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On Dodge-Romig Single Sampling Inspection Tables under Average Quality Protection

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Abstract

Objectives: This paper aims at presenting an easy method of designing Single Sampling Plans (SSP) by attributes inspection based on Dodge-Romig method under Average Quality Protection. **Methods/Statistical Analysis:** A computational algorithm is developed for obtaining the parameters of the SSP, viz., the sample size and the acceptance number. A simple computer programme under the Poisson model for the OC function is developed for the computational algorithm using which the parameters of the SSP for desired levels of Average Outgoing Quality Limit (AOQL). **Findings:** The sample size and acceptance number constituting the SSP obtained using the procedure described in this paper are found to provide closer values of the desired AOQL and the minimum Average Total Inspection (ATI) than those obtainable from Dodge and Romig. The simple computer programme replaces the huge volumes of tables of Dodge and Romig. The procedure avoids the approximations involved in the original Dodge-Romig method and thus results in better plans. **Application/ Improvements:** The Single Sampling Plans under average quality protection are applicable in attributes inspection where ever rectification is possible.

Keywords: Average Outgoing Quality Limit (AOQL), Average Total Inspection (ATI), Operating Characteristic (OC) Function, Poisson Distribution, Single Sampling Plan (SSP)

1. Introduction

Acceptance sampling by attributes inspection is applicable where, under normal conditions, it is more profitable to inspect a sample and to accept or reject a lot rather than to perform an expensive 100 percent inspection in order to screen out any nonconforming units[1]. Sampling serves for assuring that everything is in 'normal conditions'. Only if the sampling results indicate that there are abnormal conditions, 100 percent inspection is performed.

When a consumer adopts sampling inspection in place of 100 percent inspection, he foregoes the opportunity of assuring himself that each unit of product will conform to specifications. To assure economic success, the consumer must choose a sampling plan that will provide a degree of protection against non-conforming material that is consistent with his needs. This choice may be narrowed down by choosing some value of allowable proportion non-conforming and by deciding whether this value should apply to a limited quantity of product, such as a lot or to the general output comprising a more or less steady flow of lots.

In *day-to-day* inspection work, one is interested in the least amount of inspection in the light of economy. Accordingly Dodge and Romig developed a mathematical model for sampling inspection combined with 100 percent inspection of rejected lots and determined the

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optimum sampling plan within this model by minimizing the average amount of total inspection[1].

1.1 Assumptions

The Dodge-Romig tables are applicable under the following assumptions[1,3].

1. The manufacturing process is normally in binomial control with a process average equal to p_1.
2. Inspection is rectifying and rejected lots are totally inspected.
3. To make sure that the average quality of his product is satisfactory the producer chooses an AOQL= p_L , and consider only sampling plans satisfying this specification.
4. Among plans having the specified AOQL the producer chooses the one minimizing average total inspection for product of process average quality.

1.2 Operating Procedure

1. For each lot, select a sample of n units and test each unit for conformance to the specified requirements.
2. Accept the lot if d (the observed number of non-conforming units) is less than c or less nonconforming units in the sample of n units.
3. Otherwise reject the lot.

Operating characteristic function of Single Sampling Plan is given by, $P_a(p) = P(d \leq c, n)$. Single Sampling Plan is characterized by two parameters viz., the sample size, n and the acceptance number, c.

Dodge and Romig developed certain mathematical approximations and created a few nomographs using approximate relations to obtain the parameters (n,c) of single sampling plans for the above stated conditions[1]. In order to arrive at a simple solution, they have used Binomial and Poisson approximations to the hypergeometric at various levels of constraints as exact solution to hypergeometric models were impracticable. They could successfully present various tables giving the parameters for single and double sampling plans under the two kinds of consumer protection, conveniently indexed by intervals of lot size and intervals of process average for a single designed AOQL value[3].

Dodge-Romig tables have been a major source of sampling plans for the users till today for their simplicity in practical use. The sampling plans given in Dodge-Romig tables are not always optimum and exact solutions are possible under Poisson model[2]. This motivated to sort out the differences in the use of mixed Binomial and Poisson approximations together with hypergeometric in the development of tables by Dodge and Romig and develop an optimum procedure with only Poisson approximation[3].

2. Procedure of Dodge - Romig Single Sampling Inspection Tables under Average Quality Protection

The following is procedure for obtaining (n, c) which minimizes average total inspection ATI at p= $\overline{p}$ for specified values of lot size (N), process average $\overline{p}$, and AOQL (p_L)[1,3].

When the fraction nonconforming in submitted products is p, the average quality after inspection (p_A) is given by

$$p_A = \frac{p(N-I)}{N} \tag{1}$$

And the average number of pieces inspected per lot (I) for product of p quality is given by

$$I = n + (N - n)(1 - P_a(p)) \tag{2}$$

The average quality after inspection (p_A), after substituting in equation (1) the value of I given in the equation (2), under Poisson model becomes,

$$p_A = \frac{p(N-n)}{N}\sum_{m=0}^{c}\frac{e^{-np}np^m}{m!} \tag{3}$$

Differentiating p_A with respect to p gives,

$$\frac{dp_A}{dp} = \frac{N-n}{N}\left[\sum_{m=0}^{c}\frac{e^{-np}np^m}{m!} - \frac{e^{-np}np^{c+1}}{c!}\right] \tag{4}$$

Equating (4) to zero and solving for p gives the value of p say $p = p_1$ that makes p_A maximum, i.e., $p_A = p_L$.

Substituting $np_1 = x$, $p = p_1$ and $p_A = p_L$ in equation (3),

$$p_L = \frac{N-n}{Nn}x\sum_{m=0}^{c}\frac{e^{-x}x^m}{m!} \tag{5}$$

$$p_L = y\left(\frac{1}{n} - \frac{1}{N}\right) \tag{6}$$

Where, $$y = x\sum_{m=0}^{c}\frac{e^{-x}x^m}{m!} \tag{7}$$

Similarly, substituting np_1 = x in equation (4) and equating to zero implies

$$\left[\sum_{m=0}^{c}\frac{e^{-np}np^{m}}{m!}-\frac{e^{-np}np^{c+1}}{c!}\right]=0 \tag{8}$$

Substituting second term of (8) for summation term in (7) gives

$$y=\frac{e^{-x}x^{c+2}}{c!} \tag{9}$$

These relations provide a basis for determining the values of x and y, corresponding to specific values of c.

The value of *c* that minimizes average total inspection ATI (I) (at $p=\bar{p}$), is obtained directly from the equation (6) and (2). The value of *n* corresponding to value of *c* is determined from equation 6, expressed as,

$$n=\frac{yn}{p_L N+y} \tag{10}$$

A simple computer program is developed for the above procedure using which the parameters (n, c) of SSP can be obtained minimizing the ATI (I) for the desired values of N, process average $\bar{p}$, and AOQL (p_L)[3].This program replaces the entire set of Dodge-Romig tables. Table 1 is a sample table giving Single Sampling Plans for desired combination of process average $\bar{p}$ and AOQL (p_L) developed using the computer program.

3. Algorithm to Find n and c

The algorithm for obtaining the parameters (n, c)[3]:

1. Input N, $\bar{p}$, and p_L
2. Arbitrarily fix a value of c, say c=0 and vary the value of x(x=0.001(0.001)20) until equation (8) is satisfied for particular 'x'
3. Using the pair of 'x' and 'c' which satisfy the equation (8) and find the value of y from equation (9)
4. Find the value of n using equation (10) corresponding to y value found in step (3).

Table 1. Values of n and c indexed by AOQL (p_L) and Process Average ($\bar{p}$) for Single Sampling Plan, when N=10003

$\bar{p}$ / p_L	0.01 n c	0.02 n c	0.03 n c	0.04 n c	0.05 n c	0.06 n c	0.07 n c	0.08 n c	0.09 n c	0.10 n c	0.12 n c	0.14 n c	0.16 n c	0.18 n c	0.20 n c
0.001	36 0	19 0	13 0	10 0	8 0	7 0	6 0	5 0	5 0	4 0	4 0	4 0	3 0	3 0	2 0
0.002	78 1	41 1	28 1	21 1	17 1	14 1	12 1	11 1	10 1	9 1	7 1	6 1	6 1	5 1	5 1
0.003	78 1	41 1	28 1	21 1	17 1	14 1	12 1	11 1	10 1	9 1	7 1	6 1	6 1	5 1	5 1
0.004	78 1	41 1	28 1	21 1	17 1	14 1	12 1	11 1	10 1	9 1	7 1	6 1	6 1	5 1	5 1
0.005	78 1	41 1	28 1	21 1	17 1	14 1	12 1	11 1	10 1	9 1	7 1	6 1	6 1	5 1	5 1
0.006	78 1	41 1	28 1	21 1	17 1	14 1	12 1	11 1	10 1	9 1	7 1	6 1	6 1	5 1	5 1
0.007	121 2	41 1	28 1	21 1	17 1	14 1	12 1	11 1	10 1	9 1	7 1	6 1	6 1	5 1	5 1
0.008	121 2	65 2	28 1	21 1	17 1	14 1	12 1	11 1	10 1	9 1	7 1	6 1	6 1	5 1	5 1
0.009	121 2	65 2	44 2	21 1	17 1	14 1	12 1	11 1	10 1	9 1	7 1	6 1	6 1	5 1	5 1
0.01	121 2	65 2	44 2	34 2	17 1	14 1	12 1	11 1	10 1	9 1	7 1	6 1	6 1	5 1	5 1
0.015	163 3	89 3	44 2	34 2	27 2	23 2	20 2	17 2	15 2	14 2	7 1	6 1	6 1	5 1	5 1
0.02		113 4	61 3	47 3	27 2	23 2	20 2	17 2	15 2	14 2	12 2	10 2	9 2	8 2	7 2
0.025		137 5	79 4	47 3	38 3	32 3	20 2	17 2	15 2	14 2	12 2	10 2	9 2	8 2	7 2
0.03		137 5	96 5	60 4	38 3	32 3	27 3	24 3	22 3	14 2	12 2	10 2	9 2	8 2	7 2
0.04			130 7	74 5	60 5	41 4	36 4	31 4	22 3	20 3	16 3	14 3	12 3	8 2	7 2
0.05				101 7	71 6	51 5	44 5	31 4	28 4	25 4	16 3	14 3	12 3	11 3	10 3
0.06					94 8	70 7	52 6	39 5	34 5	31 5	21 4	18 4	12 3	11 3	10 3
0.07						89 9	69 8	53 7	41 6	37 6	26 5	18 4	16 4	14 4	10 3
0.08							77 9	61 8	48 7	43 7	31 6	23 5	20 5	14 4	13 4

5. Use the 'c' from step (2) and 'n' from step (4) in equation (2) and compute the value of ATI and store it.
6. Repeat step (1) to (5) and find different values of ATI for different combination of n and c. Choose the pair of (n, c) for which the ATI is minimum as the required parameters of single sampling plan

To demonstrate the above algorithm few examples are provided below[3].

Example 3.1:
Let N= 1500, AOQL (p_L) = 0.01 and process average ($\overline{p}$) = 0 .008;
Computer Program: n = 126 and c = 2 with minimum ATI = 236.87 (237)
Dodge-Romig Tables: n = 130 and c = 2 with minimum ATI = 250.29 (251)

Example 3.2:
Let N= 500, AOQL (p_L) = 0.02 and process average ($\overline{p}$) = 0.01;
Computer Program: n = 39 and c = 1 with minimum ATI = 66.145 (67)
Dodge-Romig Tables: n = 39 and c = 1 with minimum ATI = 66.145 (67)

Example 3.3:
Let N= 25000, AOQL (p_L) = 0.02 and process average ($\overline{p}$) = 0.015;
Computer Program: n =391 and c = 12 with minimum ATI = 572.51(573)
Dodge-Romig Tables: n = 395 and c = 12 with minimum ATI = 592.08 (593)

Example 3.4:
Let N= 4500, AOQL (p_L) = 0.04 and process average ($\overline{p}$) = 0.02;
Computer Program: n = 78 and c = 5 with minimum ATI = 101.07 (102)
Dodge-Romig Tables: n = 80 and c = 5 with minimum ATI = 106.69 (107)

Example 3.5:
Let N= 5000, AOQL (p_L) = 0.01 and process average ($\overline{p}$) = 0.009;
Computer Program: n = 298 and c = 5 with minimum ATI = 556.14 (557)
Dodge-Romig Tables: n = 300 and c = 5 with minimum ATI = 566.63 (567)

4. Discussion

From the above examples it is apparent that the plans obtained using the procedure outlined are the better plans. The procedure gives a plan for any desired combination of N, $\overline{p}$, and p_L. Dodge-Romig tables involve rounding off the sample size to the nearest 5 units whenever the sample size is over 50 and rounding off to the nearest 10 units for extreme large samples. In industrial situations inspecting even an additional unit than necessary is undesirable. Also Dodge-Romig tables provide sampling plans for larger intervals for lot size and process average, Dodge-Romig tables do not ensure a sampling plan for any specified N, $\overline{p}$ and p_L. The procedure presented here can be used to obtain an optimum plan that in every way protects the interest of the consumer. The computer program developed for this procedure gives a sampling plan for desired combination of N, $\overline{p}$ and p_L [4,5]. Hence the program replaces the total list of Dodge-Romig AOQL tables.

5. Computer Program

The computer program developed is available on request from the authors.

6. Conclusion

The simple alternative method for designing SSP by attributes inspection under average quality protection significantly overcomes the approximations of the tables provided by Dodge and Romig. The advantages of the procedure presented in this paper are evidenced empirically by the examples and Table 1. More over a SSP can be obtained for any combination of desired values N, $\overline{p}$ and p_L.

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