

Optimization of PI Coefficients of Permanent Magnet Synchronous Motor Drive

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Abstract

Background/Objective: This paper enhances the responses of Permanent Magnet Synchronous Motor (PMSM) drive system with design of PI controller based on Nelder-Mead optimization Method implemented in Matlab environment and Optimization Toolbox. **Methods/Statistical Analysis:** The minimization requirements on control quality such as overshoot, settling time and steady-state error have been realized by choosing objective function based on ITAE criterion. The speed error for control of the speed of PMSM based on field oriented control is considered for implementation of ITAE criterion. **Findings:** The standard AC6 model (Field oriented control of PMSM drive) of MATLAB Simpower system is analyzed for optimal design of the coefficients of the PI controller. **Improvements:** The effectiveness of responses of torque, current and speed tracking are verified with random PI and optimized PI coefficients.

Keywords: ITAE based Objective Function, Matlab Environment, Nelder-Mead Optimization Method, PI Control Design, Permanent Magnet Synchronous Motor

1. Introduction

The performance parameters such as power factor, efficiency, power density, easy control, cost and torque-to-inertia ratio of PMSM are superior compared to the conventional induction and synchronous motors for the same output power¹⁻⁴. The operation of the Permanent Magnet Synchronous Motor is based on the applications and it may be operated in constant power mode or variable power mode. In both modes with four quadrants operation, it requires easier control algorithms especially, field oriented control for generating the between the switching signals for three phase voltage source inverter. Line-start

PMSM has also been gaining popular in comparison with the induction motor for semi-hermetic drives, like in compressors with heavy loading due to better cooling methods. The performance of the PMSM is influenced by the demagnetization characteristics of a permanent magnet⁵.

Control design plays a crucial part in control engineering for control of torque, speed and position in various industrial and domestic fields. The journey from the classical control method to the modern control methods during 50 years have developed a lot of control algorithms starting from PI controller to LQ, robust, pre-

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dictive, adaptive control or artificial control. The above list is few and still the research is going on for development of different algorithms to overcome the problems faced to implement the new technologies⁶⁻¹⁰. In drive applications, though there are number of control algorithms, the industry communities prefer the simplest PI control for controlling the drives because of cost effective and easy to handle. The dynamic responses such as overshoot, settling time, good tracking and above all the stability of the system mainly depends on coefficients of the PI controller. There are number of techniques such as bode diagram, root-locus etc. for determination of coefficients of PI controller. These techniques can be used if the mathematical model of the system is known. It is difficult to find accurate mathematical model of industrial drive because of large number of components such as converters, electrical machine and its control in the system. The disturbances cause due to various reasons also affect the transfer function of an electrical drive system. There are vast number of books, scientific papers, where efforts put into the idea to develop mathematical model of drive system of finding the coefficients of PI fulfilling certain criteria regarding control qualities.

In this paper, effort has been made to find the coefficients of PI controller to improve the performances of the responses of PMSM motor in field oriented control manner using Nelder-Mead simplex direct search method. The algorithm, which one described in this paper, is simple and straight forward searching iteration method for determining the minimum of the objective function¹¹⁻¹³. The objective function is based on integral criterion of ITAE well defined in control engineering¹⁴. The Simpson's one-third rule is used for integration of objective function. The function `fminsearch` in optimization toolbox of MATLAB uses Nelder-mead algorithm is used for searching of the coefficients of PI⁹. The coefficients of PI controller have to be optimized to process the error between the command speed and the actual speed for operation of PMSM motor drive in field-oriented control mode. The PMSM drive was found in MATLAB demo model with name as AC6 is considered in this paper for optimization.

2. Motivation, Objectives and Procedures

The main motivation factor has to design the best controller to improve the performances in terms of control quality of any system. About 90% of closed loop control system uses the PID controller for error minimization with improved dynamic performances. Most of the classical methods for PID control design require the exact mathematical model of the system in term of transfer function or state-space variable form. In some system, it is difficult to find out the exact mathematical model due to non linearity of the system. Although the exact model is known but the classical control techniques will not give the accurate performances because of modification of mathematical model by the various disturbances. Therefore, this requires the robust design of PI controller. This can be achieved by simulating the unknown mathematical model repeatedly applying search method for fulfillment of the objective function.

The procedures set up by the authors can be summed up as follows

- Built the mathematical model in simpower system in MATLAB environment of your choice with proper control algorithms. The algorithms must have the PI controller whose coefficients to be optimised. The name of coefficient variables must be declared as global variables. The error which to be minimised must be stored in the workspace with name. In this paper the PMSM motor drive is used as a model. This model, named as AC6 is the built-in demo model of MATLAB simpower system environment.
- The objective function should be carefully chosen. Here, integral criterion of ITAE is considered as objective function. A function is written in MATLAB and Simpson's one-third rule is used to integrate the objective function. Inside the MATLAB objective function the model PMSM motor drive is called. The MATLAB program of objective function is given in Appendix.

- Calling Model Command: `[t,y]=sim('bkn_ac6_pides',tt);`
- Nelder-Mead simplex direct search method is used here to search and optimise the coefficients of PI controller. The built-in Nelder-Mead simplex direct search is stored in the name of `fminsearch` in optimisation toolbox of the MATLAB⁹⁻¹⁰. A main program consist of `fminsearch` function has been written in MATLAB and it should be called for running of the system. The MATLAB program of main function is given in Appendix.
- Calling objective function : `[X,FVAL,EXITFLAG,OUTPUT]=fminsearch(@byam_obj_ac6,x0,[],t_end,h)`
- Running of the main program, the `fminsearch` function calls the objective function. Since objective function contains running command of MATLAB, the model will run. The difference between the speed command and actual speed is passed into objective function. The output of objective function will processed through the iterative searching method defined in `fminsearch`. The iterative optimised searching method is discussed briefly in subsequent sections.

3. Dynamic Model and Field Oriented Control of PMSM

The difference between the PMSM and the wound rotor synchronous motor are lying in the rotor. The main flux production in PMSM is constant but, in the wound rotor synchronous motor is variable and it can be varied by changing of excitation voltage in the field coil. So, the back emf produced by permanent and that produced by an excited coil are same. This result in the mathematical model of a PMSM is similar to that of the wound rotor synchronous motor. The motor is inverter controlled and the controlled output parameters are six switching signals of the semiconductor switches of the inverter. The dynamic model of PMSM in rotor reference frame can be written without considering the effect of saturation and parameter variations. The stator q-d dynamic equations of the PMSM, in the rotor reference frame, is derived as

$$\begin{bmatrix} v_{qs}^r \\ v_{ds}^r \end{bmatrix} = \begin{bmatrix} R_s + L_q S & \omega_r L_d \\ -\omega_r L_q & R_s + L_d S \end{bmatrix} \begin{bmatrix} i_{qs}^r \\ i_{ds}^r \end{bmatrix} + \begin{bmatrix} \omega_r \lambda_{af} \\ 0 \end{bmatrix} \quad (1)$$

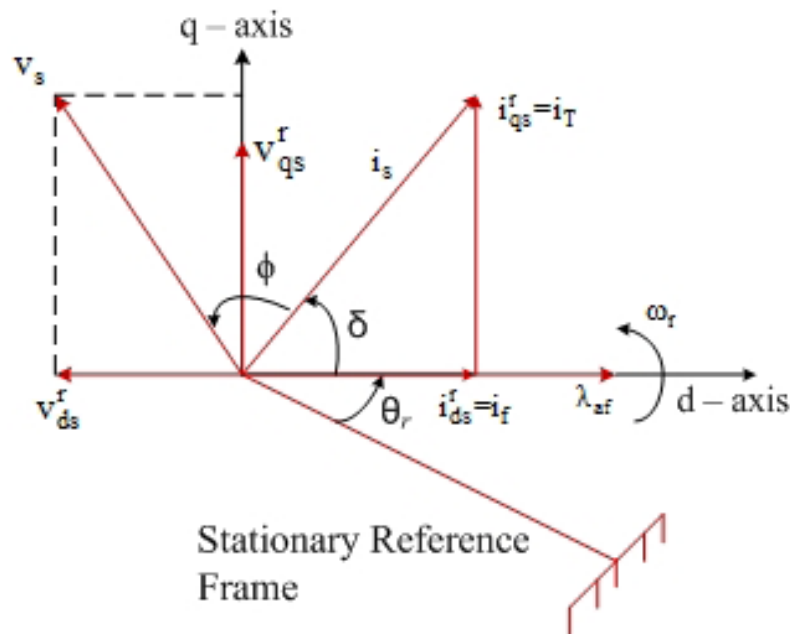


Figure 1. Phasor diagram of Field Oriented Control (FOC) for PMSM.

The torque developed in the machine is

$$T_e = \frac{3}{2} \frac{P}{2[\lambda_{af} + (L_d - L_q)i_{ds}^r]} i_{qs}^r \quad (2)$$

In order to decouple the flux component of stator and torque component of stator the flux component of stator must be oriented along the rotor flux linkage line. Vector control, by field orientation, is achieved by orienting the flux component current along rotor flux linkage at every instant of time and it should be zero.

As shown in Figure 1, the rotor flux linkage is rotated at a rotor speed ω_r and always positioned away from a stationary reference by the rotor angle θ_r . Therefore, the stator d-q current should be rotated with the same speed ω_r so that, d-axis component of stator current must be aligned to rotor flux position and rendering the stator flux current component zero by making $\delta=90^\circ$.

$$i_f = i_{ds}^r = i_s \cos \delta = 0 \quad (3)$$

$$T_e = \frac{3}{2} \frac{P}{2} \lambda_{af} i_{qs}^r = \frac{3}{2} \frac{P}{2} \lambda_{af} i_s = \frac{3}{2} \frac{P}{2} \lambda_{af} i_T \quad (4)$$

Since the machine is permanent magnet, the λ_{af} is constant. Therefore the torque can be written as

$$T_e = K_T i_T \quad (5)$$

Under this condition, the PMSM behaves exactly as the separately excited dc motor which is confirmed by the torque expression.

The rotor dynamic equations in vector control mode are

$$K_T i_T = T_L + JS + B\omega_r \quad (6)$$

$$\theta_r = \int \omega_r dt \quad (7)$$

Where i_{ds}^r, i_{qs}^r, i_f and i_T are stator direct axis, quadrature axis, field component and torque component currents respectively, S is the Laplace's operator, δ is the torque angle, ω_r and ω_m are rotor electrical and mechanical speeds in rad/s.

4. Control Procedures for Vector Control

The complete closed loop for controlling the Speed of PMSM Drive System in vector control mode is shown in Figure 2. It consists of the speed and current sensors, PI Speed Controller with limiter, hysteresis current controller and the three phase voltage source inverter. The 1st order low pass filter is used to filter out the noise signals from the output of the speed sensor. The speed error between the reference speed and actual rotor speed is processed through the PI speed controller with limiter to get the output of the torque reference. The limiter limits maximum torque production and PI controller is used to nullify the steady state error in speed and as well as, improve the dynamic behavior. The output of the PI is divided by the Torque constant (K_T) to produce the rotor reference torque component current (i_r). To achieve vector control the field component current must be zero. The above two currents are converted into three phase stator a-b-c current commands using equations (8) and (9). The three Hysteresis Current Control (HCC) are used to generate six current switching pulses for inverter operation in feedback current mode by comparing each reference current commands generated and each actual sensed stator three phase currents. The optimization of the coefficients of PI controller has been carried out by applying Nelder-Mead simplex direct search method to improve the performances of responses.

$$[T_{dq}] = \begin{bmatrix} \cos \theta_r & \sin \theta_r \\ -\sin \theta_r & \cos \theta_r \end{bmatrix} \quad (8)$$

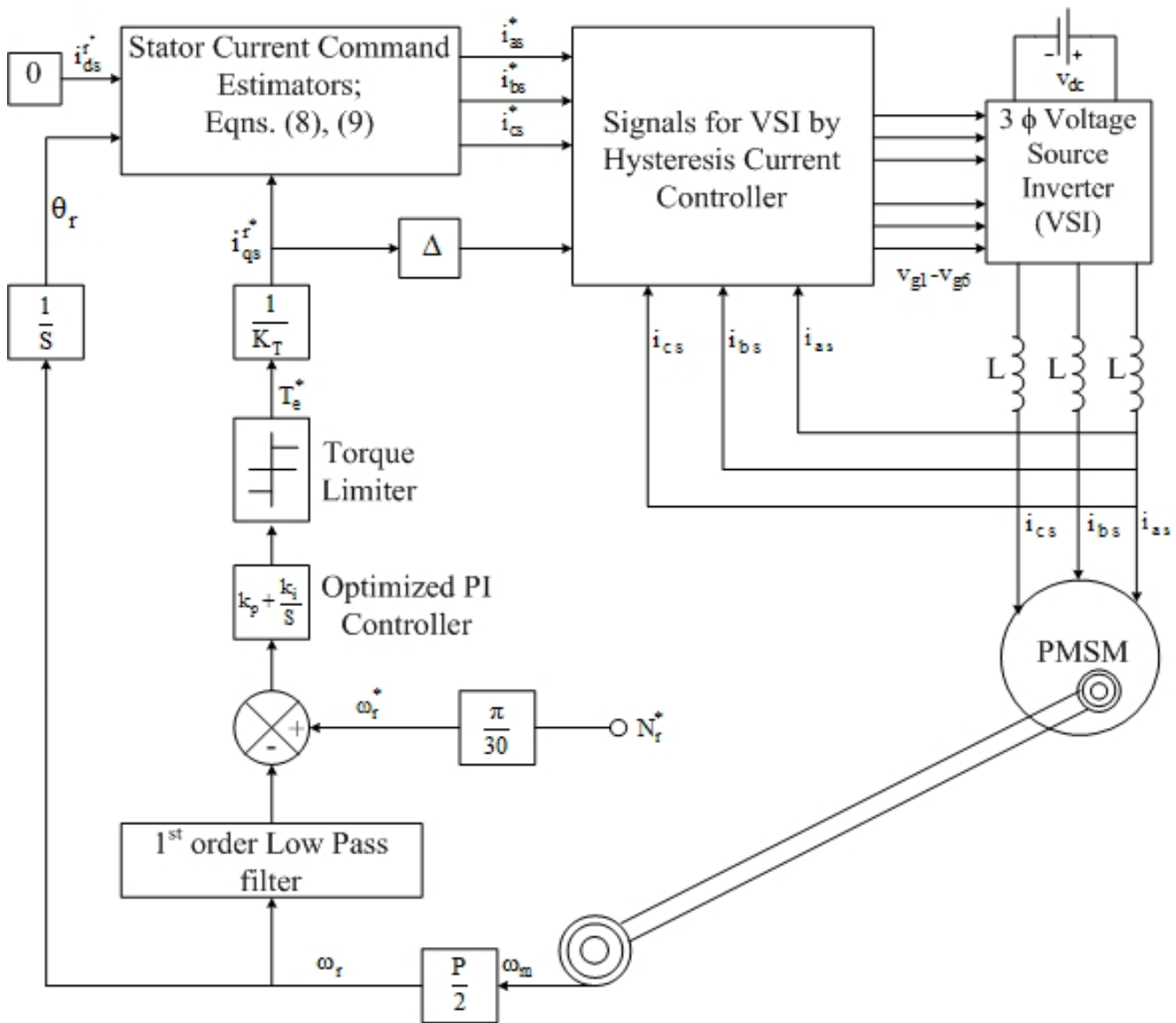


Figure 2. Complete schematic of the speed-controlled of PMSM drive in FOC mode.

$$[T_{\alpha\beta}] = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

(9)

5. Nelder-Mead “Simplex” Direct Search Method

In the mid-1960s, two English statisticians invented the Nelder–Mead “simplex” direct search method used for

solving the unconstrained optimization problem¹⁵. The Nelder-Mead method iteratively generates a sequence of interested vertex points which converge to an optimal vertex points of objective function $f(x)$ ¹⁶. At each iteration, the vertices x_i are ordered according to the objective function values

$$f(x_1) \leq f(x_2) \leq f(x_3) \leq \dots \leq f(x_{n+1}) \quad (10)$$

where x_1 is the best vertex and x_{n+1} is the worst vertex. The algorithm uses four possible operations: reflection, expansion, contraction and shrink, each being associated with a scalar parameter: α (reflection), β (expansion), γ (contraction), and δ (shrink). The values of α , β , γ and δ are lying in the range of $>0, >1$ and 0 to 1 in both γ and δ respectively.

The one iteration of Nelder-Mead algorithm is as follows^{15,16}:

- Find out worst vertices using equation- let it be x_{n+1}
- Compute the reflection point

$$x_r = \bar{x} + \alpha(\bar{x} - x_{n+1}) \text{ where } \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (11)$$

evaluate

$$f_r = f(x_r) \text{ if } f_1 \leq f_r < f_n, \text{ replace } x_{n+1} \text{ with } x_r$$

- Compute the expansion point if $f_r < f_1$

$$x_e = \bar{x} + \beta(x_r - \bar{x}) \quad (12)$$

Evaluate $f_e = f(x_e)$ if $f_e < f_r$ replace x_{n+1} with x_e otherwise by x_r

- Compute the outside contraction point if $f_n \leq f_r < f_{n+1}$

$$x_{oc} = \bar{x} + \gamma(x_r - \bar{x}) \quad (13)$$

Evaluate $f_{oc} = f(x_{oc})$ if $f_{oc} \leq f_r$ replace x_{n+1} with x_{oc} otherwise go to step-6

- Compute the inside contraction point if $f_r > f_{n+1}$

$$x_{ic} = \bar{x} - \gamma(x_r - \bar{x}) \quad (14)$$

evaluate $f_{ic} = f(x_{ic})$ if $f_{ic} \leq f_{n+1}$ replace x_{n+1} with x_{ic} otherwise go to step-6

6 shrink: for $2 \leq i \leq n+1$

define

$$x_i = x_1 + \delta(x_i - x_1) \quad (15)$$

and proceed to the next iteration.

6. Simulation Results and Discussion

The procedure used in industry for finding out the coefficients of PI controller is to adopt the lower the coefficient values and gradually tune them up until the best possible performance is achieved¹. But, actually, it is difficult to ascertain the best coefficient values and also time consuming. Therefore, the optimization of PI coefficients is the best solution.

As per procedures described above, the built-in function AC6 is simulated repetitively by modifying the coefficients of PI based on minimization of objective function. The objective function considered here for nul-

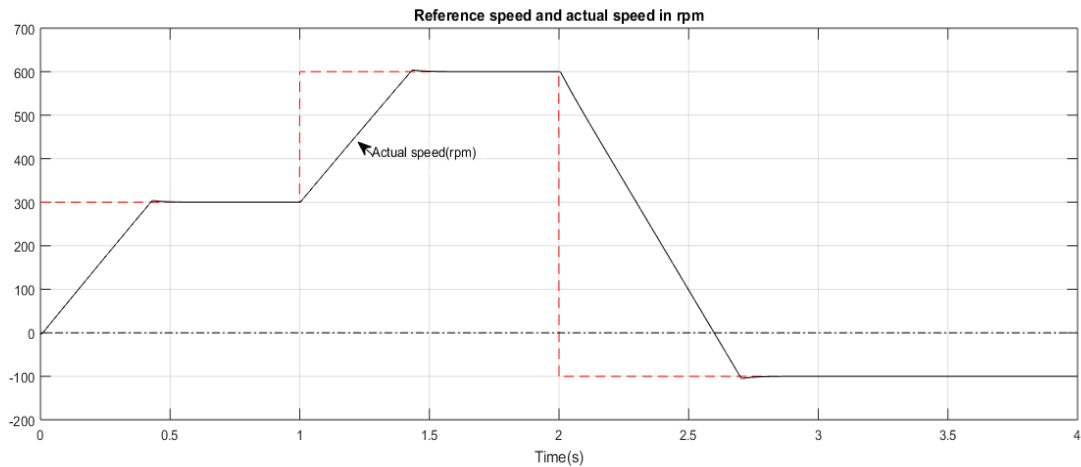


Figure 3. Reference speed and actual speed for optimized values of PI controller.

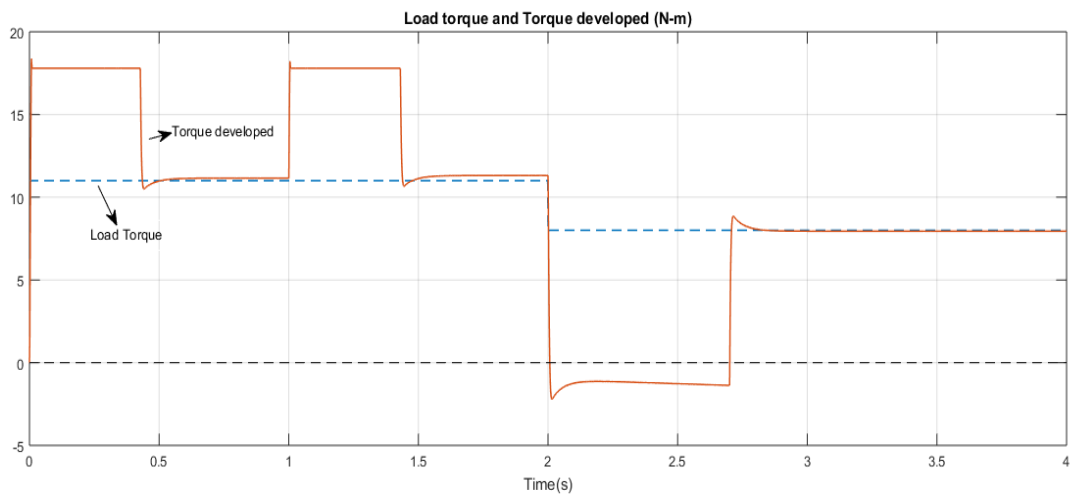


Figure 4. Load torque and actual torque developed in PMSM drive.

lify the steady-state speed error is based on Integral of Time-Weighted Absolute Error (ITAE) criterion.

$$ITAE = \int_0^{\infty} t|e(t)| dt \quad (16)$$

Where, $|e(t)|$ is the difference of absolute time dependent between actual speed and command speed and t is the time at that instant. The complete PMSM drive sys-

tem is simulated and the results presented here for step speed inputs. The datasheet of PMSM and MATLAB Programmers' for optimization are given in appendix. The various responses PMSM drive are compared with two different PI coefficients, one is randomly chosen values (initial values considered for optimization i.e. $k_p=2$ and $k_i=.2$) and other is estimated optimized vales ($k_p=1.8207$ and $k_i=35.7655$). The end of optimization, the values are found as:

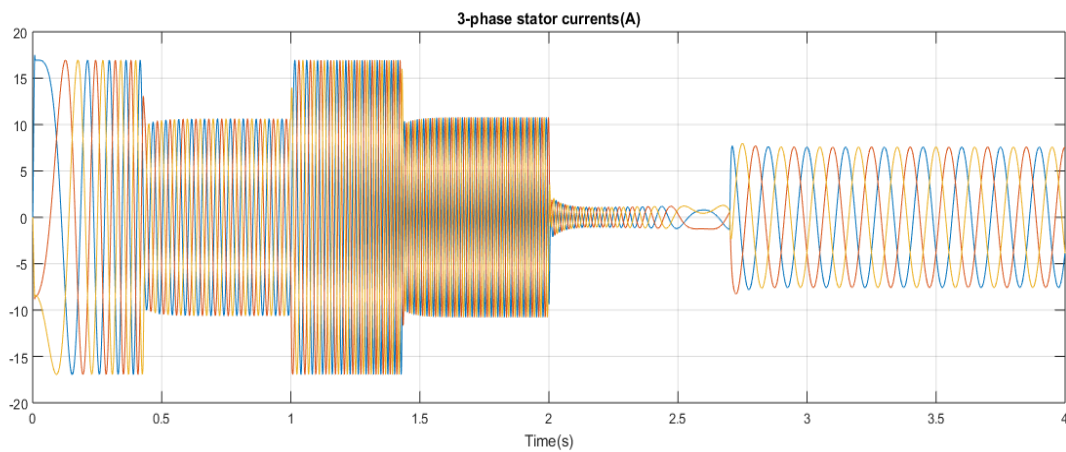


Figure 5. 3-phase stator a-b-c currents in A.

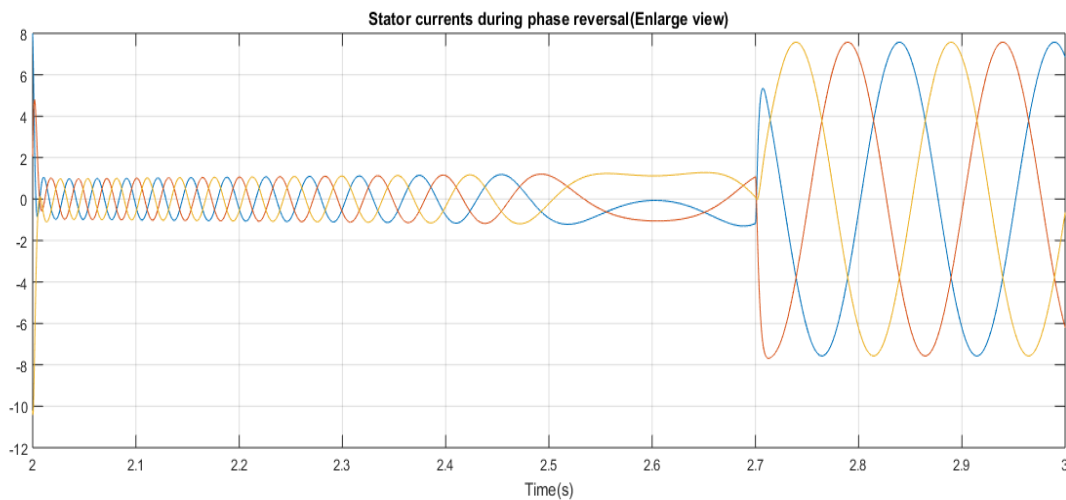


Figure 6. Enlarge stator currents during phase reversal.

X = 1.8207 35.7655 (The coefficients of PI controller)
 FVAL = 1.8062e+06
 EXITFLAG = 1
 OUTPUT =
 iterations: 48

funcCount: 117
 algorithm: 'Nelder-Mead simplex direct search'
 message: 'Optimization terminated:
 the current x satisfies the termination criteria using
 OPTIONS.TolX of 1.000000e-...'

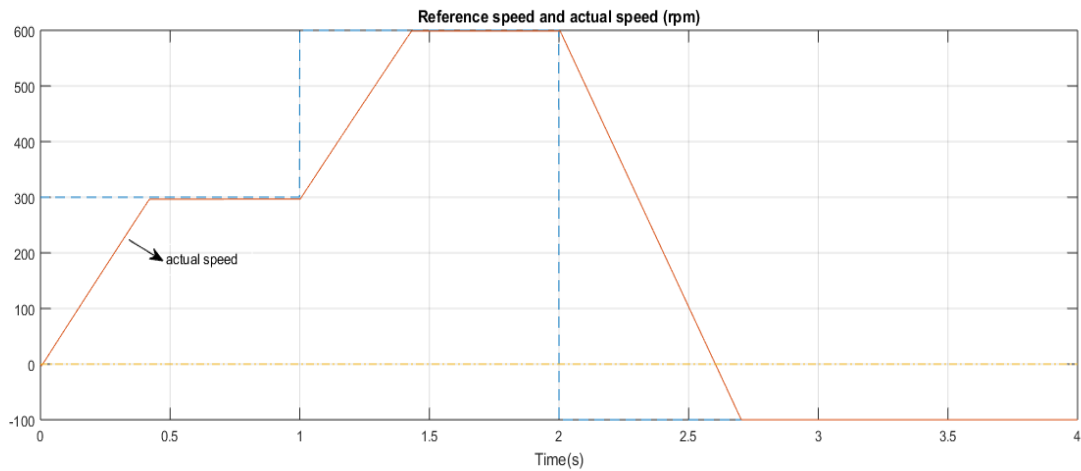


Figure 7. Reference speed and actual speed for random chosen PI controller values.

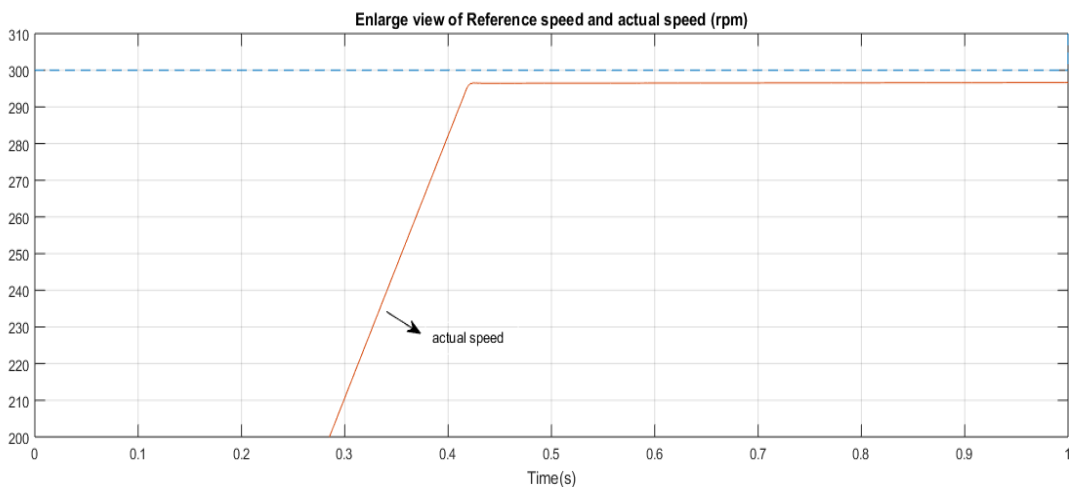


Figure 8. Enlarge view to show the steady-state error.

In order to verify the robustness of optimization values, the motor is operated at a speed command of 300 rpm at load torque of 11 N-m. At 1 second, the command speed is suddenly changed to 600 rpm without changing the load. At 2 seconds, a negative speed command of -100 is given with a load torque of 8N-m. The comparisons are made by initial parameters of PI con-

troller chosen and optimized value calculated off line by considering the initial values. It is observed in Figure 3 that the rotor speed exactly track the reference speed without any steady state error in optimized PI controller though there is slightly overshoot in the speed response. At starting the electromagnetic torque developed in the machine is equal to value of torque limiter, which is the

maximum torque capability of machine depicts in Figure 4. This ensures that the motor accelerates very quickly by increasing the torque developed far away from the load torque and stabilize to the command speed at 0.4 second approximately. Figure 5 shows the responses of 3-phase stator currents. The maximum stator current at the time of starting is approximately 17.5A far below than the rated value. In order to change the direction of rotation, the phase sequence should be changed. Figure 6 shows the phase inversion from a-b-c to c-b-a that occurs as speed changes from 600 rpm to -100 rpm. The effects of optimized PI controller and hit and trial controller is compared in Figure 7 and Figure 8. Figure 7 is the speed response of randomly chosen PI controller. There is an appreciable steady state error in randomly chosen parameters (296 rpm instead of 300 rpm) as shown in enlarge view of Figure-7 depicts in Figure 8.

7. Conclusions

A PI with hysteresis band current controlled permanent magnet synchronous motor drive based on field orientation has been considered for optimization of the coefficients of PI controller. Normally, in industries the hit and trial methods starting from lower coefficient values and gradually tuning to the values to get the best performances are used. But, this practice affects the system performances during the time of variations of PI controller's values and also difficult to get the optimized values. Therefore, offline estimation of PI controller is the best choice by building the replica of industrial model and simulated it with different values of PI controller. The different Iterative values are chosen based on the objective function. In this paper, PMSM drive is considered because of it may be the working horse in industry in near future. The gains of PI controller are optimized by using Nelder-Mead simplex optimization algorithms. The difference between the reference speed and actual speed is considered here as an objective function. The optimized values ($k_p=1.8207$ and $k_i=35.7655$) are found after 48 iterations. The specific research objectives have been achieved as shown in the simulation and analysis results.

8. References

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