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Nonlinear and Transient Heat Transfer in the Fin by a Truly Meshless Method

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Abstract

In this article, Meshless Local Petrov- Galerkin (MLPG) method is used to solve the nonlinear and transient one- dimensional heat transfer equation of a fin with the power- law temperature- dependent heat transfer coefficient. Moving least square approximants are used to approximate the unknown function of temperature T(x) with T^h (x). These approximants are constructed by using a linear basis, a weight function and a set of non- constant coefficients. Essential boundary conditions are enforced by direct method of interpolation and Penalty Method (PM) respectively. Temperature variation along the fin length over the different time range till the attainment of steady state has been demonstrated for the convective and insulated tip conditions.

Keywords: Convective Tip, Direct Method, Fins, Insulated Tip, Meshless Local Petrov-Galerkin (MLPG) Method, Penalty method, Transient

1. Introduction

Meshless methods, as alternative numerical approaches to eliminate the well-known drawbacks in the finite element and boundary element method, have attracted much attention in the past decade, due to their flexibility and potential in negating the need for the human-labor intensive process of constructing geometric meshes in a domain. The main objective of the meshless methods is to get rid of, or at least alleviate the difficulty of, meshing and remeshing the entire structure; by only adding or deleting nodes in the entire structure, instead. Various methods belonging to this family are the smooth particle hydrodynamics, the diffuse element method, the reproducing kernel particle method, the method of finite spheres, the local boundary integral method, the element free Galerkin method and meshless local Petrov- Galerkin method (MLPG).

Among all these methods, the MLPG method has been used successfully to solve variety of solid mechanics, heat transfer and fluid flow problems.

MLPG method was developed by Atluri and Zhu[1,2]. Unlike FEM and most other meshfree methods, MLPG method operates on local weak form and performs integration over overlapping simple local domains. This has removed the need of mesh at any stage of analysis. Hence, it is a truly meshfree method. It can be concluded that MLPG: (a) has a very high rate of convergence, (b) does not need any post processing technique, and (c) does not exhibit any volumetric locking. This method works on Petrov-Galerkin formulation i.e. trial and test functions are selected from different spaces which provides a large number of possible combinations to formulate MLPG method. Authors[3] have shown different variants of MLPG method in their work and presented the method as an able alternative to FEM.

Many researchers have used MLPG method to solve variety of heat transfer problems, including steady state and transient linear and nonlinear problems in regular as well as irregular domains[4-6].

Fins are extended surfaces which can provide a considerable available area for heat transfer between a solid

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and a fluid. For proper prediction and control of the fin performance, it is necessary to know the dynamic response of temperature distribution when an unpredictable or expected change occurs. Analytical solution are particular important and useful for they may be used in an on line computation. Transient and steady state analysis of fins have been addressed by meshless Element Free Galerkin (EFG)[7] method and many other numerical methods[8-10].

Heat transfer problems, in practical are transient and nonlinear. These nonlinear problems have been approximated by different numerical methods including differential transformation method[11-13], homotopy perturbation method[14] and the classical Lie point symmetry method[15] etc. Galerkin's method of weighted residual has been employed to solve thermal analysis of longitudinal fins with temperature dependent properties and internal heat generation[16]. It has been revealed that the results obtained in this analysis serve as basis for comparison of any other method of analysis of the problem and they also provide a platform for improvement in fin design of fin in heat transfer equipment. Convective-radiative fin with temperature dependent properties have been demonstrated by collocation spectral method[17]. The effect of temperature dependent properties such as thermal conductivity, surface emissivity, heat transfer coefficient, convection-conduction parameter and radiation- conduction parameter on the fin temperature distribution and efficiency have been discussed.

To the best knowledge of the authors' MLPG method has not been applied for the analysis of nonlinear heat conduction through fins.

Nonlinear heat conduction covers the problems of temperature dependent material properties or temperature dependent boundary conditions. In view of the practical importance of nonlinear heat conduction problems, particularly in the fins, we attempt their numerical simulation using MLPG method in the present work. The MLPG method has been employed to obtain discrete equations for one- dimensional nonlinear transient heat transfer through convective and insulated tip fins with the help of direct method and penalty method of interpolation.

1.1 The Meshless Local Petrov- Galerkin Method

The MLPG method operates on Petrov-Galerkin formulation i.e. it picks up test and trial functions from different function spaces. The original formulation[1,2] has subsequently evolved in various versions either by changing the meshfree approximation scheme or by selecting a new test function. Hence, the MLPG method provides a rational basis for constructing meshfree methods with a greater degree of flexibility. The discretization of the governing equation by the MLPG method requires moving least square approximants which are made up of two components of a weight associated with each node, a monomial basis and a set of non constant coefficients.

1.1.1 The Moving Least Square Approximants

The unknown function T (x) is approximated by moving least- square approximants T^h (x). In one dimensions, for linear basis T^h (x) can be written as:

$$T^h(\mathbf{x}) = \sum_{j=1}^{m} p_j(\mathbf{x}) a_j(\mathbf{x}) \equiv \mathbf{p}^T(\mathbf{x})\mathbf{a}(\mathbf{x}) \tag{1}$$

Where $\mathbf{p}^T(\mathbf{x}) = (p_1(\mathbf{x}), p_2(\mathbf{x}), \ldots\ldots, p_m(\mathbf{x}))$ is a complete monomial basis and m is the number of terms in the basis.

For example, in 1-D space the basis can be chosen as:

Linear basis: $\mathbf{p}^T(\mathbf{x}) = \{1, x\}, \quad m = 2$

Quadratic basis: $\mathbf{p}^T(\mathbf{x}) = \{1, x, x^2\}, \quad m = 3$

The unknown coefficients a_j(x) at any given point are determined by minimizing the functional J

$$J = \sum_{I=1}^{n} w(\mathbf{x}-\mathbf{x}_I)[\mathbf{p}^\mathbf{T}(\mathbf{x})\mathbf{a}(\mathbf{x}) - T_I]^2 \tag{2}$$

Where n is the number of nodes in the neighbourhood of x for which the weight function $w(x - x_I) \neq 0$, and T_I is the nodal parameter of T at x= x_I. The stationarity of J in equation (2) with respect to a_j (x) leads to the following set of linear equations:

$$\mathbf{a}(\mathbf{x}) = \mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x})T \tag{3}$$

Where, $$\mathbf{A}(\mathbf{x}) = \sum_{I=1}^{n} w(\mathbf{x}-\mathbf{x}_I)\mathbf{p}(\mathbf{x}_I)\mathbf{p}^\mathbf{T}(\mathbf{x}_I) \tag{4}$$

$$\mathbf{B}(\mathbf{x}) = [w(\mathbf{x}-\mathbf{x}_I)\mathbf{p}(\mathbf{x}_I), \ldots\ldots., w(\mathbf{x}-\mathbf{x}_n)\mathbf{p}(\mathbf{x}_n)] \tag{5}$$

$$T = [T_1, T_2, \ldots T_n] \tag{6}$$

By substituting Eq. (3) in Eq. (1), the MLS approximants can be defined as

$$T^h(x) = \sum_{i=1}^{n} \Phi_{I(x)} T_I = \Phi(x)T \quad (7)$$

Where meshless shape function $\Phi_I(\mathbf{x})$ is defined as:

$$\Phi_I(\mathbf{x}) = \sum_{j=1}^{m} p_j(\mathbf{x})\left(\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x})\right)_{jI} = p^T A^{-1} B_I \quad (8)$$

The derivatives of shape function is given by:

$$\Phi_{ix} = (p^T A^{-1} B_I)_{,x} = p_{,x}^T A^{-1} B_I + p^T (A^{-1})_{,x} B_I + p^T A^{-1} (B_I)_{,x} \quad (9)$$

1.1.2 Weight Function

The weight function $w(\mathbf{x}-\mathbf{x}_i)$ is non-zero over a small neighbourhood of $\mathbf{x}_I$ called the domain of influence of node *I*. The choice of weight function $w(\mathbf{x}-\mathbf{x}_i)$ affects the resulting approximation $T^h(x_I)$, therefore the selection of appropriate weight function is essential. In this article the fourth order spline weight function is used. It is represented by

$$w(\mathbf{x}-\mathbf{x}_i) = \begin{cases} 1-6d^2+8d^3-3d^4 & \text{if } 0 \le d \le 1 \\ 0 & \text{if } d > 1 \end{cases} \quad (10)$$

Where d = $||x-x_I||$ is the distance between two points.

2. The Discrete Equation

Let us consider the heat transfer equation with power-law temperature- dependent heat transfer coefficient (h), linear thermal conductivity (k), density (ρ) and specific heat (c). The perimeter and area of the cross-section of the fin are P_r and A_c, respectively. Heat is not generated in the solid then the governing differential equation for such condition in a 1-D domain Ω is given by

$$k\frac{\partial^2 T}{\partial x^2} - h(T)\left(\frac{P}{A}\right)(T - T_a) = \rho c \frac{\partial T}{\partial t} \quad (11)$$

Initial and boundary conditions:

$$T(x,0) = T_o \quad \text{on } \Omega \quad (12)$$

$$\begin{cases} T(x,t) = \bar{T} & \text{on } \Gamma_1 \\ q(x,t) = \bar{q} & \text{on } \Gamma_2 \\ q(x,t) = h(T_a - T) & \text{on } \Gamma_3 \end{cases} \quad (13)$$

where $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3$ is the boundary of global domain Ω, $q = k(\frac{\partial T}{\partial n})$, $\bar{T}$ is the specified temperature on essential boundary, $\bar{q}$ is the given heat flux at the natural boundary, n is the outward unit normal to the boundary, *h* is convective heat transfer coefficient and T_a is the ambient temperature.

2.1 MLPG Formulation

MLPG method is based on local weak form. Weighted residual formulation for Eq. (1) in local domain Ω_Q can be expressed as

$$\int_{\Omega_Q} v\left[k\frac{\partial^2 T}{\partial x^2} - h(T)\left(\frac{P}{A}\right)(T - T_a) - \rho c\frac{\partial T}{\partial t}\right] d\Omega = 0 \quad (14)$$

Where, *v* is the test function. Using divergence theorem, Eq. (4) yields the desired weak form given by

$$\left[k\frac{\partial T}{\partial n}v\right]_{\Gamma_Q} - \int_{\Omega_Q}\left[k\frac{\partial T}{\partial x}\frac{\partial v}{\partial x} + h(T)v\left(\frac{P}{A}\right)T - h(T)v\left(\frac{P}{A}\right)T_a + \rho c\frac{\partial T}{\partial t}v\right] d\Omega = 0 \quad (15)$$

Where Γ_Q is the boundary of the local domain .

In case of 1-D problem, boundary integrals turn to be a point value on boundaries. Taking advantage of MLPG method's flexibility, the test function *v* is selected such that it vanishes at the boundary of local domain. Hence, boundary integral remains non-zero only when local domain intersects the global boundary. Therefore, Eq. (5) can be written as follows:

$$[qv]_{\Gamma_{1Q}} + [\bar{q}v]_{\Gamma_{2Q}} + [vh(T)(T_a - T)]_{\Gamma_{3Q}} - \int_{\Omega_Q}\left[k\frac{\partial T}{\partial x}\frac{\partial v}{\partial x} + h(T)v\left(\frac{P}{A}\right)T - h(T)v\left(\frac{P}{A}\right)T_a + \rho c\frac{\partial T}{\partial t}v\right] d\Omega = 0 \quad (16)$$

Where $\Gamma_{1Q} = \Gamma_1 \cap \Gamma_Q$, $\Gamma_{2Q} = \Gamma_2 \cap \Gamma_Q$ and $\Gamma_{3Q} = \Gamma_3 \cap \Gamma_Q$.

The unknown function, *T*, at any instant of time *t*, is approximated by MLS scheme as follows:

$$T(x) = \sum_{i=1}^{n_s} \Phi_i T_i = \Phi T \quad (17)$$

where Φ is the vector of meshfree shape functions Φ_i, **T** represents the vector of nodal parameters T_i at time *t* and n_s is the number of nodes in the support domain at point x**.** Essential boundary condition is imposed by the method of direct interpolation. Substituting the approximation (17) in Eq. (16) and performing integration over all local domains corresponding to all field nodes, the semi-discrete system can be obtained as follows:

$$\mathbf{C}\dot{\mathbf{T}} + \mathbf{K}\mathbf{T} = \mathbf{F} \quad (18)$$

Where the typical matrix elements if essential boundary condition is imposed with the:

Direct Method

$$K_{ij}=\begin{cases}\Phi_j & \text{if } x_i\in\Gamma_1\\ \left[\int\limits_{\Omega_Q}\left(\frac{\partial v_i}{\partial x}k\frac{\partial\Phi_j}{\partial x}+h(T)v_i\left(\frac{P}{A}\right)\Phi_j\right)\mathrm{d}\Omega-\left[v_i\frac{\partial\Phi}{\partial n}\right]_{\Gamma_{1Q}}+\left[h(T)v_i\Phi_j\right]_{\Gamma_{3Q}}\right] & x_i\notin\Gamma_1\end{cases}\qquad(19)$$

$$C_{ij}=\begin{cases}\int\limits_{\Omega_Q}\rho c v_i\Phi_j\mathrm{d}\Omega & \text{if } x_i\notin\Gamma_1\\ 0 & \text{otherwise}\end{cases}\qquad(20)$$

$$F_i=\begin{cases}\bar{T} & \text{if } x_i\in\Gamma_1\\ \left[v_i\bar{q}\right]_{\Gamma_{2Q}}+\left[v_i h(T)T_a\right]+\int\limits_{\Omega_Q}\left(h(T)v_i\left(\frac{P}{A}\right)T_a\right)\mathrm{d}\Omega & \text{otherwise}\end{cases}\qquad(21)$$

Penalty Method

$$K_{ij}=\begin{cases}\Phi_j & \text{if } x_i\in\Gamma_1\\ \left[\int\limits_{\Omega_Q}\left(\frac{\partial v_i}{\partial x}k\frac{\partial\Phi_j}{\partial x}+h(T)v_i\left(\frac{P}{A}\right)\Phi_j\right)\mathrm{d}\Omega-\left[v_i\frac{\partial\Phi}{\partial n}\right]_{\Gamma_{1Q}}+\left[v_i h(T)\Phi_j\right]_{\Gamma_{3Q}}+\left[\alpha v_i\Phi_j\right]_{\Gamma_{1Q}}\right] & x_i\notin\Gamma_1\end{cases}\qquad(22)$$

$$C_{ij}=\begin{cases}\int\limits_{\Omega_Q}\rho c v_i\Phi_j\mathrm{d}\Omega & \text{if } x_i\notin\Gamma_1\\ 0 & \text{otherwise}\end{cases}\qquad(23)$$

$$F_i=\begin{cases}\bar{T} & \text{if } x_i\in\Gamma_1\\ \left[v_i\bar{q}\right]_{\Gamma_{2Q}}+\left[v_i h(T)T_a\right]_{\Gamma_{3Q}}+\int\limits_{\Omega_Q}\left(h(T)v_i\left(\frac{P}{A}\right)T_a\right)\mathrm{d}\Omega+\left[\alpha v_i\bar{T}\right]_{\Gamma_{1Q}} & \text{otherwise}\end{cases}\qquad(24)$$

Where α is the penalty parameter.

2.2 Time-Stepping Algorithm and Handling Nonlinearity

Spatial discretization of governing partial differential equation (11) results in a system of semi-discrete ordinary differential equations. We use two- level θ method for temporal discretization. It can vary between explicit and implicit strategies and results in the algebraic system

$$[\mathbf{C}+\theta\Delta t\mathbf{K}]\mathbf{T}^{n+1}=[\mathbf{C}+(\theta-1)\Delta t\mathbf{K}]\mathbf{T}^{n}+\Delta t\mathbf{F}\quad(25)$$

Where Δt is the time step and n denotes the time level (i.e. $t_n = n\,\Delta t$ if uniform time step is employed).

An iterative predictor-corrector scheme is used to handle nonlinearity in current work is as follows:

Predictor:

$$\begin{aligned}&\left[\mathbf{C}(\mathbf{X}^n)+\theta\Delta t\mathbf{A}(\mathbf{X}^n)\right]\mathbf{X}_*^{n+1}=\\&\left[\mathbf{C}(\mathbf{X}^n)+(1-\theta)\Delta t\mathbf{A}(\mathbf{X}^n)\right]\mathbf{X}^n+\Delta t\mathbf{B}(\mathbf{X}^n)\end{aligned}\qquad(26)$$

Corrector:

$$\begin{aligned}&\left[\mathbf{C}(\mathbf{X}_p^{\bar{n}})+\theta\Delta t\mathbf{A}(\mathbf{X}_p^{\bar{n}})\right]\mathbf{X}_{p+1}^{n+1}=\\&\left[\mathbf{C}(\mathbf{X}_p^{\bar{n}})+(1-\theta)\Delta t\mathbf{A}(\mathbf{X}_p^{\bar{n}})\right]\mathbf{X}^n+\Delta t\mathbf{B}(\mathbf{X}_p^{\bar{n}})\end{aligned}\qquad(27)$$

Where $p = 0, 1, 2, 3\ldots$up to convergence and

$$\begin{aligned}&\mathbf{X}_p^{\bar{n}}=w\mathbf{X}_p^{n+1}+(1-w)\mathbf{X}^n\qquad 0\le w\le 1\\&\mathbf{X}_0^{n+1}=\mathbf{X}_*^{n+1}\end{aligned}\qquad(28)$$

3. Numerical Results and Discussions

Consider a sample problem of one- dimensional fin as mentioned in the Figure 1.

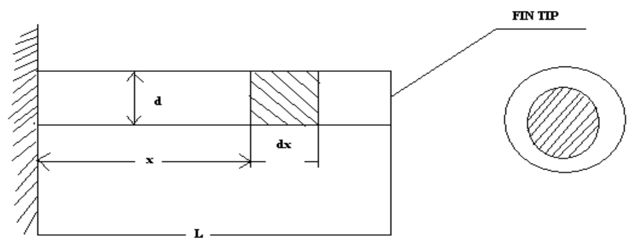

Figure 1. Schematic of cylindrical fin.

The different parameters used for the transient analysis of the model, shown in Figure 1 are tabulated in Table 1.

Table 1. Thermo- Geometric Parameters of Fin[18]

Sl. No.	Parameters	Value of Parameters
1	Fin Diameter (d)	2 cm = 0.02 m
2	Fin Length (L)	10 cm = 0.10 m
3	Fin Perimeter (P_r) =πd	0.0628 m
4	Specific heat (C)	0.48 kJ/kg.^{0}C
5	Density of the material (ρ)	7800 kg/m^3
6	Cross- sectional area (A_c)= ½πr^2	1.57 x 10^{-4} m^2
7	Thermal conductivity (k)	12 W/m.^{0}C
8	Heat transfer coefficient (h)	9.0 ΔT$^{0.175}$ W/m^2.^{0}C
9	Surrounding temperature (T_a)	30^{0}C

10	Base temperature (T_b)	200^{0}C
11	Initial temperature (T_{init})	200^{0}C
12	Time step (Δt)	10 sec

Table 2. Performance parameters of fin

Sl. No.	Parameters	Value of Parameters
1	No. of nodes taken along the fin length	21
2	Extent of quadrature domain (α_Q) and support domain (α_S)	1.66 and 2.50
4	Theta (θ)	1
5	No. of iterations	10

The solutions for the proposed cases are depicted in Figures 2-5. It can be observed that the temperature decreases with the increase in time from 500 sec to 2500 sec. for both the tip conditions- convective and insulated the temperature at the tip seems to be decreased with time. The temperature variation trend approaches towards the steady state as the time advances.

Table 3. Results of heat transfer in the convective fin by direct method of interpolation

Node Location (m)	500 sec	1000 sec	1500 sec	2000 sec	2500 sec	Steady state
0.000	200.00	200.00	200.00	200.00	200.00	200.00
0.025	129.09	118.86	116.84	116.41	116.32	116.30
0.050	102.07	84.41	80.84	80.10	79.94	79.90
0.075	90.01	68.30	63.85	62.91	62.71	62.66
0.100	84.06	62.66	58.23	57.30	57.10	57.05

Table 4. Results of heat transfer in the insulated fin by direct method of interpolation

Node Location (m)	500 sec	1000 sec	1500 sec	2000 sec	2500 sec	Steady state
0.000	200.00	200.00	200.00	200.00	200.00	200.00
0.025	129.45	119.16	117.04	116.58	116.48	116.45
0.050	103.00	85.08	81.31	80.49	80.31	80.26
0.075	91.95	69.53	64.74	63.69	63.46	63.39
0.100	87.60	64.76	59.84	58.76	58.53	58.46

Table 5. Results of heat transfer in the convective fin by penalty method of interpolation

Node Location (m)	500 sec	1000 sec	1500 sec	2000 sec	2500 sec	Steady state
0	200.00	200.00	200.00	200.00	200.00	200.00
0.025	133.07	123.81	122.02	121.66	121.58	121.57
0.050	103.68	86.87	83.57	82.90	82.76	82.73
0.075	90.67	69.67	65.48	64.63	64.45	64.41
0.100	84.47	63.70	59.52	58.67	58.49	58.45

Table 6. Results of heat transfer in the insulated fin by penalty method of interpolation

Node Location (m)	500 sec	1000 sec	1500 sec	2000 sec	2500 sec	Steady state
0.000	200.00	200.00	200.00	200.00	200.00	200.00
0.025	133.42	124.10	122.22	121.82	121.74	121.72
0.050	104.61	87.54	84.04	83.30	83.14	83.10
0.075	92.61	70.91	66.40	65.44	65.23	65.18
0.100	88.03	65.85	61.20	60.21	59.99	59.94

Table 7. Temperature- time history at 10th node

Time (Sec)	Convective tip		Insulated tip	
	Direct method	Penalty method	Direct method	Penalty method
500	103.00	104.61	102.07	103.68
1000	85.08	87.54	84.41	86.87
1500	81.31	84.04	80.84	83.57
2000	80.49	83.30	80.10	82.90
2500	80.31	83.14	79.94	82.76

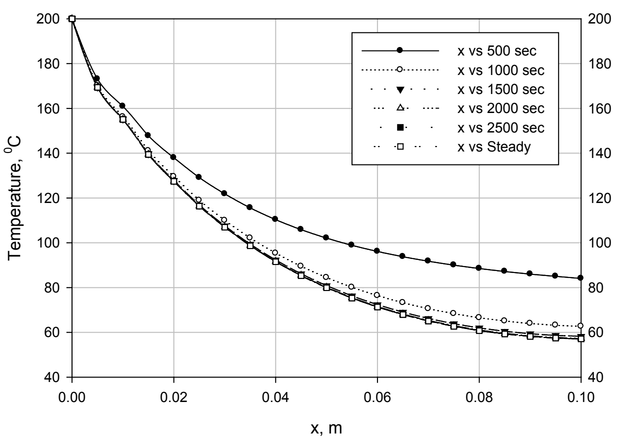

Figure 2. Heat transfer though convective fin using direct method of interpolation.

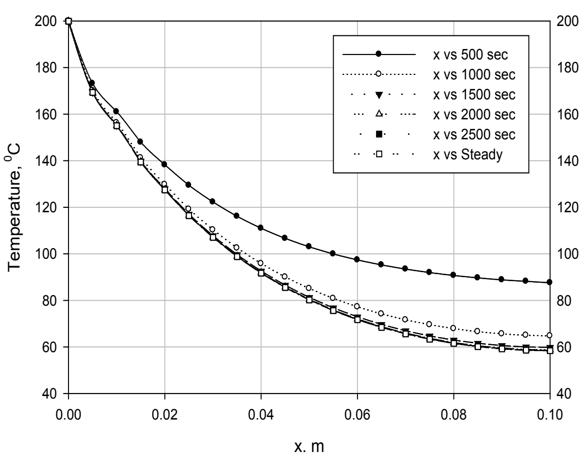

Figure 3. Heat transfer though insulated fin using direct method of interpolation.

Figure 6 depicts the temperature time history at the randomly chosen 10th node, which is located at 0.045 m from the base. It can be observed that at the initial phase of time the temperature lowers down at the faster rate but in the later phases it settles down to steady state.

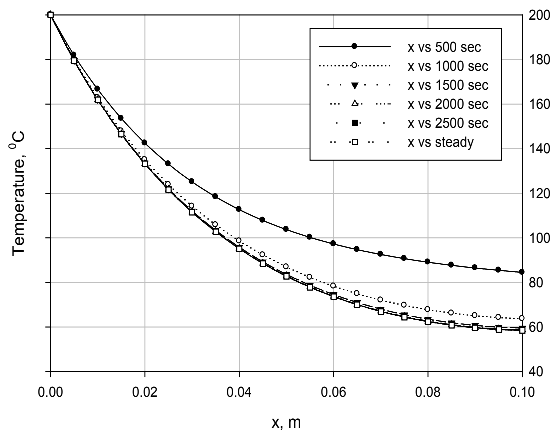

Figure 4. Heat transfer though convective fin using penalty method of interpolation.

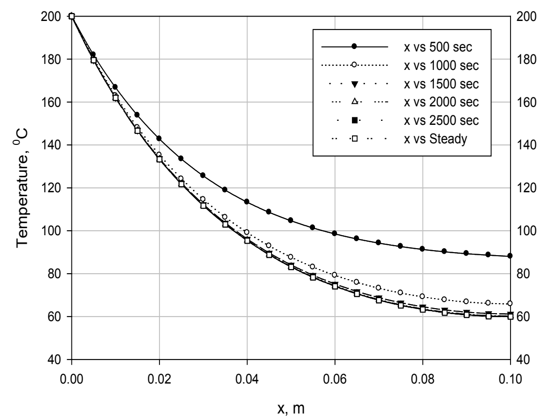

Figure 5. Heat transfer though insulated fin using penalty method of interpolation.

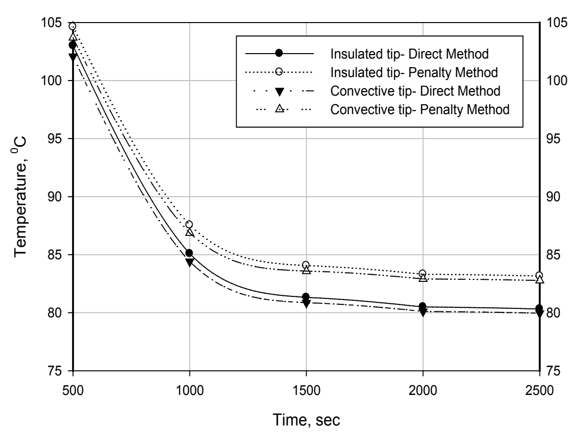

Figure 6. Temperature- time history at 10th node.

4. Concluding Remark

The transient heat transfer in a longitudinal fin was studied for convective tip and insulated tip by direct method and penalty method of interpolation respectively. The dependence of convective heat transfer coefficient on the temperature rendered the problem nonlinear. Hence, it can be depicted that MLPG method is capable enough to address nonlinear and transient heat transfer problems of the fins successfully.

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