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Reliability analysis of eccentrically loaded bolted connections in direct shear and tension using approximate procedure

Abubakar Idris and Yisa Godwin Lazhi
Dept. of Civil Engineering, Ahmadu Bello University, Zaria, Nigeria.
idrcivil1@yahoo.com

Abstract

Reliability analysis of eccentric connection fabricated from grade S275 steel plates, in direct shear and tension using approximate method of analysis in accordance with BS5950 (2000) was carried using First Order Reliability Method (FORM). Design variables such as applied load, eccentricity of applied loading, bolt diameter; considering various failure criteria of the connection were considered random and stochastic. It was shown among other findings that, the target safety indices of tensile, shear and combined failure criteria are 3.59, 5.13 and 2.26 respectively. Based on JCCS (2001), while shear design criterion is safe and uneconomical, tensile design criterion is satisfactory and combined criterion is economical and tends to unsafe region. The shear and combined design criteria therefore need to be reviewed in order to achieve a balanced design that gives a better compromise between safety and economy.

Keywords: Reliability; eccentric connection; direct shear & tension; safety index; BS5950.

Introduction

The aim of a structural design is to produce design and drawings for a safe and economical structure that fulfills its intended purpose (McGinley, 1998). Structural design is accomplished after loads estimation, by computing the internal forces and moment acting on each component of the structure, followed by the selection of appropriate cross section for a given grade of steel. Loads in industrial buildings are applied at some eccentric distance from the columns. These loads have to be transmitted to the columns by means of connections, called the eccentric connections. The eccentric connection can be achieved by the use of either off-cults of profile sections or steel plates appropriately cut and shaped into the required form. The connection has to be rigid in order that the eccentric load is transmitted suitably to columns. The primary function of the eccentric connection is in addition to transferring the eccentric load to the columns; distributes the developed stresses due to the loadings to the columns without over stressing the connectors (bolts or welds).

Depending on the method of connection, the connection may transmit either combination of direct shear and tension which is shown in Fig. 1, or direct shear and torsion as shown in Fig. 2. Both cases can be analysed (BS5950, 2000) by assuming that the centre of rotation of, for instance, the bolts group, lies either at centroid of the lowest bolt, in which case tension under this bolt is neglected because it is very small. When the tension under the lowest bolt is neglected, the analysis is termed approximate method of analysis. On the other hand, when tension under the lowest bolt is considered in the analysis, it is termed accurate method of analysis. Currently, the BS5950 considers the use of either the approximate or the accurate methods in the analysis and design of eccentric connections.

The resistance of a structural member as well as the loads applied to it is a function of several variables, most of which are random (Melchers, 1999). Therefore, the use of probabilistic approach in the design of structures enables the structural safety to be treated in a more rational manner. The study of structural reliability is concerned with the calculation and prediction of the probability of limit state violation for engineered structures at any stage during their life. In particular, the study of structural safety is concerned with the violation of the ultimate or serviceability limit states for the structure (Madsen *et al.,* 1999).

The effect of uncertainties in design is included by the use of safety factors that are based on engineering judgment and previous experience with similar structure. Due to the fact that safety involves a consideration of random variables and the realization of the limitations in design by the deterministic method, it is now generally accepted that the rational approach to the analysis of safety is through the use of probabilistic models (Morris & Plum, 1987). Under-estimation of these uncertainties sometimes leads to adverse results of collapse as reported by Igba (1996). In general, because of uncertainties, the question of safety and performance arises (McGinley & Ang, 1990). The BS5950 has been found to be very conservative (Abubakar & Sanusi, 2006; Abubakar & Mohammed, 2007). Hence it is necessary to evaluate the level of safety implied in the design criteria of eccentrically loaded bolted connections carrying direct shear and tensile stresses, analysed and designed using the approximate method in accordance with BS5950 requirements.

The main objective of a reliability-based design is to achieve an acceptable target probability of failure (or safety index). Various methods of determining target probability of failure exist (Mortensen, 1993; Whitman, 1984). The method adopted in the present study was as proposed (Ellingwood *et al.,* 1980), because of its simplicity (Sorensen *et al.,* 2001).

The work presented herein investigates the safety associated with the design criteria of grade S275 steel plates bolted connections (of grade 4.6 ordinary bolts) transmitting direct shear and tensile stresses due to the eccentric load. The connection is assumed fabricated from plates.

First order reliability procedure

Probabilistic design is concerned with the probability that a structure will realize the functions assigned to it. In this work, the reliability method employed is briefly reviewed. If R is the strength capacity and S the loading effect(s) of a structural system which are random variables, the main objective of reliability analysis of any system or component is to ensure that R is never exceeded by S. In practice, R and S are usually functions of different basic variables. In order to investigate the effect of the variables on the performance of a structural system, a limit state equation in terms of the basic design variable is required. This limit state equation is referred to as the performance or state function and expressed as:

$g(x_i) = g(x_1, x_2, \ldots, x_n) = R - S$ (1)

Where, X_i, for i = 1, 2,..., n, represent the basic design variables.

The limit state of the system can then be expressed as

$g(x_i) = 0.$ (2)

Graphically, the line $g(x_i) = 0$ represents the failure surface while $g(x_i) > 0$ represents the safe region and $g(x_i) < 0$ corresponds to the failure region. This is shown in Fig. 3. Introducing the set of uncorrelated reduced variates,

$$x'_i = \frac{(X_i - \mu_{x_i})}{\sigma_{x_i}}, i = 1,2,\ldots.n \quad (3)$$

and in terms of these reduced variates the limit state equation becomes:

$g(\sigma_{xi}X'_1 + \mu_{xi}, \sigma_{x2} X'_2 + \mu_{x2}, .., \sigma_{xn}X'_n + \mu_{xn}) = 0,$ (4)

Where, μ and σ are the means and standard deviations of the design variables. The distance D, from a point $X'_i = (X'_1, X'_2, \ldots, X'_n)$ on the failure surface $g(x'_i) = 0$ to the origin of X_i space is also given as

$$D = \sqrt{X'^2_1 + X'^2_2 + \ldots + X'^2_n}. \quad (5)$$

In matrix form:

$$D = (X'_1 X'_2 \ldots X'_n)\begin{vmatrix} X'_1 \\ X'_2 \\ . \\ . \\ . \\ X'_n \end{vmatrix} = (X'^t_i X)^{1/2}. \quad (6)$$

The point on the failure surface $(X'^*_1, X'^*_2, \ldots\ldots, X'^*_n)$, having the minimum distance to the origin may be determined by minimizing the function D and subjecting equation (6) to the constraint $g(X_i) = 0$. For this purpose, the method of Langrange's multiplier may be used. Let

$$L = D + \lambda g(X_i) \quad (7)$$

Where, D is the minimum distance to the origin of the circle in Fig. 3, λ is the value of the Langrange's multiplier and $g(X_i)$ is the limit state function.

Substituting equation (6) in (7) gives

$$L = (X'^t_i X'_i)^{1/2} + \lambda g(X_i), \quad (8)$$

Where, λ is the value of the multiplier. In scalar notation,

$$L = \sqrt{X'^2_1 + X'^2_2 + \ldots + X'^2_n} + \lambda g(x_1, x_2, \ldots, x_n), \quad (9)$$

in which $X_i=\sigma_{xi}X'_i+\mu_{xi}$ where μ_{xi} and σ_{xi} are the means and standard deviations of the design variables. Minimizing L, we obtain (n+1) equations with (n+1) unknown as

$$\frac{\partial L}{\partial x_i} = \frac{X'_i}{\sqrt{X'^2_1 + X'^2_2 + \ldots + X'^2_n}} + \ldots + \frac{\lambda \partial g}{\partial X'_i} = 0, \quad (10)$$

and,

$$\frac{\partial L}{\partial \lambda} = g(X_1, X_2, \ldots, X_n) = 0. \quad (11)$$

The solution to equations (10) and (11) would yield the most probable failure point $(X'^*_1, X'^*_2, \ldots, X'^*_n)$. Introducing the gradient vector,

$$G = \frac{\partial g}{\partial X'_1}, \frac{\partial g}{\partial X'_2}, \ldots, \frac{\partial g}{\partial X'_n}, \quad (12)$$

in which

$$\frac{\partial g}{\partial X'_i} = \frac{\partial g}{\partial X_i} . \frac{\partial X_i}{\partial X'_i} = \sigma_{X_i} \frac{\partial g}{\partial X_i}. \quad (13)$$

Therefore, in vector form we have

$$\frac{X'}{(X'^t X')^{1/2}} + \lambda G = 0. \quad (14)$$

From which

$$X' = \lambda D G. \quad (15)$$

From equation (6)

$$D = [(\lambda D G^t)(\lambda D G)]^{1/2} = \lambda D (G^t G)^{1/2}, \quad (16)$$

and,

$$\lambda = (G^t G)^{-1/2}. \quad (17)$$

Where G^t is the transpose of the gradient vector G. Substituting equation (17) into equation (15) gives

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$$X' = \frac{-GD}{(G^t G)^{1/2}}. \qquad (18)$$

Multiplying both sides of equation (18) by G^t, the transpose of the gradient vector matrix, we have

$$G^t X' = \frac{-G^t GD}{(G^t G)^{1/2}} = -(G^t G)^{1/2} D, \quad (19)$$

Which implies

$$D = \frac{-G^t X'}{(G^t G)^{1/2}}. \qquad (20)$$

The minimum distance from the origin describing the variable space to the line representing the failure surface equals β and therefore equation (20) becomes

$$\beta = \frac{-G^{*t} X'^{*}}{(G^{*t} G^{*})^{1/2}}, \qquad (21)$$

Where, G* is the gradient vector at the most probable failure point (X'^{*}_1, X'^{*}_2, ..., X'^{*}_n). It is the value of β which tells us of the safety of any given design under uncertainties in the decision variables.

Stochastic models

The calculation of the stochastic model is performed for discrete combination of basic variables into the following equation considering tension failure of the joint:

$$G(x) = 0.78\frac{\pi\phi^2}{4}.\frac{P_T}{1000} - \frac{Pey_{\max}}{2\sum y^2}, \qquad (22)$$

into equation (23) considering shear failure of the joint:

$$G(x) = 0.78\frac{\pi\phi^2}{4}.\frac{P_S}{1000} - \frac{P}{No.of\ bolts}, \qquad (23)$$

And, into equation (24) considering combined tension and shear failure of the joint:

$$G(x) = 1.4 - \left[\frac{F_S}{P_S} + \frac{F_T}{P_T}\right], \qquad (24)$$

In equations (22) to (24), P_T and P_S are the magnitudes of permissible tensile and shear loads respectively, F_T and F_S are the values of applied tensile and shear loads respectively, P is the magnitude of applied eccentric load, φ is the diameter of bolt, 'e' is the eccentricity value and y_{max} is the maximum distance of the farthest bolt from the centroid of the bolt group.

Design is said to be satisfactory if conditions set-out in the code of practice is satisfied by estimating

$$P_f = P(G(X) \le 0), \qquad (25)$$

for varying values of the relevant design variables in the limit state equation.

The procedure of the FORM in the previous section, which was coded by Gollwitzer *et al.* (1988), was

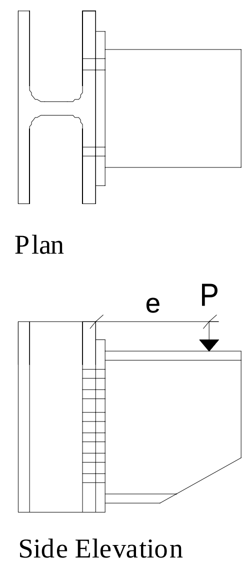


Fig. 1. Eccentric connection transmitting direct shear and tension

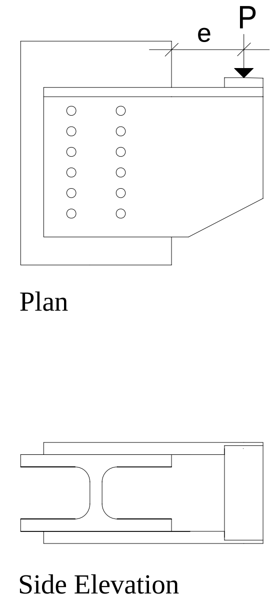


Fig. 2. Eccentric connection transmitting direct shear and torsion

Fig.3. The most likely failure point (Thoft-Christensen & Baker, 1982).

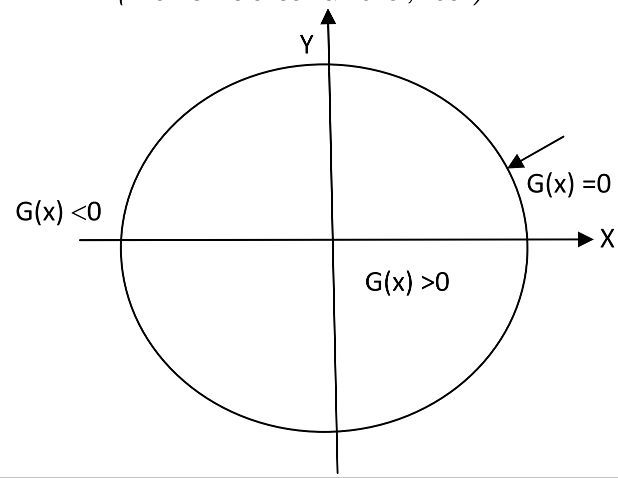
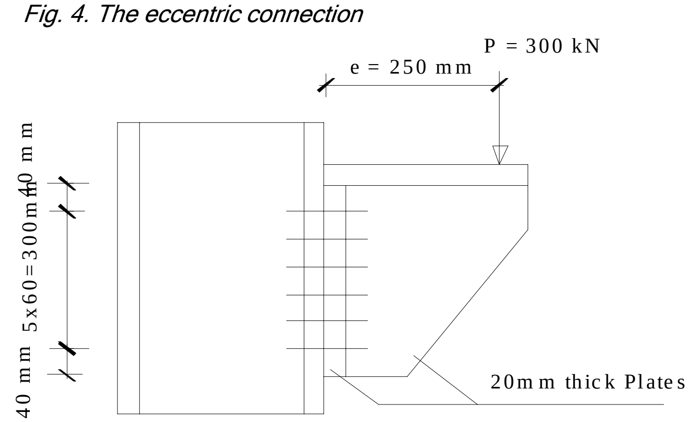


Fig. 4. The eccentric connection



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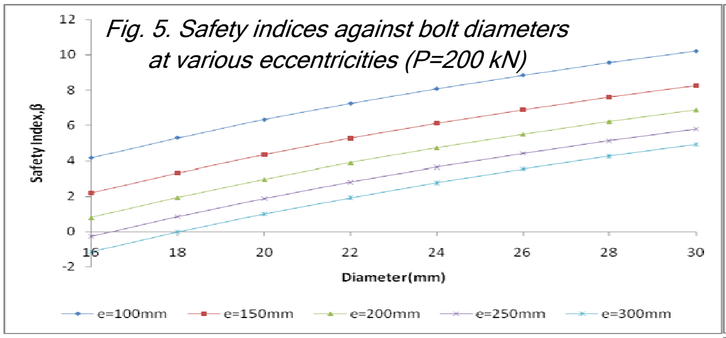

Fig. 5. Safety indices against bolt diameters at various eccentricities (P=200 kN)

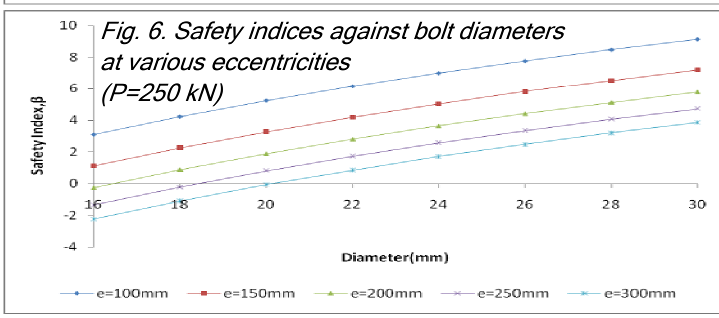

Fig. 6. Safety indices against bolt diameters at various eccentricities (P=250 kN)

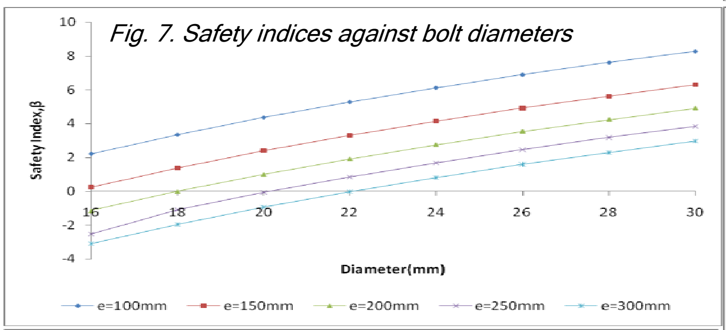

Fig. 7. Safety indices against bolt diameters

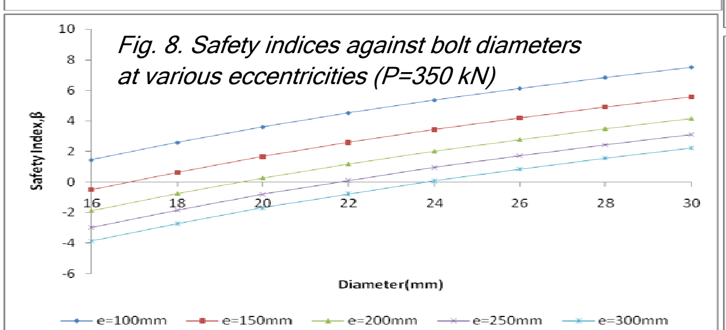

Fig. 8. Safety indices against bolt diameters at various eccentricities (P=350 kN)

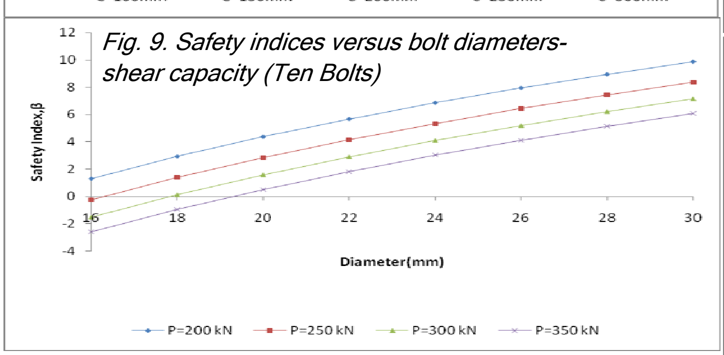

Fig. 9. Safety indices versus bolt diameters- shear capacity (Ten Bolts)

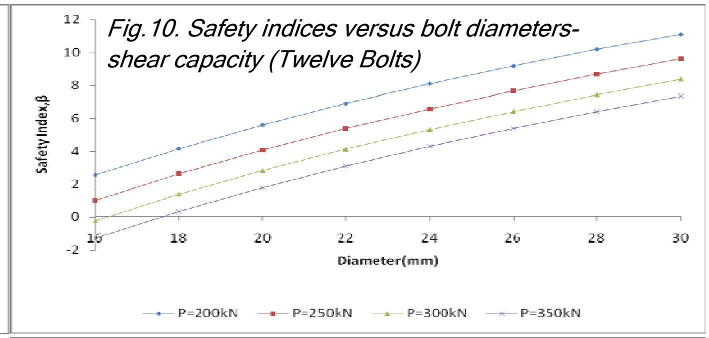

Fig.10. Safety indices versus bolt diameters- shear capacity (Twelve Bolts)

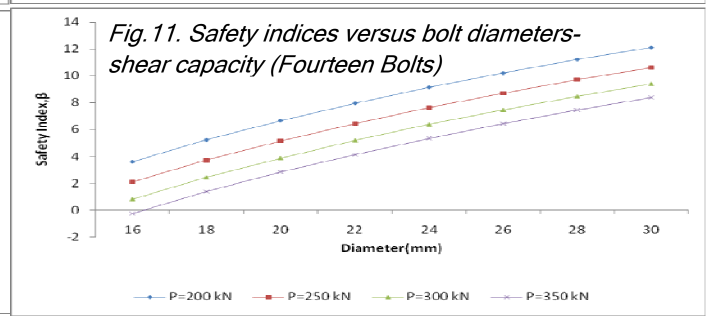

Fig.11. Safety indices versus bolt diameters- shear capacity (Fourteen Bolts)

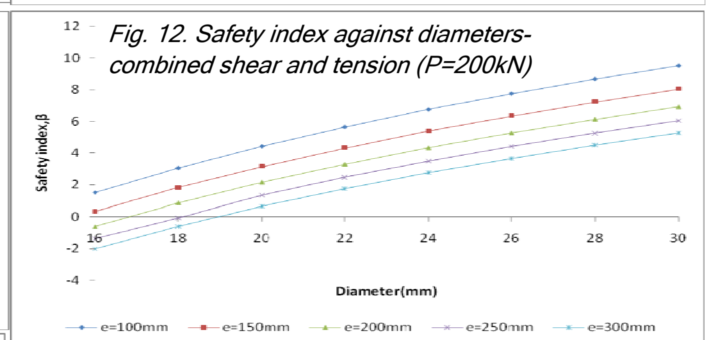

Fig. 12. Safety index against diameters- combined shear and tension (P=200kN)

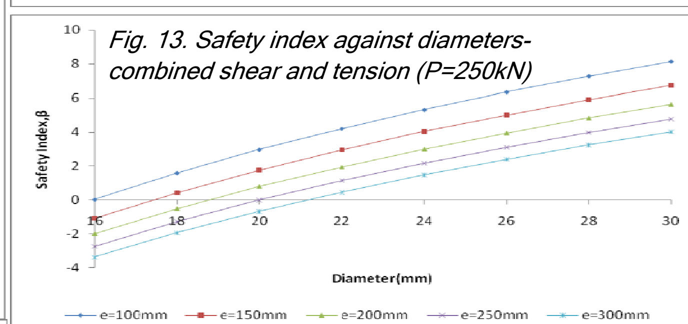

Fig. 13. Safety index against diameters- combined shear and tension (P=250kN)

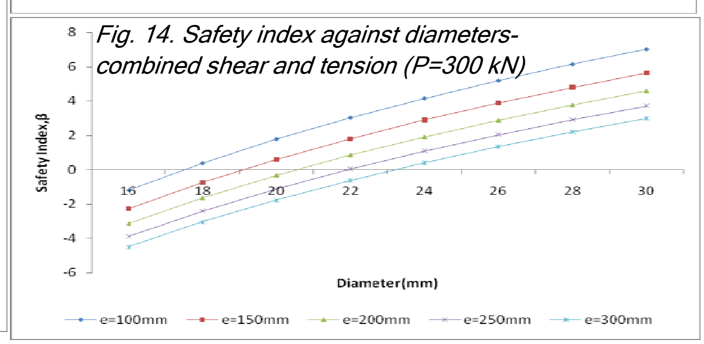

Fig. 14. Safety index against diameters- combined shear and tension (P=300 kN)

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employed for the computation of the reliability indices.

Example of eccentrically loaded connection in steel

A grade S275 steel eccentrically loaded bolted connection that transmits a factored load at a given eccentricity value, was designed using approximate method of analysis in accordance with the provisions set-out in BS5950 (2000).

Design of the Connection

Eccentric connection transmitting a factored applied load, P=300 kN at an eccentricity value of 250 mm was designed. The connection was designed with 12 No., M24 grade 4.6 ordinary bolts. Each set of row of bolts was placed at a spacing of 60mm centres, with overall depth of the connection to be 380 mm (Fig. 4). The connection was achieved by using 20mm thick plates to transmit the given loading to the 254x254UC73.

Results of reliability analysis

Reliability analyses of the connection designed in section 4 above was achieved by the use of FORM by estimating the reliability levels at varying values of bolt diameter, φ; eccentricity values, e; and applied factored load, P. Safety indices were obtained from the programs considering the failure criteria of the code (equations 22-24). Plots of the safety indices versus the varied design variables were as shown in Fig. 5-8, considering tensile failure criterion; Fig. 9-11, considering shear failure, while Fig. 12-14 when combined action of tensile and shear failure criteria was considered. From the plots it can be observed that:

a. As the magnitude of applied eccentric load, P increases, the safety of the designed section decreases.
b. Also, as the diameter of bolt increases, the safety of the designed section increases.
c. From the figures, it can be said that the design criteria of eccentrically loaded bolted connection based on the requirements of BS5950 (2000) is fairly consistent.
d. It could also be seen that with higher eccentricities of 200 to 300 mm, the negative safety indices imply that, the corresponding bolt diameters are not practicable for the connection (NKB, 1978).
e. Also, the 12 No. 24 mm bolt diameter at the constant load of 300 kN (see section 4.1), the safety indices corresponding to the tensile and combined criteria are below the recommended values (JCCS, 2001), while shear failure criterion satisfy this condition.
f. Again, at the given eccentric load and bolt diameter, the safety indices from Fig. 7 & 14, only eccentricities of 100 to 150mm, and 100mm are safe for tensile and combined failure criteria respectively.
g. The target safety indices of tensile, shear and combined failure criteria are 3.59, 5.13 and 2.26 respectively. Based on JCCS (2001), shear design criterion is safe but uneconomical, tensile design criterion is satisfactory and combined criterion is economical and tends to unsafe region. The shear and combined design criteria therefore need to be reviewed in order to achieve a balanced design that gives a better compromise between safety and economy.
h. It can therefore be inferred from the foregoing that the combined failure criterion is more critical for this joint, for range of eccentricity values, bolt diameters and applied loads considered. This implies that the margin given in eqn. (24) is too narrow for a practicable design of the joint.

Conclusion

Reliability analysis of eccentric connection fabricated from grade S275 steel plates, in direct shear and tension using approximate method of analysis in accordance with BS5950 (2000) was carried using First Order Reliability Method (FORM). Design variables such as applied load, eccentricity of applied loading, bolt diameter; considering various failure criteria of the connection were considered random and stochastic. The BS5950 design criteria of these joints were found to be fairly consistent. It was also shown among other findings that, the target safety indices of tensile, shear and combined failure criteria are 3.59, 5.13 and 2.26 respectively. Based on JCCS (2001), while shear design criterion is uneconomical, tensile design criterion is satisfactory, and combined criterion economical and tends to unsafe region. The shear and combined design criteria therefore need to be reviewed in order to achieve a balanced design that gives a better compromise between safety and economy. It was also shown that the combined failure criterion is more critical for this joint than tension and shear criteria, for range of eccentricity values, bolt diameters and applied loads considered. This is because of the margin given in the code of practice that seems too narrow for a practicable design of the joint.

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