

Fuzzy Soft BF-algebras

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Abstract

In this paper, the notion of fuzzy soft BF-algebra is given and the level subset, union and intersection of them were studied. Finally, fuzzy soft image and fuzzy soft inverse image of fuzzy soft BF-algebra are discussed.

Keywords: Fuzzy Soft BF-algebra, Level Subset, Fuzzy Soft Image, Fuzzy Soft Inverse Image.

1. Introduction

To solve complicated problems in Economics, Engineering, Environment, Sociology, Medical Science and many other fields, we cannot successfully use classical methods, because the uncertainties appearing in these domains may be of various types. There are four theories: Theory of Probability, Fuzzy Set Theory (FST), Interval Mathematics and Rough Set Theory (RST), which we can consider as mathematical tools for dealing with imperfect knowledge. All these tools require the pre-specification of some parameter to start with, e.g. probability density function in probability, membership function in FST and equivalence relation in RST. Such a requirement, seen in the backdrop of imperfect or incomplete knowledge, raises many problems. Noting problems in parameter specification Molodtsov [5] introduced the notion of soft set to deal with problems of incomplete information. Soft Set Theory (SST) does not require the specification of a parameter. This makes SST a natural mathematical formalism for approximate reasoning. Later other authors like Maji et al. [2–4] have further studied the theory of soft sets and used this theory to solve some decision making problems. In 2001, Maji et al. [2–4] introduced the concept of fuzzy soft set,

a more generalized concept, which is a combination of fuzzy set and soft set and studied its properties.

Y. Imai and K. Iseki [1] introduced two classes of abstract algebras: BCK-algebras and BCI-algebras. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. In [6], J. Neggers and H. S. Kim introduced the notion of B-algebras, which is a generalization of BCK-algebra. Recently, Andrzej Walendziak defined a BF-algebra [7].

In this paper, we define the fuzzy soft sub algebra of BF-algebra and then we discuss the union, intersection of them.

2. Preliminaries

In this section, we cite the fundamental definitions that will be used in the sequel:

DEFINITION 1.1. [7] Let X be a non-empty set with a binary operation $*$ and a constant 0 . Then $(X, *, 0)$ is called a BF-algebra if satisfies the following axioms:

$$(BF1) \ x * x = 0,$$

$$(BF2) \ x * 0 = x,$$

$$(BF3) \ 0 * (x * y) = (y * x),$$

for all $x, y \in X$.

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EXAMPLE 1.2. [7] (A) Let \mathbf{R} be the set of real numbers and $A = (\mathbf{R}; *, 0)$ be the algebra with the operation $*$ defined by

$$x * y = \begin{cases} x & \text{if } y = 0, \\ y & \text{if } x = 0, \\ 0 & \text{otherwise} \end{cases}$$

Then A is a BF-algebra.

(B) Let $A = [0; \infty)$. Define the binary operation $*$ on A as follows: $x * y = |x - y|$, for all $x, y \in A$. Then $(A; *, 0)$ is a BF-algebra.

PROPOSITION 1.3. [7] Let X be a BF-algebra. Then for any x and y in X , the following hold:

- (A) $0 * (0 * x) = x$ for all $x \in A$;
- (B) If $0 * x = 0 * y$, then $x = y$ for any $x, y \in A$;
- (C) If $x * y = 0$, then $y * x = 0$ for any $x, y \in A$.

DEFINITION 1.4. [7] A non-empty subset S of a BF-algebra X is called a subalgebra of X if $x * y \in S$ for any $x, y \in S$.

DEFINITION 1.5. A mapping $g : X \rightarrow Y$ of BF-algebras is called a BF-homomorphism if $g(x * y) = g(x) * g(y)$, for all $x, y \in X$.

Let X be a set. A fuzzy set A in X is characterized by a membership function $\mu_A : X \rightarrow [0, 1]$. Let f be a mapping from the set X to the set Y and let B be a fuzzy set in Y with membership function μ_B .

The inverse image of B , denoted by $f^{-1}(B)$, is the fuzzy set in X with membership function $\mu_{f^{-1}(B)}$ defined by $\mu_{f^{-1}(B)}(x) = \mu_B(f(x))$ for all $x \in X$.

Conversely, let A be a fuzzy set in X with membership function μ_A . Then the image of A , denoted by $f(A)$, is the fuzzy set in Y such that:

$$\mu_{f(A)}(y) = \begin{cases} \sup_{z \in f^{-1}(y)} \mu_A(z) & \text{if } f^{-1}(y) = \{x : f(x) = y\} \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

DEFINITION 1.6. Let f be a fuzzy set of X . Then f is called a fuzzy subalgebra of X if it satisfies

$$(FS) f(x * y) \geq \min\{f(x), f(y)\}, \text{ whenever } x, y \in X.$$

Molodtsov [5] defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq E$.

DEFINITION 1.7. A pair (F, A) is called a soft set over U where U is a mapping given by $F : A \rightarrow P(U)$.

That is, a soft set over U is a parameterized family of subsets of the universe U . For $x \in A$, $F(x)$ may be considered as the set of x -elements of the soft set (F, A) , or as the set of x -approximate elements of the soft set. Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [5].

DEFINITION 1.8. [7] Let U be an initial universe set and E be a set of parameters. Let $\mathcal{F}(U)$ denote the set of all fuzzy sets in U . Then (F, A) is called a fuzzy soft set over U where $A \subseteq E$ and F is a mapping given by

$$F : A \rightarrow P(U).$$

In general, for every $x \in A$, $F[x]$ is a fuzzy set in U and it is called fuzzy value set of parameter x . If for every $x \in A$, $F[x]$ is a crisp subset of U , then (F, A) is degenerated to be the standard soft set. Thus, from the above definition, it is clear that fuzzy soft sets are a generalization of standard soft sets.

DEFINITION 1.9. [3] Let (F, A) and (G, B) be two fuzzy soft sets over a common universe U . The union of (F, A) and (G, B) is defined to be the fuzzy soft set (H, C) satisfying the following conditions:

- (i) $C = A \cup B$,
- (ii) for all $e \in C$,

$$H[e] = \begin{cases} F[e] & \text{if } e \in A \setminus B, \\ G[e] & \text{if } e \in B \setminus A, \\ F[e] \cup G[e] & \text{if } e \in A \cap B. \end{cases}$$

In this case, we write $(F, A) \cup (G, B) = (H, C)$.

DEFINITION 1.10. [3] If (F, A) and (G, B) are two fuzzy soft sets over a common universe U , then ' (F, A) AND (G, B) ' denoted by $(F, A) \wedge (G, B)$ is defined by $(F, A) \wedge (G, B) = (H, A \times B)$ where $H[x, y] = F[x] \cap G[y]$ for all $(x, y) \in A \times B$.

Throughout this subsection U refers to an initial universe, E is a set of parameters; S and T are two BF-algebras.

DEFINITION 2.15. [2-4] Let $P(S)$ be the power set of S , a pair (F, A) is called a soft set over S where F is a mapping given by

$$F : A \rightarrow P(S)$$

DEFINITION 2.16 [4]. Let (F, A) be a soft set over S , (F, A) is said to be a soft sub-algebra over S if and only if $F(x)$ is a sub algebra of S for all $x \in A$.

DEFINITION 2.18 [4]. A pair (Λ, Σ) is called a fuzzy soft set over S , where $\Lambda: \Sigma \rightarrow \tilde{P}(S)$ is a mapping, $\tilde{P}(S)$ being the set of all fuzzy sets of S .

DEFINITION 2.19 [4]. Let (Λ, Σ) and (Δ, Ω) be two fuzzy soft sets over S , (Λ, Σ) is called a fuzzy soft subset of (Δ, Ω) , denoted by $(\Lambda, \Sigma) \subseteq (\Delta, \Omega)$, if

- (i) $\Sigma \subseteq \Omega$, (ii) for each $\varepsilon \in \Sigma$, $\Lambda(\varepsilon) \leq \Delta(\varepsilon)$.

DEFINITION 2.20 [4]. Let (Λ, Σ) and (Δ, Ω) be two fuzzy soft sets over S with $\Sigma \cap \Omega \neq \emptyset$. The intersection of them, denoted by $(\Lambda, \Sigma) \tilde{\wedge} (\Delta, \Omega) = (\Theta, \Xi)$, is a fuzzy soft set over S , where $\Xi = \Sigma \cap \Omega$, and for each $\varepsilon \in \Xi$, $\Theta(\varepsilon) = \Lambda(\varepsilon) \wedge \Delta(\varepsilon)$. Where $\Lambda(\varepsilon) \wedge \Delta(\varepsilon)$ means the intersection of fuzzy subsets $\Lambda(\varepsilon)$ and $\Delta(\varepsilon)$.

DEFINITION 2.21 [4]. The union of two fuzzy soft sets (Λ, Σ) and (Δ, Ω) over S , denoted by $(\Lambda, \Sigma) \tilde{\vee} (\Delta, \Omega) = (\Theta, \Xi)$, is a fuzzy soft set over S , where $\Xi = \Sigma \cup \Omega$, and for each $\varepsilon \in \Xi$,

$$\Theta(\varepsilon) = \begin{cases} \Lambda(\varepsilon), & \text{if } \varepsilon \in \Sigma - \Omega, \\ \Delta(\varepsilon), & \text{if } \varepsilon \in \Omega - \Sigma, \\ \Lambda(\varepsilon) \vee \Delta(\varepsilon), & \text{if } \varepsilon \in \Sigma \cap \Omega. \end{cases}$$

Where $\Lambda(\varepsilon) \vee \Delta(\varepsilon)$ means the union of fuzzy subsets $\Lambda(\varepsilon)$ and $\Delta(\varepsilon)$.

DEFINITION 2.22 [4]. Let U be a universe and E a set of parameters. The collection of all fuzzy soft sets over U with parameters from E , is called a fuzzy soft class and denoted as

$$\widetilde{(U, E)}$$

DEFINITION 2.23. [4] Let $\widetilde{(X, E)}$ and $\widetilde{(Y, E')}$ be classes of fuzzy soft sets over X and Y with parameters from E and E' , respectively. Let $u: X \rightarrow Y$ and $p: E \rightarrow E'$ be two mappings. For a fuzzy soft set (Λ, Σ) in $\widetilde{(X, E)}$, where $\Sigma \subseteq E$, the image of (Λ, Σ) under the fuzzy soft function $f = (u, p)$, denoted by $f(\Lambda, \Sigma)$, is the fuzzy soft set over Y defined by $f(\Lambda, \Sigma) = (u(\Lambda), p(\Sigma))$, where

$$u(\Lambda)(\beta)(y) = \begin{cases} \bigvee_{x \in u^{-1}(y)} \bigvee_{\alpha \in p^{-1}(\beta) \cap \Sigma} \Lambda(\alpha)(x), & \text{if } u^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}, \forall \beta \in p(\Sigma), \forall y \in Y.$$

DEFINITION 2.24 [4]. Let $\widetilde{(U, E)}$ and $\widetilde{(Y, E')}$ be classes of fuzzy soft sets over X and Y with parameters from E and E' , respectively. Let $u: X \rightarrow Y$ and $p: E \rightarrow E'$

be two mappings, (Δ, Ω) be a fuzzy soft set in $\widetilde{(Y, E')}$, where $\Omega \subseteq E'$. The inverse image of (Δ, Ω) under the fuzzy soft function $f = (u, p)$, denoted by $f^{-1}(\Delta, \Omega)$, is the fuzzy soft set over X defined by $f^{-1}(\Delta, \Omega) = (u^{-1}(\Delta), p^{-1}(\Omega))$, where

$$u^{-1}(\Delta)(\alpha)(x) = \Delta(p(\alpha))(u(x)), \quad \forall \alpha \in p^{-1}(\Omega), \forall x \in X.$$

3. Fuzzy Soft BF-algebra

In what follows, let X be a BF-algebra and E be a set of parameters unless otherwise specified.

DEFINITION 3.1. Let (F, A) be a fuzzy soft set over X , where A is a subset of E . If there exists $u \in A$ such that $F[u]$ is fuzzy sub algebra in X , we say that (F, A) is a fuzzy soft sub algebra based on a parameter u over X . If (F, A) is a fuzzy soft sub algebra based on a parameter u over X for all $u \in A$, we say that (F, A) is a fuzzy soft sub algebra over X .

EXAMPLE 3.2. Suppose that $U = \{0, 1, 2\}$ be a BF-algebra with the Cayley table which is given in the following table. Here U represent a set of houses and $E = \{\text{verycostly}, \text{costly}, \text{cheap}, \text{beautiful}\}$ is a parameters space and $A = \{\text{verycostly}, \text{costly}, \text{cheap}\}$.

*	0	1	2
0	0	1	2
1	1	0	0
2	2	0	0

(1) Let (F, A) be a fuzzy soft set over U . Then $F[\text{verycostly}], F[\text{costly}], F[\text{cheap}]$ are fuzzy sets defined as follows:

*	0	1	2
Very costly	0.9	0.8	0.3
costly	0.7	0.5	0.4
cheap	0.8	0.6	0.1

Then (F, A) is fuzzy soft sub algebra over U .

(2) Let (G, A) be a fuzzy soft set over U . Then $F[\text{verycostly}], F[\text{costly}], F[\text{cheap}]$ are fuzzy sets defined as follows:

*	0	1	2
Very costly	0.6	0.5	0.4
costly	0.5	0.7	0.2
cheap	0.7	0.5	0.6

Then (G, A) is not a fuzzy soft sub algebra over U , since (G, A) is not a fuzzy soft sub algebra based on a parameter “costly” over U . That is,

DEFINITION 3.3. Let (F, A) be a fuzzy soft set over X , where A is a subset of E . If there exists $u \in A$ such that (F, A) is a fuzzy ideal in X , we say that (F, A) is a fuzzy soft ideal based on a parameter u over X . If (F, A) is a fuzzy soft ideal based on a parameter u over X for all $u \in A$, we say that (F, A) is a fuzzy soft ideal over X .

EXAMPLE 3.4. Suppose that $U = \{0, 1, 2\}$ be a BF-algebra with the Cayley table which is given in the following table. Here U represent a set of houses and $E = \{\text{small, medium, big}\}$ is a parameters space and $A = \{\text{small, big}\}$

*	0	1	2
0	0	1	2
1	1	0	0
2	2	0	0

Let (F, A) be a fuzzy soft set over U . Then $F[\text{small}]$, $F[\text{big}]$ are fuzzy sets defined as follows:

*	0	1	2
small	0.9	0.3	0.3
big	0.7	0.5	0.5

Then it is easy to check (F, A) is a fuzzy soft ideal over U .

PROPOSITION 3.5. Every fuzzy soft ideal over X is fuzzy soft sub algebra over X .

However, the following example shows that the converse of proposition 3.5 is not true.

EXAMPLE 3.6. From Example 3.2(1), (F, A) is not a fuzzy soft ideal over U . Since, for $F[\text{costly}]$ is not a fuzzy soft sub algebra, we have

$$F[\text{costly}](2) \not\leq \min(F[\text{costly}](2 * 1), F[\text{costly}](1))$$

THEOREM 3.7. Let (F, A) be fuzzy soft sub algebra over X . If B is a subset of A , then $(F|_B, B)$ is fuzzy soft sub algebra over X .

The following example shows that there exists a fuzzy soft set (F, A) over X such that

- (i) (F, A) is not a fuzzy soft sub algebra over X .
- (ii) there exists a subset B of A such that $(F|_B, B)$ is a fuzzy soft sub algebra over X .

EXAMPLE 3.8. Let X be a BF-algebra as in Example 3.4. Consider a set of parameters $A := \{\text{small, medium, big}\}$.

Let (F, A) be a fuzzy soft set over U . Then $F[\text{small}]$, $F[\text{medium}]$, $F[\text{big}]$ are fuzzy sets defined as follows:

*	0	1	2
small	0.6	0.5	0.5
Medium	0.5	0.7	0.2
big	0.7	0.6	0.6

Then it is easy to check that (F, A) is not a fuzzy soft ideal over U , since $F[\text{medium}]$ is not fuzzy soft ideal over U . But if we take $B := \{\text{mall, big}\}$, then $(F|_B, B)$ is a fuzzy soft ideal over U .

DEFINITION 3.9. Let (F, A) and (G, B) be two fuzzy soft sets over a common universe U . The extended intersection of (F, A) and (G, B) is defined to be the fuzzy soft set (H, C) satisfying the following conditions:

- (i) $C = A \cup B$,
- (ii) for all $e \in C$,

$$H[e] = \begin{cases} F[e] & \text{if } e \in A \setminus B, \\ G[e] & \text{if } e \in B \setminus A, \\ F[e] \cap G[e] & \text{if } e \in A \cap B. \end{cases}$$

In this case, we write $(F, A) \cap_e (G, B) = (H, C)$.

DEFINITION 3.10. Let (F, A) and (G, B) be two fuzzy soft sets over a common universe U such that $A \cap B \neq \emptyset$. The restricted intersection of (F, A) and (G, B) is denoted by $(F, A) \cap_r (G, B)$ and is defined as $(F, A) \cap_r (G, B) = (H, C)$, the fuzzy soft set (H, C) satisfying the following conditions:

- (i) $C = A \cap B$,
- (ii) for all $e \in C$, $H[e] = F[e] \cap G[e]$

THEOREM 3.11. If (F, A) and (G, B) are fuzzy soft sub algebra over X , then the extended intersection of (F, A) and (G, B) is a fuzzy soft ideal over X .

The following two corollaries are straightforward result of Theorem 3.11.

COROLLARY 3.12. If (F, A) and (G, B) are fuzzy soft sub algebra over X , then the extended intersection of $(F, A) \cap_e (G, B)$ is a fuzzy soft sub algebra over X .

COROLLARY 3.13. The restricted intersection of two fuzzy soft sub algebra is fuzzy soft sub algebra.

THEOREM 3.14. If (F, A) and (G, B) are fuzzy soft sub algebra over X . If $A \cap B = \emptyset$, then the union of (F, A) and (G, B) is a fuzzy soft sub algebra over X .

The following example shows that Theorem 3.14 is not valid if $A \cap B = \emptyset$.

EXAMPLE 3.15. Suppose that $U = \{0, 1, 2, 3\}$ be a BF-algebra with the Cayley table which is given in the following table. Here U represents a set of houses and $E = \{\text{small}, \text{medium}, \text{big}\}$ is a parameters space. $A = \{\text{small}, \text{big}\}$ and $B = \{\text{medium}, \text{big}\}$. Clearly $A \cap B \neq \emptyset$.

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Let (F, A) be a fuzzy soft set over U . Then $F[\text{small}], F[\text{big}]$ are fuzzy sets defined as follows:

*	0	1	2	3
small	0.6	0.5	0.3	0.3
big	0.8	0.5	0.8	0.5

Let (G, B) be a fuzzy soft set over U . Then $G[\text{medium}], G[\text{big}]$ are fuzzy sets defined as follows:

*	0	1	2	3
medium	0.6	0.2	0.1	0.1
big	0.6	0.6	0.3	0.3

Then it is easy to check (F, A) and (G, B) are fuzzy soft sub algebras over U . But the union $(F, A) \cup (G, B)$ is not fuzzy soft ideals over U . Since $3 = 1 * 2$, we have

$$F[\text{big}] \cup G[\text{big}](1 * 2) = F[\text{big}] \cup G[\text{big}](3) = \max\{F[\text{big}](3), G[\text{big}](3)\} = 0.5$$

On the other hand, we have

$$\begin{aligned} \min\{F[\text{big}] \cup G[\text{big}](1), F[\text{big}] \cup G[\text{big}](2)\} &= \\ \min\{\max\{F[\text{big}](1), G[\text{big}](1)\}, \max\{F[\text{big}](2), G[\text{big}](2)\}\} &= 0.6 \end{aligned}$$

THEOREM 3.16. If (F, A) and (G, B) are fuzzy soft sub algebra over X , then $(F, A) \wedge (G, B)$ is a fuzzy soft sub algebra over X .

PROOF. By Definition 2.12, we know that $(F, A) \wedge (G, B) = (H, A \times B)$ where $H[u, v] = F[u] \cap G[v]$ for all $(u, v) \in A \times B$. For any $x, y \in X$, we have

$$\begin{aligned} H[u, v](x * y) &= (F[u] \cap G[v])(x * y) = \min\{F[u](x * y), \\ &G[v](x * y)\} \geq \min\{(F[u] \cap G[u])(x), \\ &(F[u] \cap G[u])(y)\} \end{aligned}$$

THEOREM 3.17. Let (F, A) be a fuzzy soft sub algebra over X and $\{(H_i, K_i) : i \in I\}$ be a family of fuzzy soft sub algebra of (F, A) , where I is an index set. Then

- (i) If $K_i \cap K_j = \emptyset$, for all $i, j \in I$, then $\cup(H_i, K_i)$ is a fuzzy soft ideal of (F, A) ,
- (ii) $\wedge(H_i, K_i)$ is a fuzzy soft ideal of (F, A) ,
- (iii) $\vee(H_i, K_i)$ fuzzy soft ideal of (F, A) .

DEFINITION 3.18. For two fuzzy soft sets (F, A) and (G, B) over U , we say that (F, A) is a fuzzy soft subset of (G, B) , if

- (i) $A \subseteq B$,
- (ii) For all $\varepsilon \in A$, $F[\varepsilon] \subseteq G[\varepsilon]$, and is written as $(F, A) \subseteq (G, B)$

THEOREM 3.19. Let (F, A) and (G, B) are fuzzy soft sets over X and $(F, A) \subseteq (G, B)$ with $F[u](x) \leq G[u](x)$. If (G, B) is a fuzzy soft sub algebra over X , then $G[u](0) \geq F[u](x)$ and $G[u](x * y) \geq \min\{F[u](x), F[u](y)\}$, for all $x, y \in X$ and $u \in A$.

PROOF. Let (G, B) be a fuzzy soft sub algebra over X and $(F, A) \subseteq (G, B)$. For all $x, y \in X$ and $u \in A$, we have $G[u](x * y) \geq \min\{G[u](x), G[u](y)\} \geq \min\{F[u](x), F[u](y)\}$ by putting $x = y$, $G[u](x * x) = G[u](0) \geq F[u](x)$.

In the following example we show that if (F, A) and (G, B) are fuzzy soft sets over X , $(F, A) \subseteq (G, B)$ and (G, B) is a fuzzy soft sub algebra over X , then (F, A) is not a fuzzy soft sub algebra over X .

EXAMPLE 3.20. Suppose that $U = \{0, 1, 2, 3\}$ be BF-algebra of Example 3. with the Cayley table which is given in the following table. Here U represent a set of houses and $E = \{\text{small}, \text{medium}, \text{big}\}$ is a parameters space. Consider $A = \{\text{small}\}$, $B = \{\text{small}, \text{big}\}$. Clearly $A \subseteq B$. Let (F, A) be a fuzzy

soft set over U . Then $F[\text{small}]$ is fuzzy sets defined as follows:

*	0	1	2	3
small	0.6	0.6	0.3	0.3

Let (G, B) be a fuzzy soft set over U . Then $G[\text{small}], G[\text{big}]$ are fuzzy sets defined as follows:

*	0	1	2	3
small	0.6	0.5	0.3	0.3
big	0.2	0.5	0.8	0.5

Clearly, $F[\text{small}] = \{0.6, 0.3\} \subseteq \{0.6, 0.5, 0.3\} = G[\text{small}]$ and it is easy to check (G, B) is a fuzzy soft ideal over U . But (F, A) is not a fuzzy soft ideal over U . Since

$$F[\text{small}](2 * 2) = F[\text{small}](0) = 0.2 \not\geq 0.8 = F[\text{small}](2).$$

DEFINITION 3.21. The complement of a fuzzy soft set (F, A) , is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \neg A)$, where $F^c : \neg A \rightarrow P(X)$ is a mapping given by $F^c[\alpha] = F[\neg\alpha]$, for all $\alpha \in \neg A$.

THEOREM 3.22. Every (F, A) is a fuzzy soft ideal over X if and only if $(F, A)^c$ is also a fuzzy soft ideal over X .

PROOF. Let (F, A) be a fuzzy soft ideal over X . For all $x, y \in X$ and $u \in \neg A$, we have

$$\begin{aligned} F^c[u](x * y) &= F[\neg u](x * y) \geq \min(F[\neg u](x), F[\neg u](y)) \\ &= \min(F^c[u](x), F^c[u](y)) \end{aligned}$$

The converse is straightforward.

4. Conclusion

Fuzzy soft sets are a new mathematical tool to deal with uncertainties.

In this paper, we applied the theory of fuzzy soft sets to BF-algebras. We introduced the notions of fuzzy soft sub algebras and fuzzy soft ideal over BF- algebras and many related properties were surveyed. In our future, we will apply intuitionistic fuzzy soft theory to BF-algebras.

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