Synchronization Error Investigation of an Autonomous Chaotic System in Signal Masking Application

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Abstract

Objectives: In recent years signal masking application is very popular and lot of work has been published. Most of the work suggested that to maintain synchronization between transmitter and receiver, amplitude of the message signal should be low keeping the frequency constant. The objective of this paper is to investigate the measurement of synchronization error between coupled transmitter and receiver. The paper shows how synchronization error varies with the changing amplitude and frequency of the encrypted message signal. **Methods:** The system taken in this work is described by ordinary differential equations and implemented with simple electronic components. The transmitter and receiver system are identical and coupled through a reference signal. When message signal is added to the chaotic signal, synchronization error occurs which effects the exact recovery of the message signals at receiver. Synchronization error has been calculated by keeping the frequency constant and varying the amplitude of the message signal and vice versa. An improved synchronization scheme with feedback is also implemented to see the effect of feedback on synchronization of coupled chaotic systems. **Findings:** These works gives more insight into signal masking application and conclude that the encrypted signal amplitude should be as low as possible for better synchronization and its frequency should be kept high. If feedback is used masking will be better and synchronization error is reduced because now the dynamics of transmitter is also change in accordance with the receiver. With feedback scheme the error is constant for the range of frequencies and message signal amplitude. **Applications:** The feedback scheme is useful for those signals which have changing amplitude or changing frequency. It has been observed that the synchronization error reduces considerably in case of improved synchronization scheme with feedback. This work can also be extended for masking digital signals.

Keywords: Chaotic Masking, Identical Synchronization, Synchronization Error

1. Introduction

It has been discovered that by arranging the chaotic systems in a specific way, one could achieve identical chaotic behavior even if the systems were isolated, implying that it is possible to design synchronizing systems driven by chaotic signals[1]. Synchronization of Lorenz system and its circuit implementation is also been given[2,3]. Synchronization between chaotic signals has received considerable attention and led to various communication applications such as signal masking, encryption and in signal processing. Recently many chaotic systems are designed to be used in signal masking applications[4-8]. The main problem with the chaotic masking is that when an encrypted information signal is sent along with the chaotic (reference) signal, the synchronization between transmitter and receiver system may be disturbed because the message

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signal directly affects the dynamics of the receiver system[9]. All the previous results shown in reference[1-5] put constraints on maximum amplitude of the message signal and it is found necessary that the message signal strength be too small for an appropriate synchronization to exist. However, if amplitude of the message signal is large, synchronization is disturbed causing an error in signal recovery[10]. In reference[4,5,11-14] single frequency message signal is taken to demonstrate synchronization, none of the research papers has investigated that what is the effect of message frequency on synchronization of chaotic systems which are used as a transmitter and receiver. Therefore it is required to study the effect of frequency change on synchronization.

The present paper demonstrates that the synchronization between coupled transmitter and receiver gets affected not only by the variation in amplitude of the encrypted message signal but also by varying the frequency of the message signal. These effects are investigated by evaluating synchronization error for varying message signal strength and its frequency. While analyzing the effects it is found that the synchronization is poor for lower frequency message signal. The reason behind is that the chaotic signal has dominating low frequency components. If the low frequency message signal is added to this chaotic signal, it will effects those frequencies which are dominating and thus effects the synchronization. While addition of high frequency message signal does not effects the synchronization. Synchronization can be improved if a feedback signal is being incorporated to the inputs of the master system so it can generate the chaotic signal which matches with the signal generated at the slave system. With this arrangement chaotic signal is being modified according to the sum signal which is the feedback signal and the synchronization can be achieved. In this paper a masking scheme with feedback is designed to reduce the synchronization error. The paper is organized as follows. Section 2 describe an autonomous chaotic system dynamics and signal masking. Section 3 describes the synchronization error analysis of the system .In section 4 an improved masking scheme is discussed with circuit simulation. Section 5summarize the main results obtain in this paper.

2. Signal Masking

The chaotic system used here is described by three ordinary differential equations8:

$$\dot{x} = y$$
$$\dot{y} = z$$
$$\dot{z} = -ax - by - cz - dx^2 \tag{1}$$

where x, y and z are the state variables. Basically the system behaves as a chaotic oscillator which generate three chaotic signals x(t),y(t) and z(t).The circuit Equations for the given system are:

$$\dot{x} = \frac{1}{C1R1} y$$
$$\dot{y} = \frac{1}{C2R2}$$
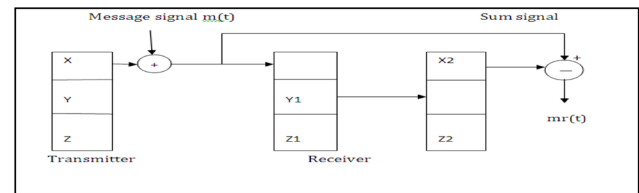$$\dot{z} = -\frac{1}{C3R4}x - \frac{1}{C3R5}y - \frac{1}{C3R6}z - \frac{1}{10C3R3}x^2 \tag{2}$$

Figure 1. Block Diagram of Chaotic Signal Masking.

The message signal taken at transmitter is a single frequency sinusoid. Chaotic masking scheme is shown in Figure 1. Reference chaotic signal x (t) and message signal m (t) is added to form a sum signal s(t), being transmitted to the receiver. Receiver consist of two stable sub systems in order to generate the chaotic signal x_2(t) which is synchronized with the transmitted reference signal x (t).At receiving end x_2(t) is subtracted from transmitted signal s(t),to recover the message signal. The receiver sub systems are defined by two sets of equations:

$$\dot{y1} = z_1 \tag{3}$$
$$\dot{z1} = -ax - by_1 - cz_1 - dx^2$$

And

$$\dot{x2} = y_1$$
$$\dot{z2} = -ax_2 - by_1 - cz_2 - dx_2^2$$

To find out the stability of the subsystem let the difference system for $\Delta y = y - y_1$ and
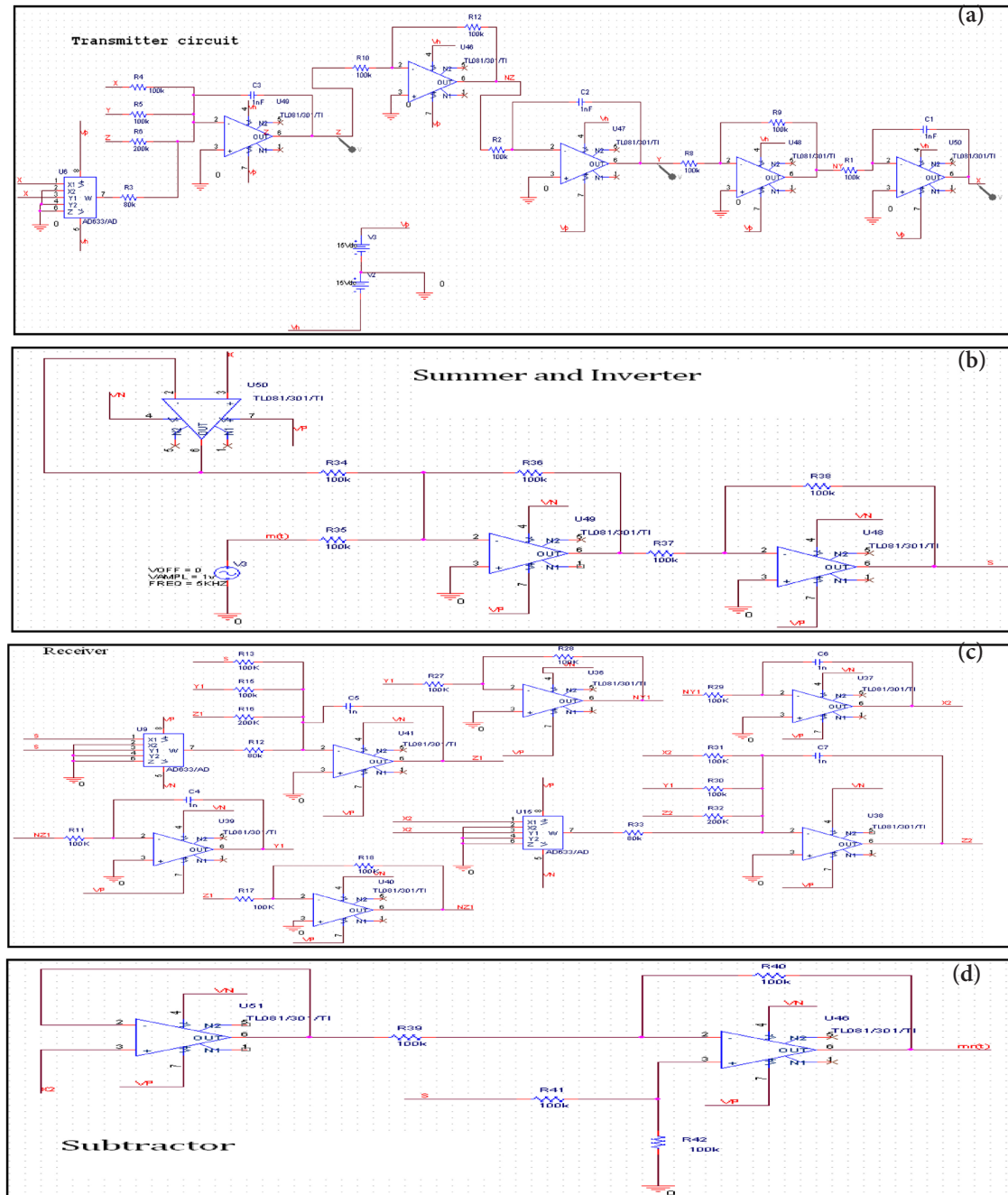$\Delta z = z - z_1$ in matrix form is given in (4).

$$\begin{bmatrix} \dot{\Delta y} \\ \dot{\Delta z} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -b & -c \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta z \end{bmatrix} \tag{4}$$

Equation (4) is the basic equation describing the stability of the perturbation transverse under x(t) drive signal from the transmitter. Here the Jacobian matrix defined for difference system is a constant matrix, thus the Eigen values for this matrix are – 0.25±0.968i. The Eigen values are negative for the onset of synchronization. The solution of (4) is given in (5).

$$\Delta y = \Delta z = e^{-0.25t}(k1\cos 0.968t + k2\sin 0.968t) \quad (5)$$

Here k1 and k2 are constants of integration. It is observed that as time t ⟶ ∞, Δy = Δz = 0, hence transmitter and receiver are synchronized if x(t) is selected as reference signal for receiver system. The system in Figure 1 is realized by the circuit diagram shown in Figure 2(a-d). The parameters of transmitter and receiver system are taken identical for implementing chaotic masking application. The values of static parameters are chosen as a = 1, b = 1, c = 0.5 and d = 0.125. The values of


Figure 2 (a). Transmitter Circuit (b) Summer and Inverter(c) Receiver Circuit and (d) Subtractor Circuit.

dynamic variables x,y and z varies with time. The initial values of x, y, z is taken as (0.1, 0.1and 0.1) at transmitter circuit and at receiver the initial conditions are (0.1, 0.1) and (0.1, 0.1).
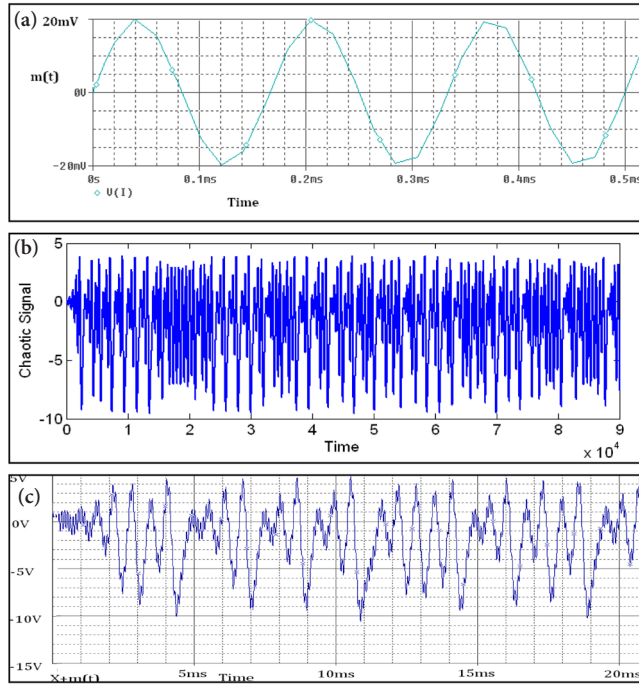
Figure 3   (a). Message Signal m(t) (b) Chaotic Signal x(t) (c) Sum Signal s(t).
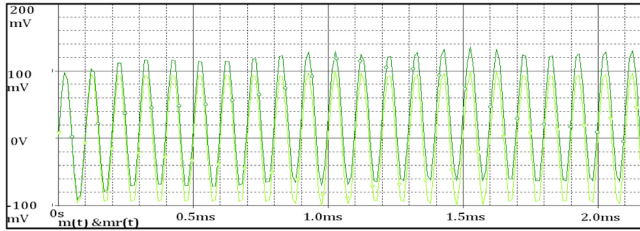
Time domain representation of message signal m(t) ,chaotic signal x(t) and the sum signal is shown in Figure 3(a-c) and their frequency domain representation is shown in Figure 4(a-c). We can observe from the Figure 3(c) that the message signal is completely hid by the chaotic signal in time domain. The message signal m(t) is a sinusoidal signal of amplitude 20mV and frequency is 6kHz, thus power spectral density is an impulse function centered at 6kHz as shown in Figure 4(a). The power spectral density of chaotic signal x(t) is broad and flat as shown in Figure 4(b). After adding the message signal with the chaotic signal the power spectral density of the sum signal s(t) is shown in Figure 4(c),in which the message signal peak is clearly identified at 6kHz frequency over the chaotic signal .Thus the message signal can be recovered without any need of chaotic receiver by simply passing the sum signal through a low pass filter. This makes the system's security weak and it can be broken without knowing the reference signal. If the message signal

frequency is kept low i.e.≤ 4kHz then it is completely hid by the chaotic signal as shown in Figure 4 (d). It is observed that if any chaotic system is used for masking purpose than its power spectrum must be drawn and looking to its spectrum, amplitude of the message signal and its frequency is chosen in such a manner that signal is totally encrypted .With these observations it is needed to investigate synchronization error with varying signal amplitude and its frequency.
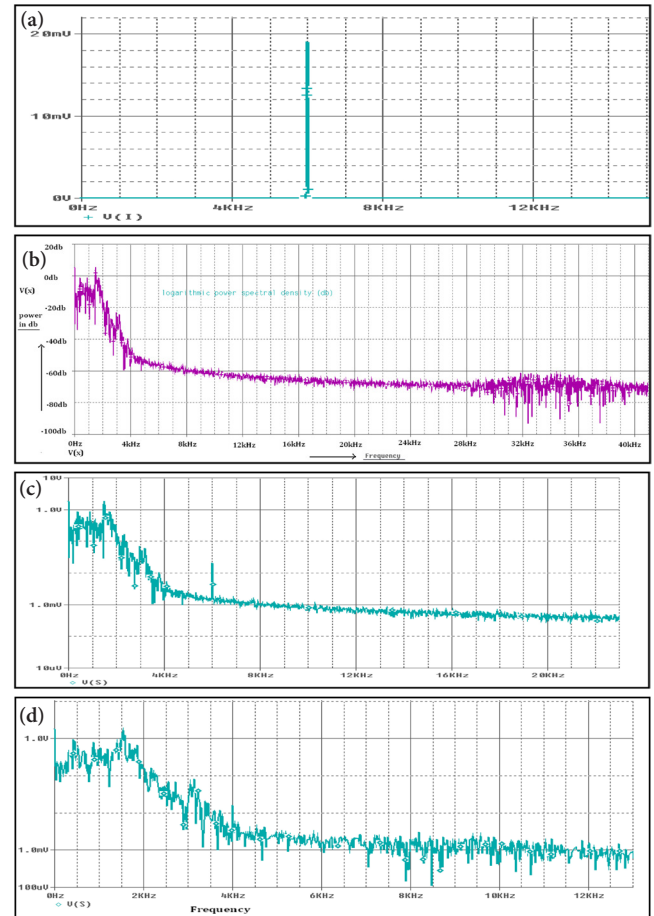
Figure 4.   Power spectral density of (a) message signal m(t) (b) chaotic signal x(t) (c) Sum signal s(t)when frequency of message signal is 6kHz (d)Sum signal s(t)when frequency of message signal is 4kHz.

For recovering hidden message signal at receiver, chaotic signal $x_2(t)$ is subtracted from incoming sum signal s(t) and message signal is recovered as shown in Figure 1. Figure 5 shows the plots of transmitted signal m(t) and received signal $m_r(t)$.The recovered signal is slightly differ with the transmitted one because of lack of synchronization.

Figure 5. Transmitted Signal m (t) and received Signal mr (t).

Figure 6. Plot of Normalized Synchronization Error VsSignal Amplitude

Figure 7. Plot of Normalized Synchronization Error VsSignal Frequency.

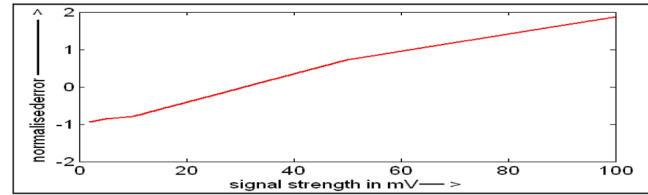# 3. Error Analysis of System with Varying Amplitude and Frequency of the Message Signal

In this section it is presented that synchronization is perturbed if signal frequency or amplitude is changed. This is because synchronization error occurs between transmitter and receiver. Let vectors d =[x, y, z] and r = [x_r, y_r, z_r] represents the transmitter and receiver state variables. Then the synchronization error is defined as the difference function of state vectors of transmitter and receiver [6] when time t turns to infinite as:
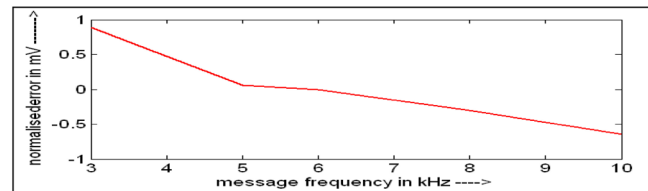
$$e = \lim_{t \to \infty} [r(t) - d(t)] \qquad (6)$$

In our case x(t) is a transmitter state variable or signal, which carries message signal and x_r(t) is a receiver state variable or signal which is required to recover the desired message signal. Therefore the synchronization error is defined as:

$$e = [xr(t) - x(t)] \qquad (7)$$

Error curve has been plotted for different message amplitudes and varying frequencies. Figure 6 shows the variation of normalized synchronization error with changing amplitude of the message signal from 2mV to 0.1V while frequency of the message signal is keep constant as f= 10KHz.Simulation time is taken as 1ms. Variation of normalized synchronization error with changing message frequency from 3 KHz to 10 KHz while keeping the amplitude of message signal constant as 20mVis shown in Figure 7.

The results show that the synchronization error increases with increasing signal strength because it directly affects the dynamic of receiver system and at low frequencies synchronization error is more than at the higher frequencies. This might be due to dominance of low frequency components in the chaotic signal and mixing at low frequencies effects more on processes of synchronization. Synchronization Error can be reduced if dynamics of transmitter system tracks the changing message signal, which can be obtained by a feedback signal.

4. Improved Synchronization with Dynamic Feedback

A system has been designed with improved synchronization mechanism which is independent of signal frequency and amplitude. Transmitter system is defined by the state equations as follows:

$$\dot{x} = f(y) \qquad (8)$$
$$\dot{y} = f(z)$$
$$\dot{z} = f(x, y, z)$$

In general the transmitter system is defined as $\dot{d}$ = f(x, y, z) where x, y, z $\in$ R^n are the state vectors. The x state is added with the message signal and a sum signal s(t) is generated and sent to the receiver sub system 1 as a

reference signal .The receiver sub system1is defined by following state equation

$$\dot{y1} = f(z1) \tag{9}$$
$$\dot{z1} = f(s, y1, z1)$$

In general the receiver sub system 1 is defined as $\dot{r1}$ = $f(s, y_1, z_1)$.The y_1 state is sent as a reference signal to receiver sub system 2 .The receiver sub system 2 is defined by following state equation:

$$\dot{x2} = f(y1) \tag{10}$$
$$\dot{z2} = f(x2, y1, z2)$$

In general the receiver sub system 2 is defined as $\dot{r2}$ = $f(x_2, y_1, z_2)$.It can be easily observed by the general state equations of transmitter and receiver system that the synchronization can be achieved if drive equation is changed to $\dot{d} = f(s, y, z)$ .This is done by taking a feedback signal after adding the message signal and apply it to the input of drive system. The block diagram of the improved scheme is shown in Figure 8.
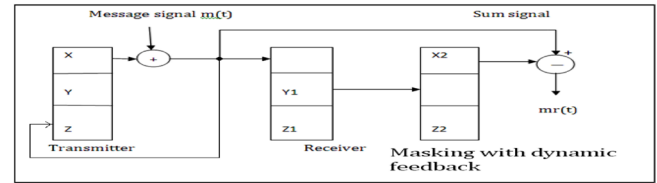

Figure 8. Block Diagram of Improved Masking Scheme

Due to feedback the dynamics of transmitter changes according to the amplitude and frequency of message signal in a same manner as it changes the dynamics of receiver. Circuit simulation of the improved system with feedback using Orcad Pspice programs are shown in Figure 9.

To investigate the effect of feedback on synchronization error ,the graph showing normalized synchronization error Vs. message signal strength is
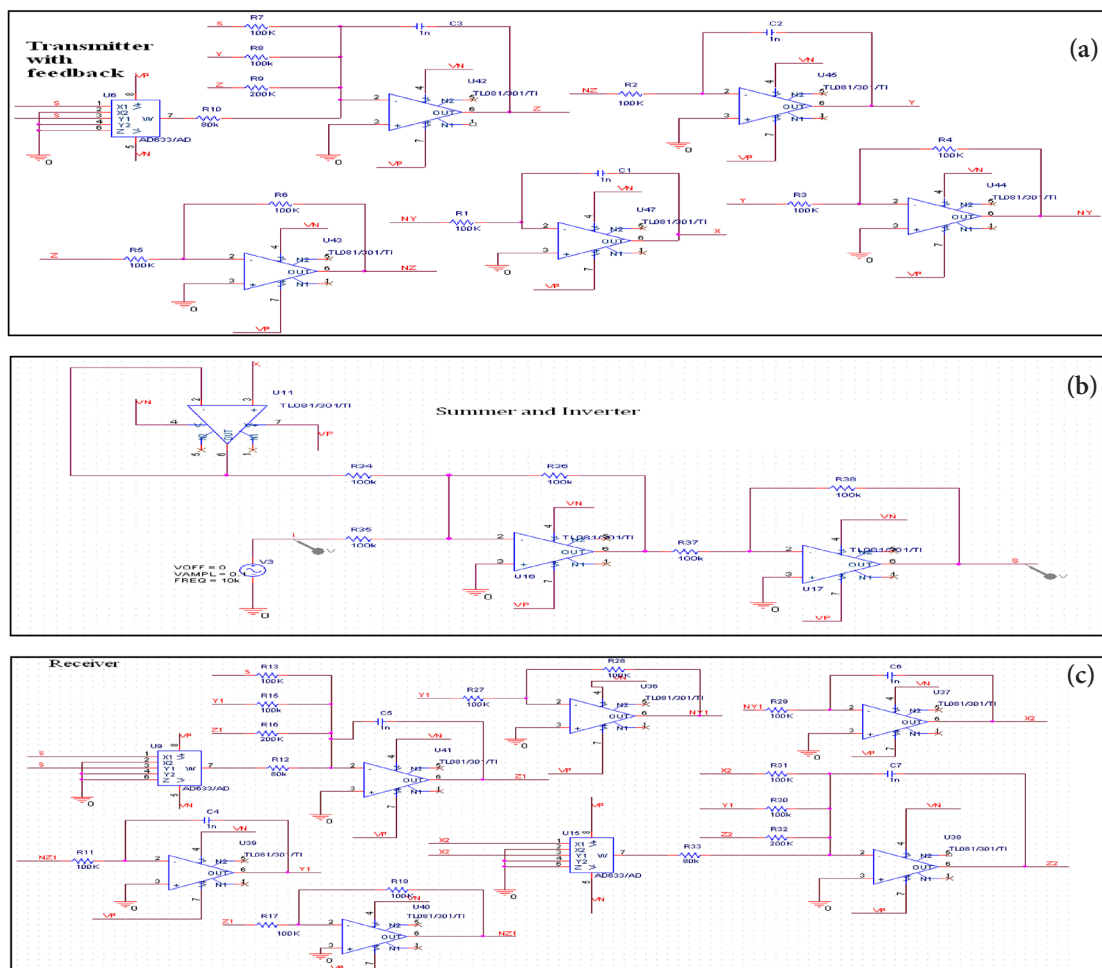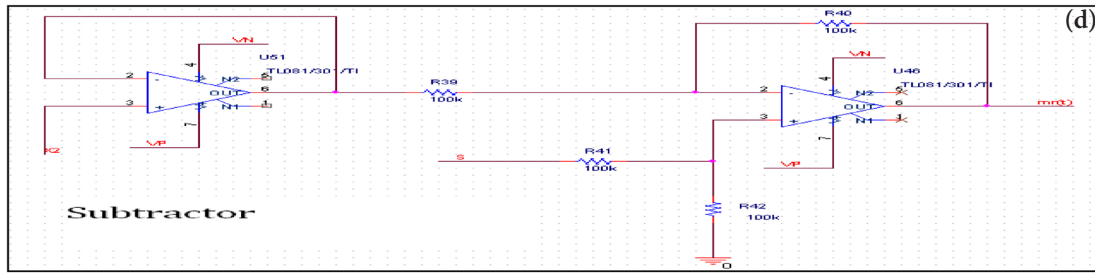
Figure 9. Improved Transmitter and Receiver circuits with Dynamic Feedback (a) Transmitter circuit (b) summer and Inverter(c)Receiver circuit and (d) Subtractor circuit.

plotted keeping the signal frequency constant as f = 10 KHz is shown below in Figure 10. The graph of normalized synchronization error Vs. frequency is shown in Figure 11 while keeping the amplitude of the signal fix at 20mV.
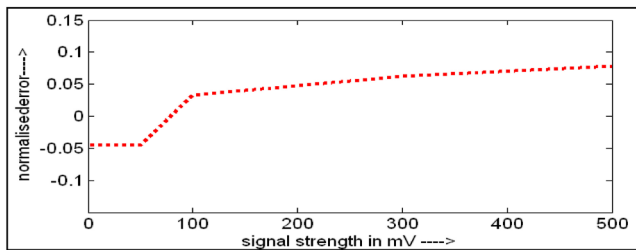
Figure 10. Plot of Improved Normalized Error Vs.Signal Strength.
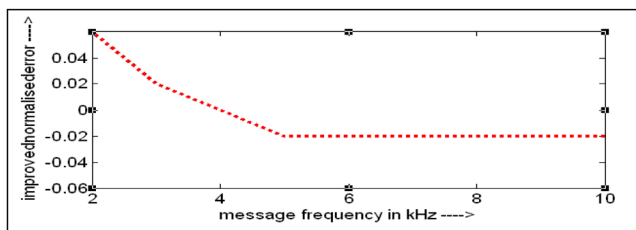
Figure 11. Plot of Improved Normalized Synchronization Error Vs.Frequency.

It is observer from the graph that synchronization error is constant for the range of message signal strength i.e.0.1volts to 0.5 volts for the fixed frequency signal of 10kHz.It can also be observed that error is constant for the range of frequencies i.e.5kHz to 10 kHz for the fixed signal strength. Synchronization error is considerably reduced because the feedback signal continuously modifies the dynamics of chaotic system used at the transmitter in a same manner as the changes occur at the receiver.

5. Conclusion

Robustness of synchronization is a major factor concerned in the chaotic masking. In this paper we have approached the problem of synchronization error resulted in the chaotic system due to increase in the amplitude and frequency of the message signal. Synchronization error is calculated and plotted with respect to changing message signals. To improve the basic chaotic masking scheme, we implemented the chaotic masking using a modified scheme to provide secure communication at higher amplitudes and lower frequencies with almost no synchronization errors. This scheme eliminates the error by changing the dynamics of the transmitter and the receiver identically. Simulation results verify it.

6. References

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