbiting delta v back to the collection orbit from DEB2:

$$\Delta V_{DE2} = \sqrt{\frac{\mu}{R_{col}}}\left(\sqrt{\frac{2R_{DEB2}}{R_{col} + R_{DEB2}}} - 1\right)$$

$$\Delta V_{DE2} = \sqrt{\frac{398600}{6760}}\left(\sqrt{\frac{2\times 7128}{6760 + 7128}} - 1\right)$$

$$\Delta V_{DE2} = 0.1010705519 Km/s$$

The de-orbiting time for the spacecraft will be the same as to reach that debris orbit from collection orbit:

$$T_{DEB2} = T_{r2} = 2879.35885\ s$$

Specific impulse of the de-orbiting process

$$I_{SPC2} = m_{S.C+DEB2}\,\Delta V_{DE2}$$

$$I_{SPC2} = 4000 \times 0.1010705519$$

$$I_{SPC2} = 404.2822076\ N\ s$$

Total impulse for the two debris object up to the collection orbit before the disposal process

$$I_{IM2} = I_{SPD1} + I_{SPC1} + I_{SPD2} + I_{SPC2}$$

$$I_{IM2} = 3830.120725 + 350.9177014 + 99.73988959 + 404.2822076$$

$$I_{IM2} = 4685.060524\ \text{N s}$$

When analyzing strategy three the question arises as to when the spacecraft is required to do the burn to maneuver itself to the collection orbit with an angle of 13 degrees to each debris. A calculation that has been worked out for a spacecraft that is at an altitude of 700km above the Earth and one which will maneuver itself down to the collect orbit which is at an altitude of 382km has been detailed below as an example.

Calculation of the waiting time for the spacecraft on each orbit prior to the return to the collection orbit so that the debris; existing and new, will at all times be at an angle of 13 degrees to the center of the Earth.

Waiting time for the spacecraft on each orbit

Since we are considering the collection orbit in strategy 3 it is important to calculate the waiting time on each orbit in order to ensure no collisions will occur during the de-orbiting process.

Waiting time for the space craft at 700Km altitude

From equation (2)

n_1 and n_2 are mean motions (angular velocity)

$$T_{W1} = \frac{-2\phi \pm 2\pi N}{n_2 - n_1}$$

where,

$$n_1 = \frac{2\pi}{T},\ n_2 = \frac{2\pi}{T_{PDEB1}}$$

Time period of the collection orbit

From equation (1)

$$T_{PC} = 5531.351598\ s$$

From equation (1)

$$T_{PD1} = \frac{2\pi}{\sqrt{\mu}} a_{DEB1}^{\frac{3}{2}}$$

$$T_{PD1} = \frac{2\pi}{\sqrt{398600}} 7078^{\frac{3}{2}}$$

$$T_{PD1} = 5926.210295\ s$$

There for the waiting time at an orbit 700Km

$$n_1 = \frac{2\pi}{T_{PD1}}$$

$$n_1 = \frac{2\pi}{5531.351598}$$

$$n_1 = 1.135922242\times 10^{-3}$$

$$n_2 = \frac{2\pi}{T_{PD1}}$$

$$n_2 = \frac{2\pi}{5926.210295}$$

$$n_1 = 1.060236643\times 10^{-3}$$

From equation (2)

$$T_{wait1} = \frac{-2\left(\frac{13\pi}{180}\right) - 2\pi N}{(1.060236643 - 1.135922242)\times 10^{-3}}$$

where,

$$n_2 < n_1$$

For the above N, figures from 0 and above should be substituted until the waiting time is a positive.

When N=0

$$T_{W1} = \frac{-2\left(\frac{13\pi}{180}\right)}{-7.5685599\times 10^{-5}}$$

$$T_{W1} = 5995.666435\ s$$

Waiting time for an altitude of 750Km orbit

$$n_3 = \frac{2\pi}{T_{PD2}}$$
$$n_3 = \frac{2\pi}{5989.116454}$$
$$n_3 = 1.04910054 \times 10^{-3}$$

where,

$$T_{PD2} = \frac{2\pi}{\sqrt{398600}} 7128^{\frac{3}{2}}$$
$$T_{PD2} = 5989.716454 \text{ s}$$

Therefore,

$n_1 > n_3$

From equation (2)

$$T_{W2} = \frac{-2\left(\frac{13\pi}{180}\right) - 2\pi N}{(1.04910054 - 1.135922242) \times 10^{-3}}$$

where, N=0

$$T_{W2} = \frac{-2\left(\frac{13\pi}{180}\right)}{-8.6821702 \times 10^{-5}}$$
$$T_{W2} = 5226.637984\ s$$

Total time for retrieval and release of debris to the collection orbit

$$T_{T2} = T_{COL} + T_{DEB1} + T_{r1} + T_{W1} + T_{DEB2} + T_{r2} + T_{W2}$$

T_{T2} = 2765.675799 + 2×2863.823321 + 5995.666435 + 2×2879.35885 + 5226.637984

$$T_{T2} = 25474.34456\ s$$
$$T_{T2} = 7.076206822\ Hours$$

Pacing maneuver calculations for the collection orbit

The calculations noted below are for the spacecraft to capture debris objects that is at an angle of 13 degrees.

Semi major axis of the collection orbit

$$a_{col} = \left(\frac{apogee + perigee}{2}\right)$$
$$a_{col} = \left(\frac{6800 + 6720}{2}\right)$$
$$a_{col} = 6760\,Km$$

Time period of the collection orbit at 382Km altitude above earth

From equation (1)

$$T_{PDC} = \frac{2\pi}{\sqrt{\mu}} a_{col}^{\frac{3}{2}}$$
$$T_{PDC} = \frac{2\pi}{\sqrt{398600}} \times (6760)^{\frac{3}{2}}$$
$$T_{PDC} = 5531.351598\ s$$

Time period of the pacing maneuver orbit

$$T_{pm} = \frac{347}{360} \times T_{pc}$$
$$T_{pm} = 5331.608346$$

From above

$$\frac{T_{pm}}{T_{pc}} = \frac{a_{pm}^{\frac{3}{2}}}{a_{col}^{\frac{3}{2}}} = \frac{347}{360}$$
$$\left[\frac{a_{pm}}{a_{col}}\right]^{\frac{3}{2}} = \frac{347}{360}$$

Therefore the semi major axis of the pacing maneuver

$$a_{pm} = \left[\frac{347}{360}\right]^{\frac{2}{3}} \times a_{col}$$
$$a_{pm} = 6596.263742\ Km$$

Apogee or the perigee of the pacing maneuver orbit should lay on the collection orbit since no additional ΔV will be required in order to retrieve the object on disposal

$$a_{pm} = \frac{6760 + h_p}{2}$$
$$a_{pm} \times 2 - 6760 = h_p$$
$$h_p = 6432.527484\ Km$$

Velocity of the collection orbit

$$V_{col} = \frac{2\pi}{T_{pc}} a_{col}$$
$$V_{col} = 7.678834354\ Km/s$$

Velocity of the pacing maneuver orbit

$$V_{pm} = \frac{2\pi}{T_{pm}} a_{pm}$$
$$V_{pm} = 7.773554383\ Km/s$$

DeltaV required to capture the debris object at the collection orbit where the orbit intersects the space craft and the Debris object in a specific time frame .

$$\Delta V_{capture} = \Delta V_{pm} - \Delta V_{col}$$
$$\Delta V_{capture} = 0.094720029\ Km/s$$

As the debris objects lay at similar distances, which are at 13 degrees from the center of the collection orbit to each other, the capturing delta-v increase will be similar to each of the debris at that similar angel.

From the above calculations we could see the perigee (h) value lies within the disposal region; which is below the altitude of 100km above the earth. Therefore, every time the pacing maneuver takes place with a debris object, the debris object will be disposed below the 100km altitude region.

Specific impulse for both the debris in the collection orbit to be undergone the pacing maneuver

$$I_{SPTP} = 2m_{S.C+DEB}\,\Delta V_{capture}$$
$$I_{SPTP} = 2 \times 4000 \times 0.94720029$$
$$I_{SPTP} = 757.760232\ N\,s$$

Therefore the total mission impulse for strategy 3

$$I_{SPS3} = I_{TM2} + I_{SPTP}$$
$$I_{SPS3} = 4685.060524 + 757.760232$$
$$I_{SPS3} = 5442.820756\ N\,s$$

The total mission time for the strategy 3 will be the above total TT 2 plus the additional pacing maneuver times

$$T_{TM3} = T_{T1} + T_{TPM}$$
$$T_{TPM} = \text{Time period of the pacing maneuver orbit} \times \text{N}$$

Since it releases an object after every cycle we could take the time for the cycle and multiply by the amount of debris to be released.

There for in this situation the total mission time

$$T_{TM3} = T_{T1} + 2 \times T_{TPM}$$
$$T_{TM3} = 25474.34456 + 2 \times 5331.608346$$
$$T_{TM3} = 36137.56125\ s$$
$$T_{TM3} = 10.03821146\ hours$$

5. Fuel Mass Calculations

We use "Tsiolkovsky Rocket equation" along with the assumptions stated below.

Assume, g = 8.7ms^{-2}

$$\Delta V = V_{ex} \ln\left(\frac{m_o}{m_i}\right)$$

Since,

$$V_{en} = I_{sp} g$$

ΔV = change in velocity V_{ex} = specific impulse of a unit mass of the fuel

m_o = initial launchpad mass

g = standard gravity

m_i = final payload mass

Therefore,

$$\Delta V = I_{sp} g\left[\frac{m_s + m_p}{m_s}\right] \tag{10}$$

5.1 Total Fuel Mass Calculation for Strategy One

Total ΔV for the first strategy

$$\Delta_{VT1} = \Delta V_{DEB1} + \Delta V_{D1\theta} + \Delta V_{DIS1} + \Delta V_{DEB2} + \Delta V_{DIS2}$$
$$\Delta_{VT1} = 0.1978003554+3.630935553+0.1717157636+0.1851845322+0.1808090053$$
$$\Delta_{VT1} = 4.36644521\ Km/s$$

Specific impulse of strategy 1

$$I_{TM1} = 5437.146101\ N\,s$$

From equation (10)

$$\Delta V_T = I_{TM1} g \ln\left[\frac{m_{s.c} + m_f}{m_{s.c}}\right]$$
$$\frac{\Delta V_T}{I_{TM1} g} = \ln\left[\frac{m_{s.c} + m_f}{m_{s.c}}\right]$$
$$m_{s.c} e^{\frac{\Delta V_{T1}}{I_{TM1} g}} - m_{s.c} = m_f$$

Therefore,

$$m_f = 1000 \times e^{\frac{4.36644521\times10^{-3}}{5437.146101\times8.7}} - 1000$$
$$m_f = 96.70218568\ Kg$$

5.2 Fuel Mass Calculation for Strategy Three

Specific impulse of strategy 3

$$I_{SP3} = 5442.820756$$

Total ΔV for strategy 3

$$\Delta V_{T3} = \Delta V_{OP1} + \Delta V_{DEB1} + \Delta V_{D1\theta} + \Delta V_{DEB2} + \Delta V_{DE1} + \Delta V_{DE2} + N\Delta V_{capture}$$

$\Delta V_{T3} = 0.1124582392 + 0.08672693323 + 3.630935553 + 0.09973988959 + 0.08772942535 + 0.101705519 + 2 \times 0.094720029$

Therefore, $\Delta V_{T3} = 4.30810065$Km/s

From equation (10)

$$m_f = m_{s.c} e^{\frac{\Delta V_{T3}}{I_{TM3} g}} - m_{s.c}$$

$$m_f = 1000 \times e^{\frac{4.30810063 \times 10^{-3}}{5442.320756 \times 8.7}} - 1000$$

$$m_f = 95.24632545\ Kg$$

6. Conclusion

6.1 Summary of Strategies One and Three

Two strategy comparisons are illustrated in Table 1. It is eminent that strategy three has a higher specific impulse of 5.674655 compared to strategy one. Even with this high specific impulse strategy three gives a less fuel mass and total delta-v than the strategy one. Total delta-v in strategy one is 0.05834456 higher than strategy three and fuel mass needed to complete the mission is also 1.45586023 higher in strategy one compared to strategy three. As clearly stated above, the process illustrated in strategy three is a relatively longer and complicated than strategy one. Time frame of space debris mitigation missions can be carried forward as long as its within the human tolerance level to complete the mission. Therefore, even with a higher time strategy three will give out an optimum output than the first strategy. Therefore, strategy three will be taken forward for further study analysis. To deduce a total minimum delta-v using the above optimization calculations, strategy three will be repeated with the below tests for different time phases inclinations and altitudes but maintaining the selected collection orbit as the same. A computerized program has been developed to run this optimization with the below order.

1. Delta-v and the specific impulse and fuel mass required to the spacecraft to reach the collection orbit.
2. Delta-v and the specific-impulse and fuel mass required to maneuver from collection orbit to the selected targeted objects and back to the collection orbit.
3. Delta-v and the specific impulse and fuel mass for the spacecraft to perform pacing maneuver and dispose the debris.
4. Total time taken for the above three steps of the mission.
5. The waiting time for the spacecraft at each orbit to maneuver itself back to the collection orbit to maintain an angle of 13 degrees to each debris object in the collection orbit.

Table 1. Comparison of Strategy 1 and Strategy 3

	Strategy 1	Strategy 3
Delta-V (ΔV)	4.36644521	4.30810065
Impulse (I_{SP})	5437.146101	5442.820756
Mission time (T_T)	3.145568617 hours	10.03821146 hours
Fuel Mass (m_f)	96.70218568	95.24632545

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8. Appendix

Table 1. Appendix

Mathematical Notation	Denotation
a_{tx}	Semi-major axis of transfer ellipse
V_{iA}	Initial velocity at point A
V_{fB}	Final velocity at point B
V_{txA}	Velocity on transfer orbit at initial orbit (point A)
V_{txB}	Velocity on transfer orbit at final orbit (point B)
ΔV	Initial velocity change
ΔV_B	Final velocity change
ΔV_T	Total velocity change
R_E	Radius of earth
μ_E	Gravitational constant of earth approximately
V_{DEB1}	Circular orbital speed 1st debris orbit
$\Delta V_{D\theta} = \Delta V_{DI\theta}$	Rotational velocity for 1st debris orbit at angle θ
R_{col}	Radius of collection orbit
R_{DEB1}	Radius of 1st dedris object
ΔV_{TO1}	Total delta-v for the 1st manuver option
V_{col}	Circular orbit speed for the collection orbit
$\Delta V_{col\theta}$	Rotational velocity for collection orbit for an angle θ
ΔV_{DEB1}	ΔV from earth to initial selected debris object
ΔV_{TO2}	Total delta-v for the 2nd maneuver option
T_{DEB1}	Time taken to reach the 1st debris orbit from current position
$\Delta V_{D1\theta}$	Rotational velocity of the s/c for 1st debris orbit at angel θ for strategy 1
I_{SPD1}	Specific impulse of the s/c from the earth to the 1st debris object
ΔV_{DIS1}	Deorbitting delta-v for the 1st debris object
T_{DIS1}	Time taken by s/c from 1st debris object to disposal region
I_{SPDIS1}	Specific impulse of the s/c for the de-orbiting the 1st debris object
ΔV_{DEB2}	Δ V from current position to 2nd selected debris object
T_{DEB2}	Time taken to reach the 2nd debris orbit from current position
I_{SPD2}	Specific impulse of the s/c from the current position to the 2 debris object
ΔV_{DIS2}	Deorbitting delta-v for the 2nd debris object
I_{TM1}	Total mission impulse for 1st strategy
T_{TM1}	Total time for mission completion for 1st strategy
$\Delta V_{T1}=\Delta V_T$	Total Δ V for the 1st strategy
ΔV_{col}	Δ V from earth to collection orbit
T_{col}	Time taken by s/c to reach the collection orbit from earth
ΔV_{DE1}	De-orbiting delta v for 1st debris object back to the collection orbit
I_{SPC1}	Specific impulse of the s/c for 1st debris de-orbiting to collection orbit
ΔV_{DE2}	De-orbiting delta v for 2nd debris object back to the collection orbit
I_{SPC2}	Specific impulse of the s/c for 2nd debris de-orbiting to collection orbit
I_{IM2}	Total impulse for the two debris object up to the collection orbit before the disposal process
T_{PD1}	Time period of the 1st debris object
$T_{W1}=T_{wait1}$	Waiting time for the s/c at 1st debris object
T_{PD2}	Time period of the 2nd debris object
T_{W2}	Waiting time for the space craft at 2nd debris object
T_{T2}	Total time for retrieval and release of debris to the collection orbit
a_{col}	Semi major axis of the collection orbit
T_{PC}	Time period of collection orbit
T_{pm}	Time period of the pacing maneuver orbit
a_{pm}	Semi major axis of the pacing maneuver orbit
h_p	Height of perigee
V_{pm}	Velocity of the pacing maneuver orbit
$\Delta V_{capture}$	ΔV required to capture the debris object at the collection orbit
I_{SPTP}	Specific impulse for both the debris in the collection orbit to be undergone the pacing maneuver
$I_{SPS3}=I_{SP3}$	Total mission impulse for strategy 3
T_{TM3}	Total mission time for strategy 3
I_{TM1}	Total Specific impulse of strategy 1
m_f	Fuel mass of the s/c
ΔV_{T3}	Total ΔV for strategy 3