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K-ordered Hamiltonian Graphs

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Abstract

In this chapter, we review the following results proved in [1, 2, 3, 4]: (i) For k ≥ 3, every (k+1)-Hamiltonian-connected graph is k-ordered. Determine f(k, n) if n is sufficiently large in terms of k. Let g(k,n) = n/2 + k/2 -1 (ii) f(k,n) = g(k,n) if n ≥ 1 lk-3 (iii) f(k,n) ≥g(k,n) for any n ≥2k and f(k,n)> g(k,n)if 2k≤n≤3k-6 (iv) if G is a graph of order n with 3≤k≤n/2 and deg(u)+deg (v) ≥n+(3k-9)/2 for every pair u, v of non-adjacent vertices of G, then G is k-ordered Hamiltonian.

Keywords: Adjacency, Connectedness, Hamilton, Vertices

1. Introduction

In this section, the concept of K-Hamiltonian connected graphs is introduced. The result is that, for k≥3, every (k+1)-hamiltonian connected graph is k-ordered is established. Also, necessary and sufficient conditions for a graph to be k-ordered are discussed. The notations used are recalled here. We write v1, (x1), v2, (x2), …, vk-1, (xk-1), vk to indicate a v1-vk path that contains the k vertices v1, v2…vk, and possibly upto k-1 additional vertices, namely x1, x2, …xk-1. Thus, for 1≤ j ≤ k-1, (x1) indicates that some vertex, which we denote by xi, may (or) may not be present on the path. So, for each i (1≤ i≤k-1), the vertices v1 and vi+1 are either adjacent or vi, vi+1 is a path. For example, u, v (x), y indicates the path u, v, y or the path u, v, x, y.

Proposition 1.1: Let G be a Hamiltonian graph of order n ≥ 3. If G is k-ordered, 3 ≤ k ≤n, then G is (k-1)-connected.

Proof: Suppose, to the contract that G is not (k-1) connected. Then, there exists a cut set S = v_1,v_2,….,v of G, where ≤ k-2. Let v_{+1} and v_{+2} be vertices belonging to distict components of G-S. Consequently, every v_{+1}-v_{+2} path in G contains vertices of S, so G contains no Hamiltonian cycle containing the vertices of the sequence v_1,v_2…, v, v_{+1}, v_{+2} in this order. Hence G is not (+2) -ordered and so is not k-ordered.

Corollary 1.2: If G is a k-ordered Hamiltonian graph, then δ(G)≥k-1.

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Sufficient conditions for k-ordered graphs

Many sufficient conditions have been given for Hamiltonian graphs. Two of the best known are by Dirac and Ore.

Theorem 1.3 (Dirac): Let G be a graph of order n ≥3. If deg v ≥ n/2 for every vertex v of G, then G is Hamiltonian.

Theorem 1.4 (Ore): Let G be a graph of order n≥3. If for every pair u, v of non-adjacent vertices of G, deg u+deg v ≥n, then G is hamiltonian.

Lemma 1.5: Let G be a graph of order n ≥3 and let S: v_1,v_2, …, v_k be a sequence of k distinct vertices of G, where 3 ≤k≤n. If

$$\deg u+\deg v \geq n+2k-7$$

For every pair u, v of non adjacent vertices of G, then G contains a v_1 - v_k path of the type v_1, (x_1), v_2,(x_2),… (x_{k-1}), v_k or a v_k-v_{k-1} path of the type v_k, v_1, (x_1), v_2, (x_2), …,(x_{k-2}), v_{k-1}.

Proof: Consider the vertices v_1 and v_2. If v_1 and v_2 are adjacent, then G contains the path v_1,vs:; otherwise, deg v_1 +deg v_2≥n+2k-7≥n-1, which implies the existence of a vertex x1 mutually adjacent to v_1 and v_2 and so v_1, x_1, v_2 is a path in G. In any case, G contains a path v_1, (x_1), v_2. Now, consider the vertex v_3. If v_2v_3 ∈ E(G), then G contains the path v_1, (x_1), v_2,v_3. Suppose, then that v_2v_2 ∈ E(G). Thus, deg v_2 + deg v_2 ≥ n + 2k-7 by hypothesis. Since G has order n, the vertices v_2 and v_3 are mutually adjacent to at least 2k-5 vertices. if k≥4, then G contains a vertex x_2 distinct from v_1 and x_1 (if it exists) such that x_2 is mutually adjacent to v_2 and v_3. Hence G contains the path v_1, (x_1), v_2, (x_2), v_3. Suppose, then that k=3 and that G contains to vertex distinct from v1 and x1 that is mutually adjacent to v_2 and v_3. If v_3 is adjacent to v_1, then v_3, v_1,(x_1), v_2 is a path of G. If v_3 is not adjacent to v_1, then v_2 and v_3 are mutually adjacent to 1 and to no other vertex, while v_2 is adjacent to v_1. However, then v_1,v_2, x_1, v_3 is a path in G.

Proceeding inductively, assume that we have constructed a path v_1, (x_1), v_2, (x_2) ,…, (x_{j-2}), v_{j-1} in G. Suppose first that j≤k-1. It is to show that G contains a path v_1 ,…, (x_1) v_2 (x_2) v_j. If v_{j-1} is adjacent to v_j. If v_{j-1} is adjacent to v_j, then G contains such a path. Suppose then that v_{j-1} $v_j \in$ E(G). If G contains a vertex x_{j-1} distinct from the vertices on the path v_1, (x_1), v_2, (x_2) ,…, (x_{j-2}) such that x_{j-1} is mutually adjacent to v_{j-1} and v_j, then G contains a path of the designed type, otherwise v_{j-1} and v_j are mutually adjacent to at most 2_{j-4} vertices and G contains at least n + 2k - 2_{j-1} vertices. Since k ≥ j + 1, this produces a contradiction, and hence the desired claim is verified.

Finally, suppose that j = k, that is, assume that there is a path v_1, (x_1), v_2, (x_2) ,…, (x_{k-2}), v_{k-1} is constructed in G. It is to show that G contains either a path v_1, (x_1), v_2, (x_2) ,…, v_{k-1}, (x_{k-1}), v_k or a path v_k, v_1, (x_1), v_2, (x_2)…, (x_{k-2}), v_{k-1}. If either $v_k v_{k-1} \in E(G)$ or $v_k v_1 \in E(G)$, then the desired result is obtained, thus assume that v_k is adjacent to neither v_{k-1} nor v_1. Also, if v_{k-1} and vk are mutually adjacent to some vertex other than the (at most) 2_{k-5} vertices on the path (x_1), v_2, (x_2), …, v_{k-2}, (x_{k-2}), the proof is complete; so assume that this is not the case. Hence v_{k-1} and v_k are mutually adjacent to at most 2_{k-5} vertices. Since G has order n, it follows that v_{k-1} and v_k are mutually adjacent to exactly 2_{k-5} vertices (so all of the vertices x_1, x_2, …, x_{k-2} exist), and every other vertex of G is adjacent to exactly one of v_{k-1}, and G contains the path v_1, x_1, v_2, x_2,…, v_{k-2}, x_{k-2}, v_k, producing the desired path. Hence, the result.

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