

Retractions of Chaotic Buchdahi Space and their Chaotic Fundamental Groups

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Abstract

The purpose of this paper is to give a combinatorial characterization and also construct representations of the chaotic fundamental groups of the chaotic submanifolds of chaotic Buchdahi space by using some geometrical transformations. The chaotic homotopy groups of the limit for chaotic Buchdahi space are presented. The chaotic fundamental groups of some types of chaotic geodesics in chaotic Buchdahi space are discussed. New types of homotopy maps are deduced.

Keywords: Chaotic Buchdahi Space, Chaotic Homotopy Groups, Chaotic Folding, Chaotic Retractions, Chaotic Deformation Retracts.

Mathematics Subject Classification: 53A35, 51H20, 57N10.

1. Introduction and Definitions

Buchdahi space represents one of the most intriguing and emblematic discoveries in the history of geometry. Although if it were introduced for a purely geometrical purpose, they came into prominence in many branches of mathematics and physics. This association with applied science and geometry generated synergistic effect: applied science gave relevance to Buchdahi space and Buchdahi space allowed formalizing practical problems [6, 8, 21].

In vector spaces and linear maps; topological spaces and continuous maps; groups and homomorphisms together with the distinguished family of maps is referred to as a category. An operator which assigns to every object in one category a corresponding object in another category and to every map in the first a map in the second in such a way that compositions are preserved and the identity map is taken to the identity map is called a functor. Thus, we may summarize our activities thus far by saying that we have constructed a

functor (the fundamental group functor) from the category of pointed spaces and maps to the category of groups and homomorphisms. Such functors are the vehicles by which one translates topological problems into algebraic problem [21–26, 28].

Most folding problems are attractive from a pure mathematical standpoint, for the beauty of the problems themselves. The folding problems have close connections to important industrial applications. Linkage folding has applications in robotics and hydraulic tube bending. Paper folding has application in sheet-metal bending, packaging, and air-bag folding [10–12, 18]. Also, used folding to solve difficult problems related to shell structures in civil engineering and aero space design, namely buckling instability [10, 11]. Isometric folding between two Riemannian manifold may be characterized as maps that send piecewise geodesic segments to a piecewise geodesic segments of the same length [4]. For a topological folding the maps do not preserves lengths [5, 6], i.e. A map $\mathfrak{S}: M \rightarrow N$, where M

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and N are c^∞ -Riemannian manifolds of dimension m and n respectively is said to be an isometric folding of M into N , iff for any piecewise geodesic path $\gamma: J \rightarrow M$, the induced path $\mathfrak{S} \circ \gamma: J \rightarrow N$ is a piecewise geodesic and of the same length as γ . If $\mathfrak{S}$ does not preserve length, then $\mathfrak{S}$ is a topological folding [1–5, 7].

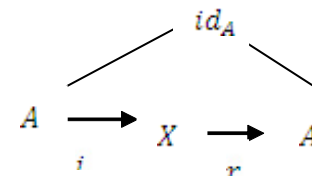
An n -dimensional topological manifold M is a Hausdorff topological space with a countable basis for the topology which is locally homeomorphic to R^n . If $h: U \rightarrow U'$ is a homeomorphism of $U \subseteq M$ onto $U' \subseteq R^n$, then h is called a chart of M and U is the associated chart domain. A collection (h_α, u_α) is said to be an atlas for M if $\bigcup_{\alpha \in A} U_\alpha = M$. Given two charts h_α, h_β such that $U_{\alpha\beta} = U_\alpha \cap U_\beta \neq \emptyset$, the transformation chart $h_\beta \circ h_\alpha^{-1}$ between open sets of R^n is defined, and if all of these charts transformation are c^∞ -mappings, then the manifolds under consideration is a c^∞ -manifolds. A differentiable structure on M is a differentiable atlas and a differentiable manifold is a topological manifold with a differentiable structure [29–31]. M may have other structures as colors, density or any physical structures. The number of structures may be infinite. In this case the manifold is said to be a chaotic manifold and may become relevant to vacuum fluctuation and chaotic quantum field theories. The magnetic field of a magnet bar is a kind of chaotic 1-dimensional manifold represented by the magnetic flux lines. The geometric manifold is the magnetic bar itself [1–5].

Fuzzy manifolds are special type of the category of chaotic manifolds. Usually we denote by $M = M_{012\dots h}$ to a chaotic manifolds [1–3, 6, 7, 19], where M_{0h} is the geometric (essential) manifold and the associated pure chaotic manifolds, the manifolds with physical characters, are denoted by $M_{1h}, \dots, M_{\infty h}$ [11, 13, 18, 19].

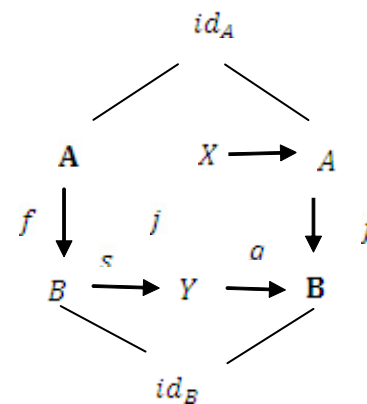
The aim of this paper is to describe the connection between the chaotic fundamental groups and the chaotic homotopy group geometrically, specifically concerned with the study of the new type of chaotic retractions, chaotic deformation retracts, chaotic foldings and the chaotic fundamental groups of chaotic Buchdahi space $\beta^4_{0123\dots h}$ as presented by El-Ahmady [1–24] and Abu-Saleem [25–38]. The set of chaotic homotopy classes of chaotic loops based at the point $x_0(\mu)$ with the product operation $[f(\mu)][g(\mu)] = [f(\mu).g(\mu)]$ is called the chaotic fundamental groups and denoted by $\pi_1(x(\mu), x_0(\mu))$ [7, 8, 18–39].

A subset A of a topological space X is called a retract of X if there exists a continuous map $r: X \rightarrow A$ such that $r(a) = a, \forall a \in A$ where, A is closed and X is open [3, 7].

Also, let X be a space and A a subspace. A map $r: X \rightarrow A$ such that $r(a) = a$, for all $a \in A$, is called a retraction of X onto A and A is called a retract of X . This can be restated as follows. If $i: A \rightarrow X$ is the inclusion map, then $r: X \rightarrow A$ is a map such that $ri = id_A$. If, in addition, $ri = id_X$, we call r a deformation retract and A a deformation retract of X . Another simple-but extremely useful-idea is that of a retract. If $A, X \subset M$, then A is a retract of X if there is a commutative diagram.



If $f: A \rightarrow B$ and $g: X \rightarrow Y$, and $g: X \rightarrow Y$, then f is a retract of g if there is a commutative diagram [9, 19, 20, 25, 26, 27, 28].



2. Main Results

THEOREM 1: The chaotic fundamental groups of types of the chaotic deformation retracts of chaotic Buchdahi space $\beta^4_{0123\dots h}$ are isomorphic to chaotic identity group $0 = 0_{0123\dots h}$.

PROOF: Consider the chaotic Buchdahi space $\beta^4_{0123\dots h}$. Used cylindrical coordinates $z(\mu), r(\mu), \theta(\mu)$ and $t(\mu)$, with metric

$$ds^2(\mu) = -\gamma^2(\gamma^{-1}dr^2(\mu) + r^2(\mu)d\theta^2(\mu) + r^2(\mu)\sin^2\theta(\mu)d\phi^2(\mu)) + p^{-1}dt^2(\mu) \quad (1)$$

The coordinates of chaotic Buchdahi space $B^4_{0123\dots h}$ are given by

$$\left. \begin{aligned} x_1(\mu) &= \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \cos\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right) \\ x_2(\mu) &= \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \sin\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right) \\ x_3(\mu) &= \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \\ x_4(\mu) &= \frac{A_4}{1-\sqrt{p-1}} \end{aligned} \right\} (2)$$

where A_1, A_2, A_3 and A_4 are the constant of integration, $p = 1 - \frac{2m}{r(\mu)}$, $\gamma = \gamma(t(\mu))$ and $\mu = \mu_{0123\dots h}$. Now, we use Lagrangian equations $\frac{d}{ds} \left(\frac{\partial T}{\partial G_i} \right) - \frac{\partial T}{\partial G_i} = 0, i = 1, 2, 3, 4$. To find a chaotic geodesic which is a subset of the chaotic Buchdahi space $\beta_{0123\dots h}^4$. Since

$$T = \frac{1}{2} \{ -\gamma^2 (\gamma^{-1} r'^2(\mu) + r^2(\mu) \theta'^2(\mu) + r^2(\mu) \sin^2 \theta(\mu) \phi'^2(\mu)) + p^2 t'^2(\mu) \}$$

Then the Lagrangian equations for chaotic Buchdahi space $\beta_{0123\dots h}^4$ are

$$\frac{d}{ds} (\gamma r'(\mu)) + (-\gamma^2 r(\mu) \theta'^2(\mu) - \gamma^2 r(\mu) \sin^2 \theta(\mu) \phi'^2(\mu)) = 0 \quad (3)$$

$$\frac{d}{ds} (\gamma^2 r^2(\mu) \theta'(\mu)) + (\gamma^2 r^2(\mu) \sin \theta(\mu) \cos \theta(\mu) \phi'(\mu)) = 0 \quad (4)$$

$$\frac{d}{ds} (\gamma^2 r^2(\mu) \sin^2 \theta(\mu) \phi'(\mu)) = 0 \quad (5)$$

$$\frac{d}{ds} (p^{-1} t'(\mu)) = 0 \quad (6)$$

$$\frac{d}{ds} (-\gamma^2 ((\gamma^{-1} r'(\mu) \mu') + r(\mu) \mu' \theta'^2(\mu) + r^2(\mu) \theta'(\mu) \mu' + r(\mu) \mu' \sin^2 \theta(\mu) \phi'(\mu) + r^2(\mu) (\sin \theta(\mu) \cos \theta(\mu) \mu' \phi'(\mu) + \sin^2 \theta(\mu) \phi'(\mu) \mu')) + p^2 t'(\mu) \mu' = 0 \quad (7)$$

From equation (5) we obtain $\gamma^2 r^2(\mu) \sin^2 \theta(\mu) \phi'(\mu) =$ constant say β_1 , if $\beta_1 = 0$, we obtain the following cases. If $\gamma^2 = 0$, then the coordinates of chaotic Buchdahi space $\beta_{0123\dots h}^4$ are given by $(A_1 \sin A_2 \cos A_3, A_1 \sin A_2 \sin A_3, A_1 \cos A_2, \frac{A_4}{1-\sqrt{p-1}})$,

which is a chaotic hypersphere $S_1^3(\mu) \subset \beta_{0123\dots h}^4$ which is a chaotic geodesic retraction. If $r^2(\mu) = 0$, hence we get the chaotic hypersphere $S_1^0(\mu) \subset \beta_{0123\dots h}^4$ on the null cone which is a chaotic geodesic retraction. If $\phi'(\mu) = 0$, then $\phi(\mu) =$ constant say β_2 , if $\beta_2 = 0$, then we obtain the following chaotic geodesic retraction $S_2^3(\mu) \subset \beta_{0123\dots h}^4$.

The chaotic deformation retracts of the chaotic Buchdahi space $\beta_{0123\dots h}^4$ is defined by $\eta : \{\beta_{0123\dots h}^4 - (\delta_i)\} \times I(\mu) \rightarrow \{\beta_{0123\dots h}^4 - (\delta_i)\}$, where $\{\beta_{0123\dots h}^4 - (\delta_i)\}$ is the open chaotic Buchdahi space $\beta_{0123\dots h}^4$ and $I(\mu)$ is the chaotic closed interval $[0(\mu), 1(\mu)]$. The chaotic retractions of the open chaotic Buchdahi space $\beta_{0123\dots h}^4$ is $R : \{\beta_{0123\dots h}^4 - (\delta_i)\} \rightarrow S_1^3(\mu), S_1^0(\mu), S_2^3(\mu)$.

The chaotic deformation retract of $S_1^3(\mu) \subset \beta_{0123\dots h}^4$ is given by

$$\eta(m, c) = \cos \frac{\pi c}{2} \left\{ \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \cos\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right), \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \sin\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right), \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_4}{1-\sqrt{p-1}} - (\delta_i) \right\} + \sin \frac{\pi c}{2} \{ A_1 \sin A_2 \cos A_3, A_1 \sin A_2 \sin A_3, A_1 \cos A_2, \frac{A_4}{1-\sqrt{p-1}} \},$$

where

$$\eta(m, 0) = \left\{ \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \cos\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right), \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \sin\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right), \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_4}{1-\sqrt{p-1}} - (\delta_i) \right\},$$

and

$$\eta(m, 1) = \{ A_1 \sin A_2 \cos A_3, A_1 \sin A_2 \sin A_3, A_1 \cos A_2, \frac{A_4}{1-\sqrt{p-1}} \}.$$

The chaotic deformation retract of $S_1^0(\mu) \subset \beta_{0123\dots h}^4$ is defined as

$$\eta(m, c) = \frac{1-c}{1+c} \left\{ \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \cos\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right), \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \sin\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right), \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_4}{1-\sqrt{p-1}} - (\delta_i) \right\} + c(2c-1) \{ 0, 0, 0, 0 \}.$$

The chaotic deformation retract of $S_2^3(\mu) \subset \beta_{0123\dots h}^4$ is defined as

$$\eta(m, c) = \ln e^{(1-c)} \left\{ \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \cos\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right), \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \sin\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right), \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_4}{1-\sqrt{p-1}} - (\delta_i) \right\} + \ln e^c \left\{ \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right), 0, \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_4}{1-\sqrt{p-1}} \right\}.$$

Thus, $\pi_1\{\beta_{0123...h}^4 - (\delta_i)\} \approx \pi_1(S_1^3(\mu))$, $\pi_1\{\beta_{0123...h}^4 - (\delta_i)\} \approx \pi_1(S_1^0(\mu))$ and $\pi_1\{\beta_{0123...h}^4 - (\delta_i)\} \approx \pi_1(S_2^3(\mu))$. Therefore $\pi_1(S_1^3(\mu))$, $\pi_1(S_1^0(\mu))$ and $\pi_1(S_2^3(\mu))$ are isomorphic to the chaotic identity group $0_{0123...h}$.

COROLLARY 1. The chaotic fundamental group of types of the chaotic deformation retracts of $S_1^3(\mu) \subset \beta_{0123...h}^4$ and any chaotic manifold homeomorphic to chaotic Buchdahi space $\beta_{0123...h}^4$ is isomorphic to the chaotic identity group.

THEOREM 2: The chaotic fundamental group of the limit limit of chaotic foldings of the chaotic hypersphere $S_1^3(\mu) \subset \beta_{0123...h}^4$ and any chaotic manifold homeomorphic to it is isomorphic to $Z(\mu)$.

PROOF: Consider the chaotic hypersphere $S_1^3(\mu)$ and let $\eta_1: S_1^3(\mu) \rightarrow S_1^3(\mu)$ be a chaotic folding map, now we can define a series of chaotic folding maps by $\eta_2: \eta_1(S_1^3(\mu)) \rightarrow \eta_1(S_1^3(\mu))$, $\eta_3: \eta_2(\eta_1(S_1^3(\mu))) \rightarrow \eta_2(\eta_1(S_1^3(\mu)))$ $\dots \eta_n: \eta_{n-1}(\eta_{n-2} \dots (\eta_1(S_1^3(\mu))) \dots) \rightarrow \eta_{n-1}(\eta_{n-2} \dots (\eta_1(S_1^3(\mu))) \dots)$, then $\lim_{n \rightarrow \infty} (\eta_n(\eta_{n-1}(\eta_{n-2} \dots (\eta_1(S_1^3(\mu))) \dots))) = S_1^2(\mu)$, is a great chaotic sphere (of dimension two. Therefore, $\pi_1(S_1^2(\mu))$ is isomorphic to chaotic identity group. Also, if we consider

$$\gamma_1: S_1^2(\mu) \rightarrow S_1^1(\mu), \gamma_2: \gamma_1(S_1^2(\mu)) \rightarrow \gamma_1(S_1^1(\mu)),$$

$$\gamma_3: \gamma_2(\gamma_1(S_1^2(\mu))) \rightarrow \gamma_2(\gamma_1(S_1^1(\mu))), \dots, \gamma_m: \gamma_{m-1}(\gamma_{m-2} \dots$$

$$(\gamma_1(S_1^2(\mu))) \dots) \rightarrow \gamma_{m-1}(\gamma_{m-2} \dots (\gamma_1(S_1^2(\mu))) \dots), \text{ then}$$

$$\lim_{m \rightarrow \infty} \gamma_m(\gamma_{m-1}(\gamma_{m-2} \dots (\gamma_1(S_1^2(\mu))) \dots) = S_1^1(\mu) \subset \beta_{0123...h}^4,$$

which is the great chaotic circle of dimension one. Therefore $\pi_1(S_1^1(\mu) \subset \beta_{0123...h}^4)$ is isomorphic to $Z(\mu)$.

THEOREM 3. Under the chaotic folding $\Pi_m(x_1(\mu), x_2(\mu), x_3(\mu))$, $x_4(\mu) = (x_1(\mu), x_2(\mu), x_3(\mu), \left| \frac{x_4(\mu)}{m} \right|)$, the chaotic fundamental group of the limit of chaotic foldings and any chaotic manifold homeomorphic to this type of chaotic folding of $S_2^3(\mu)$ is isomorphic to $Z(\mu)$.

PROOF. Consider the chaotic geodesic retraction hypersphere $S_2^3(\mu)$ and let $\Pi_m: S_2^3(\mu) \rightarrow S_2^3(\mu)$ be given by

$$\Pi_m(x_1(\mu), x_2(\mu), x_3(\mu), x_4(\mu)) = (x_1(\mu), x_2(\mu), x_3(\mu), \left| \frac{x_4(\mu)}{m} \right|).$$

Then, the isometric chain chaotic folding of $S_2^3(\mu)$ into itself defined by

$$\Pi_1: \left\{ \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right), 0, \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_4}{1-\sqrt{p-1}} \right\} \rightarrow$$

$$\left\{ \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right), 0, \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \left| \frac{A_4}{1-\sqrt{p-1}} \right| \right\},$$

$$\Pi_2: \left\{ \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right), 0, \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \left| \frac{A_4}{1-\sqrt{p-1}} \right| \right\} \rightarrow$$

$$\left\{ \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right), 0, \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \left| \frac{\frac{A_4}{1-\sqrt{p-1}}}{2} \right| \right\}, \dots,$$

$$\Pi_m: \left\{ \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right), 0, \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \left| \frac{\frac{A_4}{1-\sqrt{p-1}}}{m-1} \right| \right\} \rightarrow$$

$$\left\{ \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right), 0, \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \left| \frac{\frac{A_4}{1-\sqrt{p-1}}}{m} \right| \right\}$$

Then we get

$$\lim_{m \rightarrow \infty} \Pi_m \left\{ \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right), 0, \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_4}{1-\sqrt{p-1}} \right\}.$$

Thus, $-x_1^2(\mu) - x_2^2(\mu) - x_3^2(\mu) + x_4^2(\mu) = \left(\frac{A_1}{1-i\sqrt{\gamma}}\right)^2$, which is the chaotic geodesic retraction great circle $S_2^1(\mu) \subset S_2^3(\mu)$ with $x_2(\mu) = x_4(\mu) = 0$. Therefore $\pi_1(S_2^1(\mu) \subset S_2^3(\mu))$ is isomorphic to $Z(\mu)$.

COROLLARY 2. Under the chaotic folding $\Pi_m(x_1(\mu), x_2(\mu), x_3(\mu), x_4(\mu)) = (x_1(\mu), x_2(\mu), x_3(\mu), \left| \frac{x_4(\mu)}{m} \right|)$, the chaotic fundamental group of the limit of chaotic foldings of the chaotic hypersphere $S_1^3(\mu)$, be a chaotic hypersphere $S_1^2(\mu) \subset S_1^3(\mu)$ is isomorphic to the chaotic identity group.

COROLLARY 3. Under the chaotic folding $\Pi_m(x_1(\mu), x_2(\mu), x_3(\mu), x_4(\mu)) = (x_1(\mu), x_2(\mu), x_3(\mu), \left| \frac{x_4(\mu)}{m} \right|)$ and $A_3 = 0$, the chaotic fundamental group of the limit of chaotic foldings of the a chaotic hypersphere $S_1^3(\mu) \subset \beta_{0123...h}^4$, is a chaotic great circle $S_1^1(\mu) \subset S_1^3(\mu)$, which is isomorphic to $Z(\mu)$.

COROLLARY 4. Under the chaotic folding $\Pi_m(x_1(\mu), x_2(\mu), x_3(\mu), x_4(\mu)) = (x_1(\mu), x_2(\mu), x_3(\mu), \left| \frac{x_4(\mu)}{m} \right|)$ and $A_3 = 0$, the

chaotic fundamental group of the chaotic deformation retract of $S_1^3(\mu)$ onto the chaotic great circle $S_3^1(\mu) \subset S_1^3(\mu)$ is isomorphic to $Z(\mu)$.

THEOREM 4. Under the chaotic folding $\Pi_m(x_1(\mu), x_2(\mu), x_3(\mu), x_4(\mu)) = \left(\left| \frac{x_1(\mu)}{m} \right|, \left| \frac{x_2(\mu)}{m} \right|, x_3(\mu), x_4(\mu) \right)$ the chaotic fundamental group of the limit of chaotic foldings of the chaotic Buchdahi space $\beta_{0123...h}^4$ is isomorphic to the chaotic identity group.

PROOF. Now consider the chaotic Buchdahi space $\beta_{0123...h}^4$ of dimension four and let $\Pi_m: \beta_{0123...h}^4 \rightarrow \beta_{0123...h}^4$ be given by

$$\Pi_m(x_1(\mu), x_2(\mu), x_3(\mu), x_4(\mu)) = \left(\left| \frac{x_1(\mu)}{m} \right|, \left| \frac{x_2(\mu)}{m} \right|, x_3(\mu), x_4(\mu) \right).$$

Then, the isometric chain chaotic folding of the chaotic Buchdahi space $\beta_{0123...h}^4$ into itself may be defined by

$$\begin{aligned} \Pi_1: & \left(\frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \sin\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right), \right. \\ & \left. \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_4}{1-\sqrt{p-1}} \right) \rightarrow \left(\frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \right. \\ & \left. \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \sin\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right), \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_4}{1-\sqrt{p-1}} \right), \\ \Pi_2: & \left(\frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \sin\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right), \right. \\ & \left. \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_4}{1-\sqrt{p-1}} \right) \rightarrow \left(\frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \right. \\ & \left. \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \sin\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right), \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_4}{1-\sqrt{p-1}} \right), \\ \Pi_m: & \left(\frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \sin\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right), \right. \\ & \left. \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_4}{1-\sqrt{p-1}} \right) \rightarrow \left(\frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \right. \\ & \left. \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \sin\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right), \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_4}{1-\sqrt{p-1}} \right), \dots \\ \Pi_m: & \left(\frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \sin\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right), \right. \\ & \left. \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_4}{1-\sqrt{p-1}} \right) \rightarrow \left(\frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \right. \\ & \left. \frac{A_1}{1-i\sqrt{\gamma}} \sin\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \sin\left(\frac{A_3}{1-ir(\mu)\gamma \sin(\mu)}\right), \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_4}{1-\sqrt{p-1}} \right), \dots \end{aligned}$$

Then we get

$$\lim_{m \rightarrow \infty} \Pi_m \left\{ 0, 0, \frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right), \frac{A_4}{1-\sqrt{p-1}} \right\}.$$

Thus,

$$-x_1^2(\mu) - x_2^2(\mu) - x_3^2(\mu) + x_4^2(\mu) = \left(\frac{A_1}{1-i\sqrt{\gamma}} \cos\left(\frac{A_2}{1-ir(\mu)\gamma}\right) \right)^2 + \left(\frac{A_4}{1-\sqrt{p-1}} \right)^2,$$

which is chaotic hypersurface $\beta_{0123...h}^1 \subset \beta_{0123...h}^4$ with $x_1(\mu) = x_2(\mu) = 0$. Therefore $\pi_1(\beta_{0123...h}^1 \subset \beta_{0123...h}^4)$ is isomorphic to the chaotic identity group.

THEOREM 5. Under the chaotic folding $\Pi_m(x_1(\mu), x_2(\mu), x_3(\mu), x_4(\mu)) = \left(\left| \frac{x_1(\mu)}{m} \right|, \left| \frac{x_2(\mu)}{m} \right|, \left| \frac{x_3(\mu)}{m} \right|, \left| \frac{x_4(\mu)}{m} \right| \right)$, the chaotic fundamental group of the limit of chaotic foldings of the chaotic Buchdahi space $\beta_{0123...h}^4$ is the chaotic identity group.

PROOF. Consider the four dimension chaotic Buchdahi space $\beta_{0123...h}^4$ and let $\Pi_m: \beta_{0123...h}^4 \rightarrow \beta_{0123...h}^4$ be given by

$$\Pi_m(x_1(\mu), x_2(\mu), x_3(\mu), x_4(\mu)) = \left(\left| \frac{x_1(\mu)}{m} \right|, \left| \frac{x_2(\mu)}{m} \right|, \left| \frac{x_3(\mu)}{m} \right|, \left| \frac{x_4(\mu)}{m} \right| \right).$$

Then, the isometric chain chaotic folding of the chaotic Buchdahi space $\beta_{0123...h}^4$ into itself get $\Pi_m\{0, 0, 0, 0\}$, which a zero-dimensional chaotic Buchdahi space $0_{0123...h}$. Thus, it is a chaotic point and the chaotic fundamental group of a chaotic point is the chaotic identity group.

COROLLARY 5. The chaotic fundamental group of the end limits of chaotic foldings of the n-dimensional chaotic manifold $F^n(\mu)$ homeomorphic to n-dimensional chaotic Buchdahi space $\beta_{0123...h}^n$ into itself is the chaotic identity group.

THEOREM 6. The chaotic fundamental group of the minimal chaotic retraction of the n-dimensional chaotic manifold $F^n(\mu)$ homeomorphic to n-dimensional chaotic Buchdahi space $\beta_{0123...h}^n$ is the chaotic identity group.

PROOF. Let $r_i: \{F^n(\mu) - (\beta_i^n)\} \rightarrow F^{n-1}(\mu)$ be the chaotic retraction map. Then, we have the following chains

$$\begin{aligned} \{F^n(\mu) - (\beta_i^n)\} & \xrightarrow{r_1^1} \{F_1^n(\mu) - (\beta_1^n)\} \xrightarrow{r_2^1} \{F_2^n(\mu) - (\beta_2^n)\} \rightarrow \dots \\ \{F_2^n(\mu) - (\beta_2^n)\} & \xrightarrow{\lim_{i \rightarrow \infty} r_i^1} F^{n-1}(\mu) \\ \{F^{n-1}(\mu) - (\beta_i^{n-1})\} & \xrightarrow{r_1^2} \{F_1^{n-1}(\mu) - (\beta_1^{n-1})\} \xrightarrow{r_2^2} \{F_2^{n-1}(\mu) - (\beta_2^{n-1})\} \rightarrow \dots \\ \{F_2^{n-1}(\mu) - (\beta_2^{n-1})\} & \xrightarrow{\lim_{i \rightarrow \infty} r_i^2} F^{n-2}(\mu), \dots, \\ \{F^1(\mu) - (\beta_i^1)\} & \xrightarrow{r_1^n} \{F_1^1(\mu) - (\beta_1^1)\} \xrightarrow{r_2^n} \{F_2^1(\mu) - (\beta_2^1)\} \rightarrow \dots \\ \{F_2^1(\mu) - (\beta_2^1)\} & \xrightarrow{\lim_{i \rightarrow \infty} r_i^n} F^0(\mu). \end{aligned}$$

Thus from the above chain the minimal chaotic retractions of the n -dimensional chaotic manifold $F^n(\mu)$ coincides with the zero-dimensional chaotic space which is the limit of chaotic retractions. Thus, it is a chaotic point and the chaotic fundamental group of a chaotic point is the chaotic identity group.

THEOREM 7. The chaotic fundamental group of the chaotic retraction of chaotic Buchdahi plane $\beta_{0123\dots h}^2$ is isomorphic to $Z(\mu)$.

PROOF. Since $\gamma = \frac{1}{2} \ln BC$ [6], if $B=C$, then $\gamma = \ln BC$. When $\ln B = 1$, implies $\gamma = 1$. Hence (1) becomes $ds^2(\mu) = (-dr^2(\mu) + r^2(\mu)d\theta^2(\mu) + r^2(\mu)\sin^2\theta(\mu)d\phi^2(\mu)) + p^{-1}dt^2(\mu)$. Also, under the condition $t(\mu) = \phi(\mu) = 0$, then $ds^2(\mu)$ implies that $ds^2(\mu) = -dr^2(\mu) - r^2(\mu)d\theta^2(\mu)$. Now $S_\sigma^1(\mu)$ is a chaotic retract of $\beta_{0123\dots h}^2 - \{(0,0)\}$, where $r : \beta_{0123\dots h}^2 - \{(0,0)\} \rightarrow S_\sigma^1(\mu)$ defined by $r(x) = \frac{x(\mu)}{\|x(\mu)\|}$ is a continuous map. Therefore $\pi_1(S_\sigma^1(\mu))$ is isomorphic to $Z(\mu)$.

THEOREM 8. The chaotic fundamental group of the chaotic deformation retract of the chaotic Buchdahi plane $\beta_{0123\dots h}^2$ is isomorphic to $Z(\mu)$.

PROOF. Since $S_\sigma^1(\mu)$ is a chaotic retract of $\beta_{0123\dots h}^2 - \{(0,0)\}$, but a subset A of a chaotic Buchdahi plane $\beta_{0123\dots h}^2$ is said to be a chaotic deformation retract of the chaotic Buchdahi plane $\beta_{0123\dots h}^2$ if there exists a chaotic homotopy map $F : (\beta_{0123\dots h}^2 - \{(0,0)\}) \times [0,1] \rightarrow (\beta_{0123\dots h}^2 - \{(0,0)\})$ defined as $F(x(\mu), t) = (1-t)x(\mu) + t(\frac{x(\mu)}{\|x(\mu)\|})$ such that $F_0 = id_B$ and $F_1 : (\beta_{0123\dots h}^2 - \{(0,0)\}) \rightarrow (\beta_{0123\dots h}^2 - \{(0,0)\})$ satisfies $F_1(x(\mu)) \in A$ for every $x \in (\beta_{0123\dots h}^2 - \{(0,0)\})$ and $F_1(S_\sigma^1(\mu)) = S_\sigma^1(\mu)$ for every $S_\sigma^1(\mu) \in A$. Hence F a chaotic deformation retract of the chaotic Buchdahi plane onto $S_\sigma^1(\mu)$ and $\pi_1(S_\sigma^1(\mu))$ is isomorphic to $Z(\mu)$.

THEOREM 9. The chaotic fundamental group of any types of chaotic folding of $S^n(\mu) \subset \beta_{0123\dots h}^{n+1}$ such that $\dim \mathfrak{S}(S^n(\mu)) = \dim(S^n(\mu))$, $n \geq 2$, is the chaotic identity group.

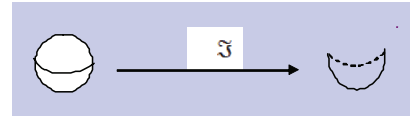


Figure 1. Chaotic folding with singularity.

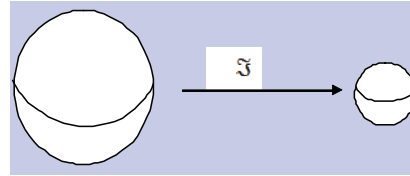


Figure 2. Chaotic folding without singularity.

PROOF. Now, consider the chaotic foldings with singularity of $S^n(\mu) \subset \beta_{0123\dots h}^{n+1}$ to a subset of $S^n(\mu) \subset \beta_{0123\dots h}^{n+1}$. Such that $\dim \mathfrak{S}(S^n(\mu)) = \dim(S^n(\mu))$, then all chaotic loops of $\mathfrak{S}(S^n(\mu))$ are homotopic to the chaotic identity loop, and hence the chaotic fundamental group of this types of chaotic folding is the chaotic identity group as in Figure 1. Again, consider the chaotic foldings without singularity of $S^n(\mu) \subset \beta_{0123\dots h}^{n+1}$ and also the chaotic fundamental group of this types of chaotic foldings is the chaotic identity group as in Figure 2.

THEOREM 10. Let $M(\mu) \subset \beta_{0123\dots h}^2$ be the union of the chaotic circles $C^n(\mu) = \bigcup_{i \in Z - \{0\}} S^1(\mu)((\frac{1}{i}, 0), \frac{1}{i}) \subset \beta_{0123\dots h}^2$. Then there are chaotic folding $F : M(\mu) \rightarrow M(\mu)$ and chaotic retractions $r : M(\mu) \rightarrow C^n(\mu)$ such that $\pi_1(F_n(M(\mu))) = \pi_1(r_n(M(\mu)))$ and $\pi_1(F_n(M(\mu)))$ is either chaotic identity group or isomorphic to $Z(\mu)$.

PROOF. Let $F_n : M(\mu) \rightarrow M(\mu)$ be a chaotic folding such that $F_n(C^n(\mu)) = C^n(\mu)$, then $F_n(M(\mu)) = C^n(\mu)$. Also, consider the chaotic retraction $r_n : M(\mu) \rightarrow C^n(\mu)$, which collapsing all $C_j(\mu)$ except $C^n(\mu)$ to the origin and too $r_n : M(\mu) = C^n(\mu)$, then $\pi_1(F_n(M(\mu))) = \pi_1(r_n(M(\mu)))$. Now, if $n \rightarrow \infty$ then $C^n(\mu)$ is a chaotic point and so $\pi_1(F_n(M(\mu))) = 0_{0123\dots h}$ otherwise if $n \rightarrow \infty$ then $C^n(\mu)$ is a chaotic circle and $\pi_1(F_n(M(\mu)))$ is isomorphic to $Z(\mu)$. See Figure 3.

THEOREM 11. Let $M(\mu) \subset \beta_{0123\dots h}^2$ be the union of the chaotic circles $C^n(\mu) = \bigcup_{i \in Z - \{0\}} S^1(\mu)((\frac{1}{i}, 0), \frac{1}{i}) \subset \beta_{0123\dots h}^2$. Then there are chaotic folding $F : M(\mu) \rightarrow M(\mu)$ which induces a chaotic folding $\bar{F} : \pi_1(M(\mu)) \rightarrow \pi_1(M(\mu))$ such that $\bar{F}(\pi_1(M(\mu))) = \pi_1(F(M(\mu)))$ and $\bar{F}(\pi_1(M(\mu)))$ is uncountable.

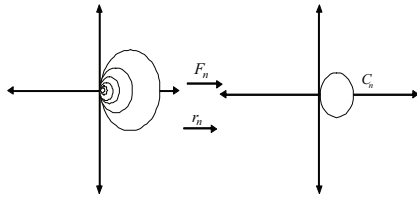


Figure 3. Chaotic retractions of union of chaotic circles.

PROOF. Let $F_n : M(\mu) \rightarrow M(\mu)$ be a chaotic folding such that

$$F_n(C^n(\mu)) = F_n\left(\bigcup_{i \in Z - \{0\}} S^1(\mu)\left(\left(\frac{1}{i}, 0\right), \frac{1}{i}\right) \subset \beta_{0123\dots h}^2\right) = C^n(\mu)$$

Also,

$$F_n(C_1^n(\mu)) = F_n\left(\bigcup_{i \in Z - \{0\}} S^1(\mu)\left(\left(\frac{1}{i}, 0\right), \frac{1}{i}\right) \subset \beta_{0123\dots h}^2\right) = C_1^n(\mu) \text{ i.e.}$$

$$F_n(C_i^n(\mu)) = F_n\left(\bigcup_{i \in Z - \{0\}} S^1(\mu)\left(\left(\frac{1}{i}, 0\right), \frac{1}{i}\right) \subset \beta_{0123\dots h}^2\right) = C_i^n(\mu). \text{ Thus}$$

we get the induced chaotic folding

$\bar{F} : (\pi_1(M(\mu))) \rightarrow \pi_1(M(\mu))$ such that $\bar{F}(\pi_1(M(\mu))) = \pi_1(F(M(\mu)))$ since, $\pi_1(F(M(\mu)))$ is uncountable, it follows that $\bar{F}(\pi_1(M(\mu)))$ is uncountable. See Figure 4.

COROLLARY 6. Let $M(\mu) \subset \beta_{0123\dots h}^2$ be the union of the chaotic circles $C^n(\mu) = \bigcup_{i \in Z - \{0\}} S^1(\mu)\left(\left(\frac{1}{i}, 0\right), \frac{1}{i}\right) \subset \beta_{0123\dots h}^2$.

If $\mathfrak{I} : M(\mu) \rightarrow M(\mu)$ be a chaotic folding defined as $\mathfrak{I}(x(\mu), y(\mu)) = (|x(\mu)|, |y(\mu)|)$. Then $\pi_1(\mathfrak{I}(M(\mu))) = 0_{0123\dots h}$.

3. Conclusion

In this paper we achieved the approval of the important of the chaotic fundamental groups in the submanifolds of chaotic Buchdahi Space by using some geometrical transformations. The relations between chaotic folding, chaotic retractions, chaotic deformation retracts, limits of chaotic foldings and limits of chaotic retractions of the chaotic fundamental groups in the submanifolds of chaotic Buchdahi Space are discussed. The connection between limits of the chaotic foldings and the chaotic fundamental groups are obtained. New types of minimal chaotic retractions on the chaotic fundamental groups are deduced.

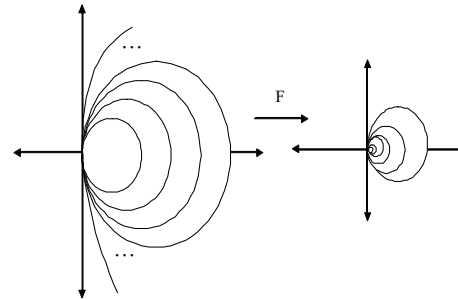


Figure 4. Chaotic foldings of union of chaotic circles.

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