

# Bipartite Entanglement for Two Electronic Qubits in Double Photoionization of Xeon

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## Abstract

**Objectives:** To study of entanglement between two electronic qubits, without observing spin orbit interaction (SOI), produced by double photo ionization from Xeon atom following the absorption of a photon. **Methods:** In absence of SOI, Russel-Salunders coupling (L-S coupling) is applicable. As the estimations of entanglement we have considered Peres-Horodecki condition and negativity. In case of L-S coupling, all the properties of electronic qubits are predicted merely with the knowledge of the spins of the target atom, the residual photoion, emitted electrons and the state of polarization of the absorbed photon. **Findings:** We have found that the electronic qubits may be totally entangled, partially entangled or separable depending on spin states of target Xe, residual cation  $Xe^{2+}$ , emitted electrons and the state of polarization of incident photon as well as of the direction of spin quantization and ejection of the photoelectrons. **Applications/Improvements:** Studies of quantum entanglement and its paradoxical properties hold the key to various Quantum information (QI) tasks such as quantum teleportation, quantum cryptography and quantum computer technology.

**Keywords:** Density Operator PACS No. 32.30, L-S Coupling, Negativity, Peres-Horodecki Condition

## 1. Introduction

Quantum entanglement is the resource needed for QI processing<sup>1,2</sup>. The basic unit of QI is quantum bit or qubit. Production and characterization of qubits<sup>3,4</sup> is the essential ingredient for any quantum information process. Although, photons are the fastest carriers of information<sup>5,6</sup> but they are not suitable for storing information for long durations of time as the detection of a photon always leads to its destruction. QI science requires entangled states of two or more particles having rest mass different from zero and such particles can be detected without being destroyed. Both atoms and their ions can store information for long periods, but are not suitable for transmission because they cannot travel with high speeds. Electrons are good to carry information because they can move with sufficiently high speeds—even with those approaching to that of light. Speed of an electron is readily maneuvered with a suitable combination of electric and magnetic fields. It is not destroyed until it interacts

with a positron or lost its identity unless it is captured by an atom or positive ion. Moreover it can be detected without affecting any of its properties. For such properties of electrons, it is already been suggested<sup>1,7,8</sup> that electronic qubits are suitable both for quantum computation and quantum communication.

One of the simplest processes for producing electronic qubits is simultaneous ejection of two electrons produced by absorption of a single photon in an atom or molecule. This process is known as<sup>9</sup> double photoionization (DPI), is one of the most direct manifestation of electron-electron correlation<sup>10,11</sup>. In this article, we have studied the entanglement between bipartite states of two electronic qubits ( $\varepsilon_1$  and  $\varepsilon_2$ ) produced by single-step double photoionization from Xeon atom following the absorption of a single photon. The entanglement is quantified by Peres-Horodecki's negative partial transpose (NPT) condition<sup>12</sup> of the density matrix (DM) of bipartite states and also negativity<sup>13-16</sup>.

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In section 2, we briefly describe the density operator (DO), states for DPI of Xeon (Xe) atom and the criteria and measure of entanglement. This operator corresponds to the case when the ionizing electromagnetic radiation is in a pure state of polarization and the target Xe atom is in its ground state before DPI. In section 3, we study the entanglement in DPI for electronic qubit system. The conclusion part is given in section 4.

## 2. Preliminaries

### 2.1 Density Operators

We have studied here the Coulombic entanglement of two freely moving electrons  $\varepsilon_1, \varepsilon_2$  ejected from Xeon (Xe) atom produced in single step DPI process by single photon ( $\nu_r$ ) with energy  $E_r$ . Our process can schematically be represented by

$$h\nu_r + Xe(4d^{10}5s^25p^6^1S_0) \rightarrow Xe^{2+}(4d^85s^25p^6^1S_0) + \varepsilon_1(\vec{k}_1; \mu_1\hat{u}_1) + \varepsilon_2(\vec{k}_2; \mu_2\hat{u}_2). \quad (1)$$

Let us denote by  $\rho_0 = |0\rangle\langle 0|$  and  $\rho_r = |m_r\rangle\langle m_r|$  the respective density operators of Xe before DPI and of the ionizing radiation. This means the density operator of the combined system before photoionization of Eq. (1) is given by the direct product

$$\rho_i = \rho_0 \otimes \rho_r. \quad (2a)$$

In equation (1), the bound electronic state of Xeon is  $|0\rangle \equiv |L_0 S_0 M_{L_0} M_{S_0}\rangle$  and that of the dication  $Xe^{2+}$  is

$|2^+\rangle \equiv |L_{2^+} S_{2^+} M_{L_{2^+}} M_{S_{2^+}}\rangle$ . The orbital angular momenta

are  $l_1, l_2$  and spin angular momenta are  $\left(\frac{1}{2}\right)_1, \left(\frac{1}{2}\right)_2$  of

the photoelectrons  $\varepsilon_1, \varepsilon_2$  respectively. The symbols

$M_{L_0}, M_{S_0}, M_{L_{2^+}}$  and  $M_{S_{2^+}}$  are used to represent the

respective projections of  $\vec{L}_0, \vec{S}_0, \vec{L}_{2^+}$  and  $\vec{S}_{2^+}$  along the polar axis of the space frame (XYZ). As LS-coupling is applicable here, the total orbital angular momenta ( $\vec{L}_0, \vec{L}_{2^+}$ ) and the spin angular momenta ( $\vec{S}_0, \vec{S}_{2^+}$ ) of Xe

and  $Xe^{2+}$  are the conserved quantities, we then have

$$\vec{L}_0 + \vec{L}_r = \vec{L}_{2^+} + \vec{L}_1 + \vec{L}_2 \text{ and } \vec{S}_0 = \vec{S}_{2^+} + \vec{S}_r (= \left(\frac{1}{2}\right)_1 + \left(\frac{1}{2}\right)_2) \quad (2b)$$

The density operator (2a) becomes

$$\rho_i = \frac{1}{(2L_0 + 1)(2S_0 + 1)} \sum_{M_{L_0} M_{S_0}} |0; 1m_r\rangle\langle 0; 1m_r| \quad (2c)$$

Where we have defined  $|0; 1m_r\rangle = |0\rangle|1m_r\rangle$ . Let us denote by  $F_p$  the photoionization operator<sup>17</sup> in the E1 approximation. Then the density operator of the combined ( $Xe^{2+} + \varepsilon_1 + \varepsilon_2$ ) system<sup>18</sup> in equation (1) after DPI becomes

$$\rho_{2^+} = K_p F_p \rho_i F_p^\dagger. \quad (3)$$

Here,  $K_p = 3\pi(e^2 / \alpha_0 E_r)^2$  with  $\alpha_0$  the dimensionless fine structure constant<sup>17</sup>.

### 2.2 Criteria and Measure of Entanglement

Nowadays quantum entanglement is recognized as resources following various applications such as quantum teleportation and quantum key distribution in the field of quantum information and quantum computation. So, the task of characterizing and quantifying entanglement has emerged as one of the prominent challenge of quantum information theory. The condition for entanglement given by Peres and is that the partial transpose (PT), with respect to either of the two particles, of its density matrix (DM) remains negative. We have applied Peres-Horodecki NPT condition in order to characterization of entanglement.

As a measure of the degree entanglement we consider the *negativity*, which is an additive and operational measure of entanglement. It can be defined as

$$N = \max(0, -\lambda_{neg}) \quad (4)$$

where  $\lambda_{neg}$  is the sum of the negative eigenvalues of  $\rho_{AB}^{\tau_B}$ , is the partial transpose of  $\rho_{AB}$ . Additivity is a very desirable property that can reduce the complexity of computation of entanglement. The negativity of a state indicates to which extent a state violates the positive partial transpose separability criterion.

### 3. Entanglement between Electronic Qubits of Xeon Atom for DPI

#### 3.1 Density Matrix

Here, we calculate the DM for the angle- and spin-resolved DPI of Xeon atom without considering SOI (L-S coupling) into account in either of the bound electronic states of Xe and  $Xe^{2+}$  or in the continua of the two photoelectrons ( $\varepsilon_1, \varepsilon_2$ ) ejected in the process (1).

In order to calculate the DM for the ( $Xe^{2+} + \varepsilon_1 + \varepsilon_2$ ) system in process (1), we need to calculate the matrix elements of  $\rho_{2^+}$ . Following the procedures given in reference<sup>19,20</sup> the DM (4) in the present case are given by

$$\langle 2^+; \vec{k}_1, \mu_1 \hat{u}_1; \vec{k}_2, \mu_2 \hat{u}_2 | \rho_{2^+} | 2^+; \vec{k}_1, \mu'_1 \hat{u}_1; \vec{k}_2, \mu'_2 \hat{u}_2 \rangle = \frac{K_p}{(2L_0+1)(2S_0+1)} \sum_{M_{L_0} M_{S_0}} \langle 2^+; \vec{k}_1, \mu_1 \hat{u}_1; \vec{k}_2, \mu_2 \hat{u}_2 | F_p | 0; 1m_r \rangle \times \langle 2^+; \vec{k}_1, \mu'_1 \hat{u}_1; \vec{k}_2, \mu'_2 \hat{u}_2 | F_p | 0; 1m_r \rangle^* \quad (5)$$

The DM of Eq. (5) is Hermitian, i.e.,

$$\langle 2^+; \vec{k}_1, \mu_1 \hat{u}_1; \vec{k}_2, \mu_2 \hat{u}_2 | \rho_{2^+} | 2^+; \vec{k}_1, \mu'_1 \hat{u}_1; \vec{k}_2, \mu'_2 \hat{u}_2 \rangle = \langle 2^+; \vec{k}_1, \mu'_1 \hat{u}_1; \vec{k}_2, \mu'_2 \hat{u}_2 | \rho_{2^+} | 2^+; \vec{k}_1, \mu_1 \hat{u}_1; \vec{k}_2, \mu_2 \hat{u}_2 \rangle^* \quad (6a)$$

In order to calculate the matrix elements present in the right hand side of Eq. (5), we introduce  $L_0 = L_{2^+} = 0$  for DPI process in Xe and angular momentum coupling given in Eq. (2b). We therefore can write

$$|0; 1m_r\rangle = |(01); S_0 M_{S_0}\rangle \quad (6b)$$

and

$$\begin{aligned} |2^+; \vec{k}_1, \mu_1 \hat{u}_1; \vec{k}_2, \mu_2 \hat{u}_2\rangle &= (-1)^{-s_1} \sum_{l_1 l_2 m_1 m_2} \sum_{v_1 v_2} i^{l_1+l_2} e^{-i(\sigma_1+\sigma_2)} (-1)^{-l_1+l_2+s_1-v_1-m_1-M_S} \\ &\times \sqrt{(2l_1+1)(2s_1+1)(2S_0+1)} \begin{pmatrix} l_1 & l_2 & l \\ m_1 & m_2 & m \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & s_1 \\ v_1 & v_2 & v \end{pmatrix} \begin{pmatrix} S_{2^+} & s_1 & S \\ M_{2^+} & v - M_S & S \end{pmatrix} \\ &\times Y_{l_1}^{m_1}(\hat{k}_1) Y_{l_2}^{m_2}(\hat{k}_2) D_{\mu_1 v_1}^{s_1}(\omega_1) D_{\mu_2 v_2}^{s_2}(\omega_2) |(01); (S_{2^+}, s_1); SM_S\rangle. \end{aligned} \quad (6c)$$

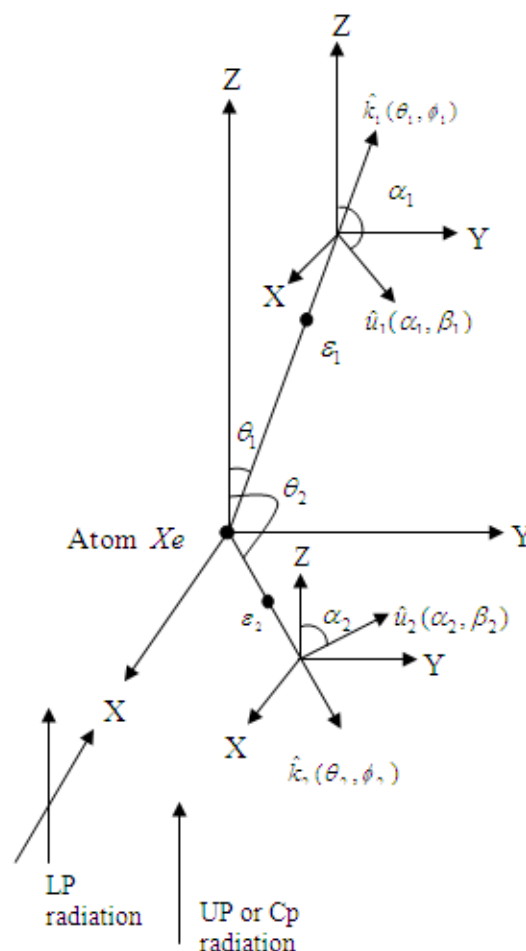
Here,  $D_s$  are the rotational harmonics<sup>21</sup>,  $\sigma_{l_i}$  and  $\sigma_{l_2}$  are the Coulomb phases for  $l_1$ th and  $l_2$ th partial waves of the photoelectrons and  $\omega_1(\alpha_1, \beta_1, 0)$  and  $\omega_2(\alpha_2, \beta_2, 0)$  the Euler angles which rotate the axis of the space-frame into the spin-polarization directions  $\hat{u}_1$  and  $\hat{u}_2$  (Figure 1), respectively.

Furthermore, in (6c), the properly anti symmetric and asymptotically normalized<sup>19</sup> ket  $|(01); (S_{2^+}, s_1); SM_S\rangle$  represents the electronic state of  $Xe^{2+}$  and the two photoelectrons with their total orbital and spin angular momenta coupled according to Eq. (2b).

Now we substitute in equation (5) the condition<sup>20</sup>

$$\langle (01); (S_{2^+}, s_1); SM_S | F_p | (01); S_0 M_{S_0} \rangle = \delta_{S_0 S} \delta_{M_{S_0} M_S} \langle 01 | F_p | 01 \rangle, \quad (7)$$

which arises from the conservation conditions of (2b).



**Figure 1.** Two Electrons ( $\varepsilon_1$  and  $\varepsilon_2$ ) with Directions of Propagation  $[\hat{k}_1(\theta_1, \phi_1), \hat{k}_2(\theta_2, \phi_2)]$  and of Spin Quantization  $[\hat{u}_1(\alpha_1, \beta_1), \hat{u}_2(\alpha_2, \beta_2)]$  along with Energies  $[\xi_1, \xi_2]$  are Emitted Simultaneously following Photoabsorption.

The resulting equation (5) is simplified by substituting Eqs. (6), (7) and analytically evaluating as many sums as possible present therein by the use of Racah algebra. It requires, for example, use of (a) the addition theorems (i.e., Eqs. (4.3.2) and (4.6.5) from<sup>21</sup> for rotational and spherical harmonics, (b) Eq. (6.2.5)<sup>21</sup> for converting a single sum of the product of three 3-j symbols into a product of one 3-j and one 6-j symbols, (c) identity (5) given on page 453 in Ref. 22 for converting a single sum of the product of two 3-j symbols into a product of two 3-j and one 6-j symbols summed over two variables, (d) Eq. (14.42) from Ref. 23 which transforms a quadruple sum of the product of four 3-j symbols into a double sum containing two 3-j and one 9-j symbols, (e) Eq. (3.7.9)<sup>21</sup> for changing a phase factor into a 3-j symbol, (f) orthogonality (3.7.7)<sup>21</sup> of 3-j symbols, (g) relation (6.4.14)<sup>21</sup> for writing a 9-j symbol (whose one of the nine arguments is zero) in terms of a 6-j symbol, and (h) relation (6.4.14)<sup>21</sup> for converting a single sum of the product of two 6-j symbols into one 6-j symbol. These and some other simplifications lead one to write the total DM (5) in the following form:

$$\langle 2^+; \vec{k}_1, \mu_1 \hat{u}_1; \vec{k}_2, \mu_2 \hat{u}_2 | \rho_2^+ | 2^+; \vec{k}_1, \mu'_1 \hat{u}_1; \vec{k}_2, \mu'_2 \hat{u}_2 \rangle = \frac{d^3 \sigma(m_r)}{d\varepsilon_1 d\vec{k}_1 d\vec{k}_2} \sigma(S_0; S_{2^+}; \hat{u}_1, \hat{u}_2)_{M_{S_0}, \mu_1 \mu_2; M_{S_{2^+}}, \mu'_1 \mu'_2} \quad (8)$$

The first term i.e., the normalized triple differential cross section (TDCS) on the right-hand side of (6) depends upon the orbital angular momenta of Xe and Xe<sup>2+</sup>, energies, the direction of ejection of the emitted electrons, the state of polarization of the ionizing photon and the photoionization dynamics. It does not include spins of the photoelectrons or the target atom or the residual dication and can be written as

$$4\sigma_{\varepsilon_1 \varepsilon_2} =$$

$\mu'_1 \mu'_2$ $\mu_1 \mu_2$	$\frac{1}{2} \frac{1}{2}$	$\frac{1}{2} - \frac{1}{2}$	$-\frac{1}{2} \frac{1}{2}$	$-\frac{1}{2} - \frac{1}{2}$
$\frac{1}{2} \frac{1}{2}$	$1 - c_1 c_2 - s_1 s_2 c$	$c_1 s_2 - s_1 c_2 c$	$s_1 c_2 - c_1 s_2 c$	$ic - s_1 s_2 - c_1 c_2 c$
$\frac{1}{2} - \frac{1}{2}$	$c_1 s_2 - s_1 c_2 c$	$1 + c_1 c_2 + s_1 s_2 c$	$-ic - s_1 s_2 - c_1 c_2 c$	$-s_1 c_2 + c_1 s_2 c$
$-\frac{1}{2} \frac{1}{2}$	$s_1 c_2 - c_1 s_2 c$	$-ic - s_1 s_2 - c_1 c_2 c$	$1 + c_1 c_2 + s_1 s_2 c$	$-c_1 s_2 + s_1 c_2 c$
$-\frac{1}{2} - \frac{1}{2}$	$ic - s_1 s_2 - c_1 c_2 c$	$-s_1 c_2 + c_1 s_2 c$	$-c_1 s_2 + s_1 c_2 c$	$1 - c_1 c_2 - s_1 s_2 c$

(12a)

$$\begin{aligned} \frac{d^3 \sigma(m_r)}{d\varepsilon_1 d\vec{k}_1 d\vec{k}_2} &= (-1)^{m_r} \frac{K_p}{36\pi} \sum_{l_1 l_2} \sum_{L_1 L_2 M} (-1)^{l'_1 + l'_2 + 1} (2L_r + 1) \\ &\times \sqrt{(2L_1 + 1)(2L_2 + 1)} \begin{pmatrix} l_1 & l'_1 & L_1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_2 & l'_2 & L_2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & L_r \\ m_r - m_r & 0 & 0 \end{pmatrix} \begin{pmatrix} L_1 & L_2 & L_r \\ M & -M & 0 \end{pmatrix} \\ &\times \begin{Bmatrix} l_1 & l_2 & 1 \\ l'_1 & l'_2 & 1 \\ L_1 & L_2 & L_r \end{Bmatrix} Y_{l_1}^{m_1}(\hat{k}_1) Y_{l_2}^{m_2}(\hat{k}_2) \langle 0 | F(1) | 0 \rangle \langle 0 | F(1) | 0 \rangle^* \end{aligned} \quad (9)$$

The second term (i.e.,  $\sigma(S_0; S_{2^+}; \hat{u}_1, \hat{u}_2)_{M_{S_0}, \mu_1 \mu_2; M_{S_{2^+}}, \mu'_1 \mu'_2}$ ) is the spin-correlation DM. It completely determines the properties of the Coulombic entanglement of the bipartite system. It can be written as

$$\begin{aligned} \sigma(S_0; S_{2^+}; \hat{u}_1, \hat{u}_2)_{M_{S_0}, \mu_1 \mu_2; M_{S_{2^+}}, \mu'_1 \mu'_2} &= (-1)^{1+\mu_1+\mu'_2+S_{2^+}} (2S_{2^+} + 1) \sum_{\substack{M_{S_0}, s_p s_{p'} s_1 \\ s_2 q m_p m_{p'}}} (-1)^{s_2 - M_{S_0}} \\ &\times \begin{pmatrix} S_0 & S_0 & q \\ M_{S_0} & -M_{S_0} & 0 \end{pmatrix} \begin{pmatrix} S_0 & S_0 & q \\ s_1 & s_1 & S_{2^+} \end{pmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & s_1 \\ \frac{1}{2} & \frac{1}{2} & s_1' \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & s_p \\ \mu_1 - \mu'_1 & m_p \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & s_{p'} \\ \mu_2 - \mu'_2 & m_{p'} \end{Bmatrix} \begin{Bmatrix} s_p & s_{p'} & q \\ m_p & m_{p'} & n \end{Bmatrix} \\ &\times \begin{pmatrix} S_0 & S_0 & q \\ M_{S_0} & -M_{S_0} & 0 \end{pmatrix} \begin{pmatrix} S_0 & S_0 & q \\ s_1 & s_1 & S_{2^+} \end{pmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & s_1' \\ \frac{1}{2} & \frac{1}{2} & s_1 \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & s_p' \\ s_p & s_{p'} & q \end{Bmatrix} D_{m_p, n}^{s_p}(\omega_1) D_{m_{p'} - n}^{s_{p'}}(\omega_2). \end{aligned} \quad (10)$$

In the DPI of Xenon  $S_0 = S_{2^+} = 0$ , therefore the spin-correlation DM of Eq. (10) becomes

$$\begin{aligned} \sigma(0; 0; \hat{u}_1, \hat{u}_2)_{\mu_1 \mu_2; \mu'_1 \mu'_2} &= \frac{1}{2} (-1)^{1+\mu_1+\mu'_2} \sum_{q m_p m_{p'}} (-1)^{q-n} (2q+1) \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & q \\ \mu_1 - \mu'_1 & m_p \end{pmatrix} \\ &\times \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & q \\ \mu_2 - \mu'_2 & m_{p'} \end{pmatrix} D_{m_p, n}^{q*}(\omega_1) D_{m_{p'} - n}^{q*}(\omega_2). \end{aligned} \quad (11)$$

Before writing the spin-correlation DM, we first consider

$$i = \sqrt{-1}, \alpha = \alpha_1 - \alpha_2, c_1 = \cos \beta_1, c_2 = \cos \beta_2, \\ s_1 = \sin \beta_1, s_2 = \sin \beta_2, s = \sin \alpha, c = \cos \alpha \\ \text{and } \sigma(0; 0; \hat{u}_1, \hat{u}_2)_{\mu_1 \mu_2; \mu'_1 \mu'_2} = \sigma_{\varepsilon_1 \varepsilon_2}.$$

We obtain the following spin-correlation DM from (9) using Racah algebra<sup>22,23</sup>:

Its partial transpose (PT)<sup>24,25</sup> is given as

We calculate from (12a) that

$$\det(\sigma_{\varepsilon_1 \varepsilon_2}) = 0 \text{ [with eigenvalues: } 0, 0, \frac{1}{2}(1 - \cos \alpha), \\ \frac{1}{2}(1 + \cos \alpha) \text{]} \quad (13a)$$

and the value of its PT from Eq. (12b) is given by

$$\det(\sigma_{\varepsilon_1 \varepsilon_2}^T) = -\frac{1}{16} \cos^2 \alpha \text{ [with eigenvalues: } \frac{1}{2}, \frac{1}{2}, \\ -\frac{1}{2\sqrt{2}}(\sqrt{1 + \cos 2\alpha}), \frac{1}{2\sqrt{2}}(\sqrt{1 + \cos 2\alpha}) \text{]}. \quad (13b)$$

Using the definition of negativity [Eq. (4)], we get the value spin-correlation negativity from Eq. (13b)

$$N = \frac{1}{2\sqrt{2}}(\sqrt{1 + \cos 2\alpha}) \quad (13c)$$

So, the spin-correlation DM of equation (13a) has more than one nonzero eigenvalues (provided

that  $\alpha \neq (2n+1)\frac{\pi}{2}$  where  $n$  is integer). Thus, all the

states, except  $\alpha = (2n+1)\frac{\pi}{2}$ , in DPI in Xe except

$\alpha = (2n+1)\frac{\pi}{2}$  form mixed states [18]. The determinant

and eigenvalues of PT clearly mean that the spin-correlation states are entangled, depending on  $\alpha$ .

In order to how TDCS affects the properties entanglement in present case, we use the values of TDCS of Xenon given in Ref. 24 for photon energy (E) 1eV for the geometry  $\theta_{12} = 180^\circ$ , where  $\theta_{12}$  is the angle between two ejected electrons. The TDCS is

$$\left. \frac{d^3 \sigma(\theta_{12} = 180^\circ)}{d\varepsilon_1 d\hat{k}_1 d\hat{k}_2} \right|_p = 4a^2 E^{n-\frac{3}{2}} \sin^2 \theta_1 \quad (14a)$$

and

$$\left. \frac{d^3 \sigma(\theta_{12} = 180^\circ)}{d\varepsilon_1 d\hat{k}_1 d\hat{k}_2} \right|_u = 4a^2 E^{n-\frac{3}{2}} (1 + \cos^2 \theta_1) \quad (14b)$$

for polarized and unpolarized photons respectively ( $a=1$  and  $n=1.056$ ).

The variation of total PT

$$[\det \sigma_{\varepsilon_1 \varepsilon_2}^T (total) = \frac{d^3 \sigma(m_r)}{d\varepsilon_1 d\hat{k}_1 d\hat{k}_2} \times \det \sigma_{\varepsilon_1 \varepsilon_2}^T]$$

calculated using equations (13b) and (14) is shown in Figure 2.

We can see from Figure 2 that DPI for xeon atom the states are entangled (the values of PT are negative) and separable (for the zero value of PT) depending on the spins of the target atom, the residual photoion, electronic qubits and the state of polarization of the absorbed pho-

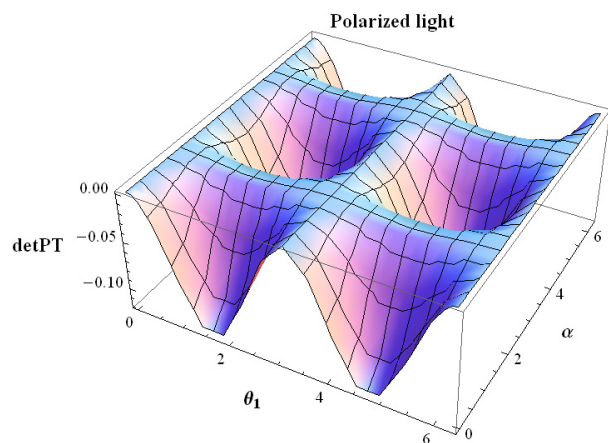
$$4\sigma_{\varepsilon_1 \varepsilon_2}^T =$$

$\mu'_1 \mu'_2$ $\mu_1 \mu_2$	$\frac{1}{2} \frac{1}{2}$	$\frac{1}{2} - \frac{1}{2}$	$-\frac{1}{2} \frac{1}{2}$	$-\frac{1}{2} - \frac{1}{2}$
$\frac{1}{2} \frac{1}{2}$	$1 - c_1 c_2 - s_1 s_2 c$	$c_1 s_2 - s_1 c_2 c$	$s_1 c_2 - c_1 s_2 c$	$-ic - s_1 s_2 - c_1 c_2 c$
$\frac{1}{2} - \frac{1}{2}$	$c_1 s_2 - s_1 c_2 c$	$1 + c_1 c_2 + s_1 s_2 c$	$ic - s_1 s_2 - c_1 c_2 c$	$-s_1 c_2 + c_1 s_2 c$
$-\frac{1}{2} \frac{1}{2}$	$s_1 c_2 - c_1 s_2 c$	$ic - s_1 s_2 - c_1 c_2 c$	$1 + c_1 c_2 + s_1 s_2 c$	$-c_1 s_2 + s_1 c_2 c$
$-\frac{1}{2} - \frac{1}{2}$	$-ic - s_1 s_2 - c_1 c_2 c$	$-s_1 c_2 + c_1 s_2 c$	$-c_1 s_2 + s_1 c_2 c$	$1 - c_1 c_2 - s_1 s_2 c$

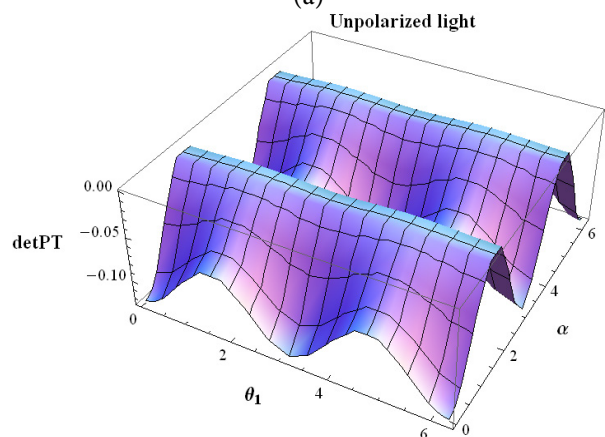
(12b)



ton as well as direction of ejection and spin quantization direction of electronic qubits.

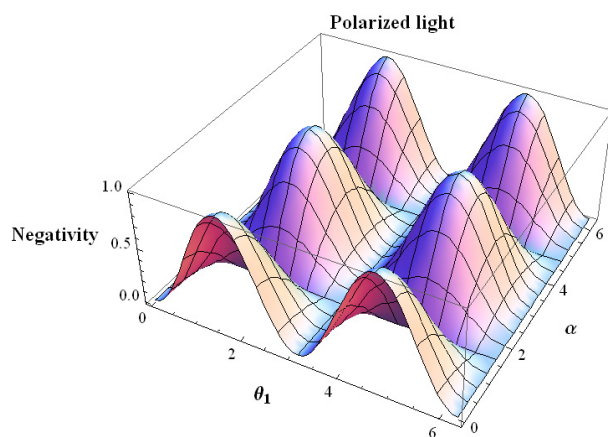


(a)

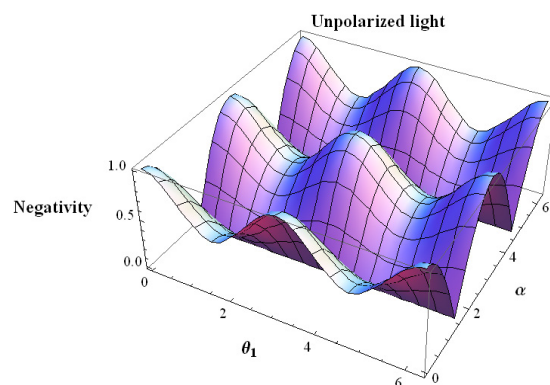


(b)

**Figure 2.** Variation of Partial Transpose (polarized light in (a) and Unpolarized Light (b)) with respect to Direction of Ejection and Spin Quantization Direction of the Photoelectrons.



(a)



(b)

**Figure 3.** Variation of Negativity (Polarized Light in (a) and Unpolarized Light (b)) with respect to Direction of Ejection and Spin Quantization Direction of the Photoelectrons.

We have also calculated total negativity

$$[N(total) = \frac{d^3 \sigma(m_r)}{d\varepsilon_1 d\hat{k}_1 d\hat{k}_2} \times N] \text{ using equations (13b) and (14)}$$

in order to measure the degree of entanglement. The variations of negativity with respect to the direction of ejection and direction of spin quantization of the photoelectrons are shown Figure 3.

From Figure 3 we see that depending on the spins of the target atom, the residual photoion, electronic qubits and the state of polarization of the absorbed photon as well as direction of ejection of photoelectrons for in xeon atom are entangled (when  $N(total) = 1$ ), partially entangled (when  $0 < N(total) < 1$ ) and separable (when  $N(total) = 0$ ).

## 4. Conclusion

The basic building block of QI is the presence of entangled states of two or more particles but entanglement between a pair of particles is a very delicate thing and is easily destroyed. Electrons, the lightest massive particles, have been used to carry information. As electrons can easily be made to travel with sufficiently high speed comparable to light and able to store information for a long time, it was already suggested that electron spin can be used for quantum computation.

DPI is extremely an simple process for simultaneously producing two electrons in continuum in a single step. This process is also the most direct manifestation of electron- electron correlation in an atom. Two electrons

emerged simultaneously in a single step shared the energy of a absorbed photon and the potential energy of the target. Here we have tried to show that the DPI process is a powerful tool for investigations of bipartite entanglement between two electronic qubits. The entanglement in DPI of xeon is quantified by Peres-Horodecki's NPT condition and negativity. This bipartite system may possess pure or mixed entanglement. Electronic qubits may be totally entangled, partially entangled or separable depending on spin states of target Xe, residual dication Xe, emitted electrons and the state of polarization of incident photon as well as of the directions spin quantization and ejection of the photoelectrons.

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