

Picosecond Optical Pulse Generation by Nonlinear Mirror Mode-Locking: A Review

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Abstract

Background/Objectives: This article provides comprehensive review on nonlinear mirror mode-locked laser which is compact, efficient, simplest and stable source of picosecond optical pulse and is of immense research interest. **Methods/Analysis:** Starting with the basics of nonlinear mirror mode-locking technique, a chronological development of the subject is presented. Issues with the efficiency and stability of the nonlinear mode-locking are addressed in both theoretical and experimental perspective. Different schemes of nonlinear mirror mode-locking, reported till date, are illustrated and compared in reference to the obtained pulse durations, power scaling, self starting and self sustained efficient mode-locking. **Findings:** Nonlinear mirror, comprising a second harmonic generating crystal and a dichroic output coupler, can mode-lock a laser due to its behaviour analogous to fast saturable absorber. Efficiency of the laser is increased with the diode pumping; however nonlinear mirror is observed prone to passive Q-switching instability. Simultaneously Q-switched and mode-locked operation gives a simple way of increasing the peak power but is of no use because of rapid fluctuations of pulse amplitude and repetition rate. Incorporation of an acousto-optic Q-switch in the laser cavity helps run the laser in actively Q-switched and passively mode-locked regime which provides enormous stability of the Q-switched and mode-locked pulse envelopes and the enhanced peak power becomes useful. Pure contentious wave mode-locking is obtained by incorporation of additional intensity dependent loss mechanism by way of introducing third harmonic generation in the laser cavity and it gives an inverse saturable nonlinear loss modulation. Inverse saturable nonlinear mirror produces efficient, self sustained and stable contentious wave mode-locked picosecond optical pulse train. **Application /Improvement:** Nonlinear mirror mode-locked lasers have wide application in micromachining, optical frequency conversion, pumping optical parametric oscillators etc. Propositions are made for getting improved nonlinear mirror mode-locking to get bandwidth limited pulse width.

Keywords: Laser, Mode-Locking, Nonlinear Mirror, Q-switching, Second Harmonic Generation, Third Harmonic Generation

1. Introduction

Over the last three decades there has been remarkable progress in the area of ultra-short optical pulse generation from a laser oscillator and till date an intensive research is going on towards all solid-state ultrafast laser technology. Success came from material scientists for developing of high power, reliable diode laser to be used as pump source for the laser and also superior quality solid-state laser materials with very high quantum efficiency has been developed. These made it possible to realize compact, high power and efficient all solid state laser-heads.¹ Several mode-locking techniques have been employed

to generate ultra-short pulse train with increased peak power. The most stunning feature of the laser is that it can produce optical pulse of extremely short duration by passive mode-locking.² The term Mode-Locking (ML) means: locking the phases of numerous simultaneously oscillating longitudinal modes of a laser resonator. The process can be understood by describing the correspondence between the output of a laser in frequency and time domain.²⁻⁴ The laser cavity can support series of axial frequency modes separated by $\Omega = 2\pi / T_R$, where T_R is the cavity round trip time. Normally the relative phases of the axial modes are quite random in time, which leads to contentious wave (cw) laser operation. For a homoge-

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neously broadened gain medium and having no spatial whole burning, a few axial modes near the centre of the gain bandwidth would saturate the gain and thus the laser output will consist of those few modes only. The output power is almost constant with time. However, all the axial modes come in same phase after each cavity roundtrip time. Once the phases are locked by some technique, the modes interfere constructively at a certain point in the cavity, resulting in a formation of intense light pulse. Since the mode amplitude changes with time due to dispersion, the point at which the modes interfere constructively changes and it makes the pulse to propagate through the cavity. Thus the evolution of the intense optical pulse occurs by locking the modes and the separation between the two pulses is just T_R . The gain bandwidth of laser material plays the key role and limits the number of modes that can be locked together. Thus the lower limit of the duration of the mode-locked pulse is primarily determined by the gain bandwidth of the laser material. In time domain the mode-locking can be viewed as the periodic modulation of intra-cavity gain and loss that generates ultra-short pulses. Though it can be realized by both active and passive techniques, passive mode-locking is always preferred since it can support shorter pulses. Passive mode-locking requires some kind of saturable absorption behaviour inside the laser cavity. The laser loss should decrease nonlinearly with the intra-cavity intensity or fluence depending on fast or slow absorber.^{5,6} For a slow saturable absorber the transmission reacts to the pulse fluence while a fast saturable absorber reacts to the instantaneous intensity of the pulse. In general, saturable absorbers help form a net gain window to assist building up of a pulse and wipe out perturbations. Several passive mode-locking techniques like Kerr-lens mode-locking, Semiconductor saturable absorber mirror mode-locking has been used for efficient ultra-short pulse generation.^{7,8} However, Nonlinear Mirror Mode-Locking (NLM) is the simplest and cheapest technique to generate picosecond (ps) pulse train and can be developed indigenously in any laser laboratory with a very compact geometry.² The aim of this paper is to start with the basics of NLM and then gives a chronological development of the subject. The issues with NLM in respect of its stability and efficiency are addressed and different schemes of NLM for efficient mode-locking are elaborated. This article also addresses the challenges with NLM and gives some proposition for future development.

2. Nonlinear Mirror

Nonlinear Mirror (NLM), as the name suggests, has an intensity dependent reflection coefficient.¹⁰ The intensity dependence of the reflection coefficient is achieved by combination of a dichroic mirror and a second order nonlinear optical frequency conversion process, mostly Second Harmonic Generation (SHG).¹¹ The SHG NLM is described schematically in Figure 1.

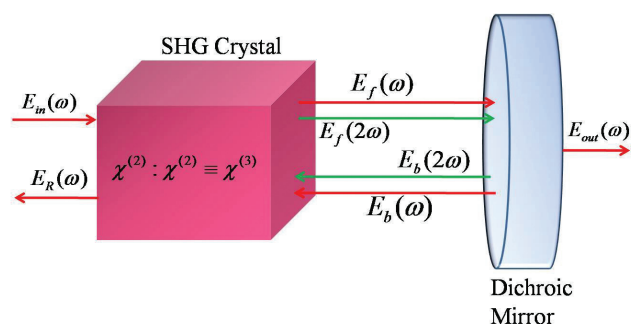


Figure 1. Second harmonic generation based nonlinear mirror.

Electric field $E_{in}(\omega)$ corresponding to the fundamental wave (FW) at frequency ω , and intensity $I_{in}(\omega)$ is incident on a SHG crystal at phase-matched condition and generates Second Harmonic (SH). The phase difference between the FW and the SH at the exit of the crystal becomes $\pi/2$ and it can be shown by solving the coupled amplitude equations.¹¹ Now, the unconverted FW and the SH are incident on a dichroic mirror, which reflects the SH completely and the FW partially. In the return pass if an additional phase difference of $-\pi$ is introduced between FW and the SH by air dispersion, the SH will be completely reconverted back in to the fundamental by difference frequency generation with the FW. The nonlinear reflectivity of the system comprising the SHG crystal and the dichroic mirror can be shown to be $R_{NL} = B \left\{ 1 - \tanh^2 \left[\sqrt{B} \tanh^{-1}(\sqrt{\eta}) - \tanh^{-1}(\sqrt{\eta R_{2\omega}}/B) \right] \right\}$; with $B = \eta R_{2\omega} + (1 - \eta)R_{\omega}$. Here η is the SHG efficiency, R_{ω} and $R_{2\omega}$ are respectively the reflectivity of the dichroic mirror at FW and SH. Since the SH efficiency is proportional to the incident FW intensity, the reflectivity of the system increases nonlinearly with the $I_{in}(\omega)$. If the nonlinear mirror system is used instead of usual output coupler of a laser the laser loss decreases nonlinearly with $I_{in}(\omega)$ and behaves as fast saturable absorber owing to the fast electronic second order nonlinearity. Figure 2 shows typical nonlinear loss ($L_{NL} = 1 - R_{NL}$) saturation behaviour for a NLM

comprising of 15 mm long LBO crystal, cut for type-I SHG at 1064 nm ($\theta=90^\circ$ and $\varphi=11.6^\circ$) and a dichroic mirror having reflectivity 100% at the SH (532 nm) and different reflectivity (70%, 78% and 81%) for the FW (1064 nm).

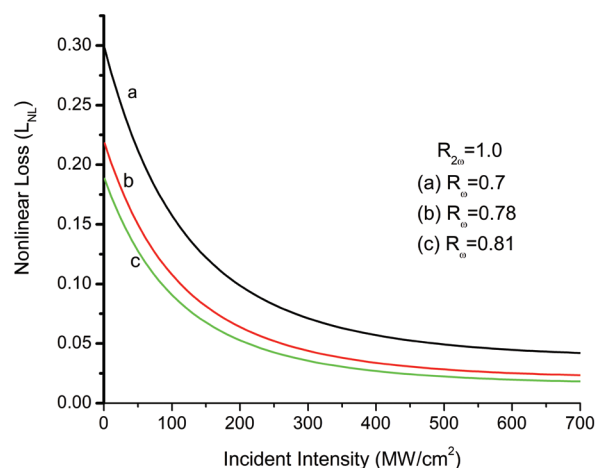


Figure 2. Nonlinear loss saturation of nonlinear mirror

Nonlinear decrease of laser loss with the intensity of FW favours mode-locking pulse formation. The overall frequency conversion process in forward and return pass through the crystal can be looked at as a cascaded second order interaction.¹² When two second order nonlinear optical process occurs simultaneously and being dependent on each other, the process called cascaded second order process and it induces a third order nonlinear susceptibility $\chi_{ind}^{(3)}$. In SHG NLM, the SHG in forward pass and the DFG in return pass produces cascaded second order interaction. The imaginary part of the $\chi_{ind}^{(3)}$ is utilized to provide direct amplitude modulation of the incident wave. There is another type of mode-locking called cascaded second order mode-locking (CSM) utilises the real part of the $\chi_{ind}^{(3)}$.^{13,14} Here instead of getting direct amplitude modulation, a phase shift is imprinted on the FW which intern deforms the FW wave front. By placing a suitable aperture in the laser cavity the nonlinear phase shift can be converted in to amplitude modulation and there by the laser loss modulation. The CSM is also a powerful technique for ps pulse generation and will be discussed elsewhere.

3. Nonlinear Mirror Mode-Locked Laser

Mode-locking by NLM was first demonstrated.² In a linear cavity, flash lamp pumped Nd:YAG laser. Beta Barium

Borate (BBO) was used as SHG crystal and train of 100 ps pulses was generated. The same group tried to reduce the mode-locked pulse width by using different SHG crystal and got 45 ps mode-locked pulse train using Potassium Titanyl Phosphate (KTP) as SHG crystal from a side flash lamp pumped Nd:YAG laser.¹⁵ Using different cavity configuration and intra-cavity telescope for efficient SH generation, 25 ps mode-locked pulses were generated from a NLM mode-locked Nd:YAG laser with BBO as SHG crystal.¹⁶ There was a continuous effort to get near transform limited pulse width from NLM mode-locked laser and different gain medium was also used for efficient NLM mode-locking. Using Nd:YALO₃ as gain medium and LiIO₃ as SHG crystal 15 ps mode-locked pulses were generated at 1.34 μm wavelength.¹⁷ To make NLM more efficient as pulse shortening device, acousto-optic modulator (AOM) run at mode-locked pulse retention frequency ($\sim\text{MHz}$) was tried in different cavity configuration. Buchvarov *et.al* shown that while active ML by AOM can generate 450 nanosecond (ns) mode-locked pulse from a Nd:YAG oscillator, the incorporation of NLM as pulse shortening device in the same cavity could shorten the pulse width up to 130 ps.¹⁸ AOM assisted NLM was also used in BBO based NLM mode-locked Nd:YAG laser and 31 ps pulse train was efficiently generated to pump a PPLN based OPO.¹⁹ This configuration was effective since AOM could assist self starting of mode-locking while NLM used as pulse shortening device. Mode-locked pulse train having pulse width 12 ps was generated from flash lamp pumped Nd:YAG laser oscillator using active-passive mode-locking.²⁰ Mode-locked laser, pumped by a flash lamp has an intrinsic limitation in terms of stability and long term performance due to unnecessary heat load and thermal birefringence. With the advent of laser diode, selective pumping was possible and the heat load to the gain medium was significantly reduced and efficiency was enhanced in terms of stability and output power scaling. The first end-diode-pumped Nd:YAG laser, mode-locked by BBO based NLM was reported by Cerullo *et.al* and 10 ps mode-locked pulse train with 750 mW output power was achieved for 3 W of pump power at 780 nm wavelength.²¹ Quasi continuous diode pumping in side-pumped geometry was tried for better power scaling, however, the pulse width was observed to be increased.¹⁹ A new laser crystal called Neodymium doped Yttrium Vanadate (Nd:YVO₄) came up with superior quality to be used in efficient NLM mode-locked laser at 1064.3 nm

and also around 1342 and 946 nm. Nd:YVO₄ has some distinct advantages over the most common Nd doped laser crystal YAG. It has the stimulated emission cross section five times larger than the Nd:YAG and eleven times larger than Nd:YLF.^{22,23} The fluorescence life time of Nd:YVO₄ is 3.2 \approx times smaller than Nd:YAG and thus relaxation oscillation damps more rapidly.^{22,24} Nd:YVO₄ has a broader band pump absorption around 808 nm than Nd:YAG and capable of generating polarized laser radiation.²² The thermal properties of Nd:YVO₄ is inferior to the Nd:YAG or Nd:YLF. However this drawback can be taken care of by controlling the doping concentration and restricting the pump power less than 20 W.^{25,26} Because of the higher gain, the Nd:YVO₄ is suitable crystal for all solid state efficient multi Watt laser with diode end pumped configuration.

Our group developed diode end pumped Nd:YVO₄ laser, mode-locked by LBO based SHG NLM, which could produce mode-locked pulse train having pulse width 8.4 ps, pulse repetition rate of 178 MHz and maximum output power of 3.23 W obtained corresponding to 10 W of pump power.^{27,28} The typical layout of end diode pumped NLM mode-locked laser is shown in Figure 3.

Here the pump source is a laser diode array and for Neodymium (Nd) doped laser AlGaAs diode should be used which emits radiation at 808 nm. A multi-mode optical fiber coupled with the laser diode array transmits the 808 nm radiation, which then focussed on to the laser crystal by two lenses. The laser crystal usually mounted tilted to avoid any back reflection. The rear mirror for Nd doped laser crystal should be such that it has very high transmission at 808 nm and 100% reflectivity

at lasing wavelength i.e. 1064 nm. The output coupler is a dielectric coated mirror having reflectivity \sim 78% at laser wavelength 1064 nm and 100% reflectivity at the second harmonic i.e. at 532 nm. A Z-shaped cavity, as shown in figure 3, to be formed by rear mirror, output coupler and two concave mirror to form an astigmatism free stable resonator capable of sustaining TEM₀₀ laser mode. Length of different arms of the resonator cavity is to be optimized by well known ABCD matrix calculation so that laser mode size at the laser crystal matches well with the focal spot size of 808 nm radiation at the gain medium.² The Z-shaped cavity has minimum cavity mode size near the output coupler where the SHG crystal is to be placed for maximum SH generation. The distance between the SHG crystal and the output coupler is varied to achieve mode-locking. Typical mode-locked pulse train as recorded by 500 MHz digital oscilloscope as shown in Figure 4(a).

However it is to be noted that even with a fast photo diode it is not possible to determine mode-locked pulse width from oscilloscope, rather a second order intensity auto-correlation is to be set to measure the mode-locked pulse width.²⁹

NLM mode-locked efficient self-starting of mode-locking process, but has a tendency to pass over to passive Q-Switched and Mode-Locked (QML) regime. In QML regime the mode-locked ps pulses are underneath of ns Q-switched envelope of repetition rate \sim kHz² and since it originates from a passive Q-switching instability, pulse to pulse fluctuation is very high as shown in Figure 4(b). Although a very high peak power is achieved in the passive QML mode, it is undesirable from a mode-locked laser to be used in scientific application.

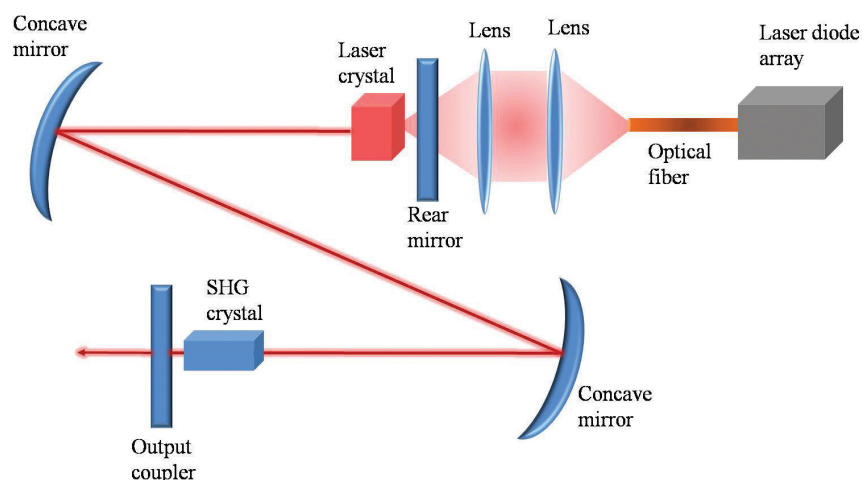


Figure 3. Schematic of diode end pumped nonlinear mirror mode-locked laser.

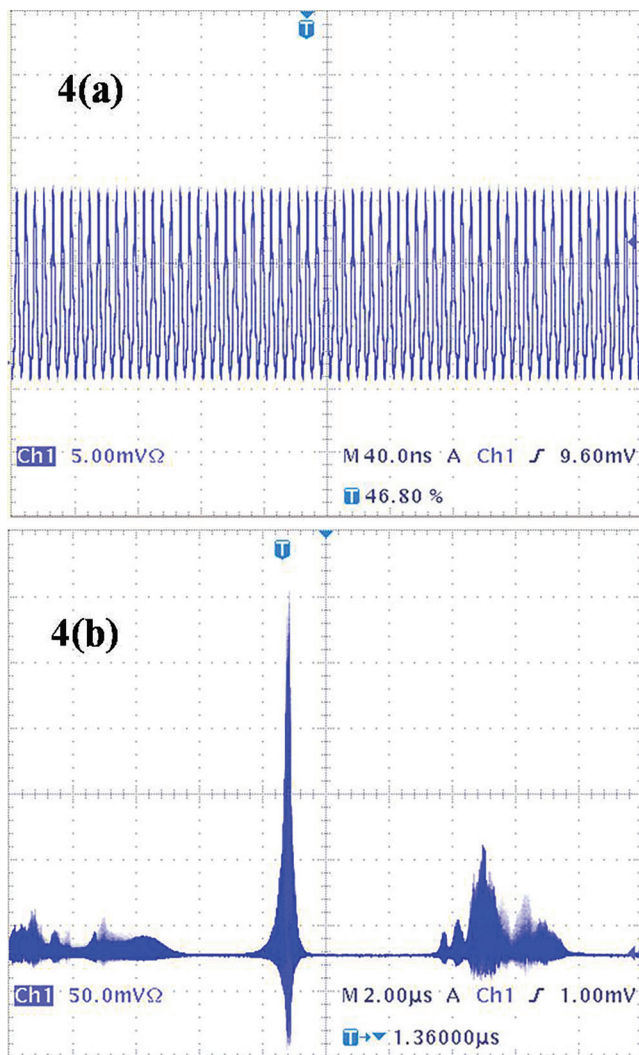


Figure 4 (a). Continuous wave mode-locked pulse train in ns time scale; (b) Passive Q-switching instability

4. Actively Q-Switched and Mode-Locked Operation of NLM

In mode-locked laser the passive Q-switching instability is always an issue because of rapid amplitude and repetition rate fluctuation of the mode-locked pulses. Nevertheless QML provides a simple way to increase the pulse energy by orders of magnitude in comparison to the continuous wave mode-locking regime. This opportunity can be conveniently exploited by incorporating in the cavity an acousto-optic Q-switch operating at frequencies of several kilohertz. In previous reports AOM was used as mode-locking device, operated at cavity roundtrip frequency (\sim MHz) and NLM was used for pulse short-

ening but the achieved pulse width was far away from bandwidth limited value.¹⁸⁻²⁰ Our group introduced an Acousto-Optic Q-Switch (AOQS) operated at frequency range tens of kHz.³⁰ The laser configuration remained same as in figure 3, except that an additional AOQS was introduced in the cavity near the gain medium which had very low insertion loss. The Q-switch was driven by a radio frequency signal of 27.2 MHz with a modulation in the frequency range 0-50 kHz. First the laser was optimized for stable Q-switched operation and then the LBO crystal was inserted to get the passive mode-locking. The MHz signal modulated by a kHz frequency creates a transmission grating having grating period determined by the kHz signal. The translating transmission grating help actively Q-switch the laser and the Q-switching repetition rate were fixed by the kHz signal. The stable Q-switched and mode-locked operation obtained for modulation frequency range of 35 – 50 kHz. The oscilloscope trace of QML pulse train is shown in Figure 5(a).

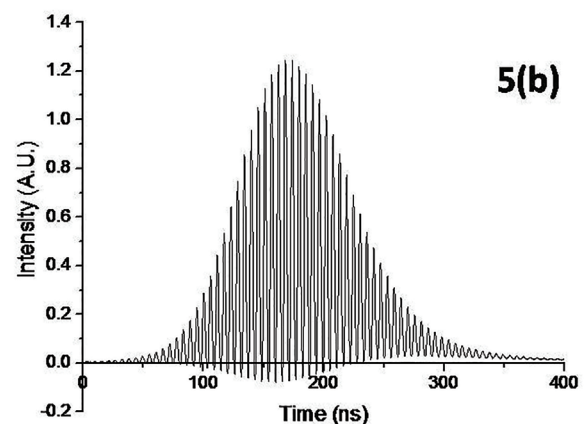
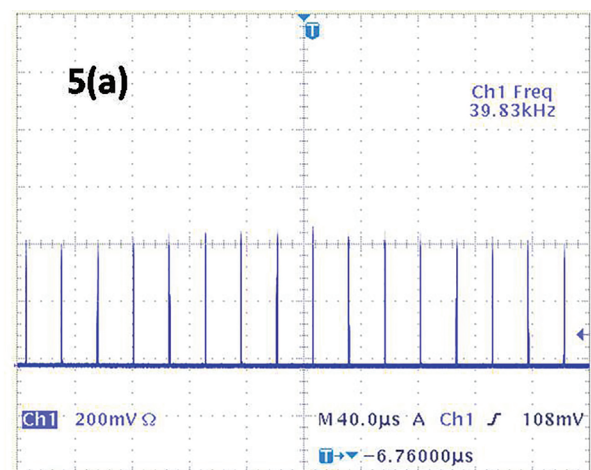


Figure 5 Actively Q-switched and nonlinear mirror mode-locked pulse train; (b) stabilized single QML pulse envelope.

It is clear from Figure 5(a) that actively Q-switched and mode-locked pulse train is stable and almost free from amplitude and repetition rate fluctuation in contrast to unstable passive QML pulse train Figure 4(b). The ps mode-locked pulse lays underneath of the Q-switch envelope of width 120 ns as shown in Figure 5(b). The mode-locked pulse width as measured by SHG intensity autocorrelation was found to be 8 ps and separated from each other by 5.4 ns corresponding to cavity round trip time. The mode-locked average power was measured to be 2.44 W corresponding 10 W of optical pumping. The peak power achieved in actively Q-switched and mode-locked operation was 380kW giving a 200 times of enhancement compared to the peak power achievable in continuous wave mode-locked regime.

Actively Q-switched and passively mode-locked regime of operation became very promising because of its stability and enhanced peak power. Actively Q-switched and nonlinear mirror mode-locked laser has demonstrated later on by various groups, using different laser gain medium as well as different SHG crystal. Using KTP as SHG crystal for NLM in an Nd:YVO₄ laser, mode-locked pulses of width 9 ps under the Q-switched envelope of 110 ns was obtained with an average output power of 2.44 W corresponding to 10 W of pump power.³¹ NLM mode-locked and actively Q-switched Nd:GdVO₄ laser with KTP based NLM was reported by Lin *et.al* where the mode-locked pulse width observed to increase to a value of 38 ps with an average power of 2.65W corresponding to a pump power of 10W.³² In other report on actively Q-switched and KTP based NLM in Nd:GdVO₄ laser a mode-locked pulse width of 190 ps was reported corresponding to Q-switched modulation frequency of 140 kHz.³³ It is thus to be noticed that increasing the Q-switch modulation frequency increases the mode-locked pulse width. On the contrary decreasing the modulation frequency to a value 5kHz – 10 kHz the laser gets also very inefficient as well as broadening of the mode-locked pulse width far above the bandwidth limited value observed.³⁴ It is thus very necessary to control the laser dynamics by optimizing the modulation frequency of the AOQS for efficient and near bandwidth limited pulse width.

It is to be mentioned here that, in practice, the back conversion of the SH to the FW does not happen completely during the return pass through the SGH crystal in a NLM. In fact complete back conversion is not necessary because even a fraction of the back conversion is sufficient to provide enough loss modulation to achieve

mode-locking. Thus the air dispersion can be controlled by varying the distance between the SHG crystal and the dichroic output coupler to optimize mode-locking as well as extract maximum SH power from the laser. Due to order of magnitude increase of the intra-cavity peak power the SHG efficiency would be very high and thus actively Q-switched and NLM mode-locked laser can be used to generate efficient SH directly from the laser head. Li *et.al* generated 185 ps mode-locked green pulses with 600mW of average power for a incident pump power of 5W from Nd:GdVO₄ laser with KTP based NLM.³⁵

The technique of actively Q-switched and passively mode-locked was later extended to SESAM mode-locked laser to exploit its stability and enormous peak power compared to cw mode-locking. 12ps optical pulses with maximum pulse energy of 0.55μJ was extracted from a SESAM mode-locked Nd:YVO₄ laser simultaneously Q-switched by AOQS.³⁶ This scheme has also been used in mode-locked fiber laser efficiently.³⁷ Actively Q-switched and mode-locked laser has been proved to be very suitable device for micromachining applications, intra-cavity frequency conversion and pumping picosecond optical parametric oscillator.³⁸⁻⁴²

5. Inverse Saturable Nonlinear Mirror

Solid state lasers, due to the long upper state life time of the gain medium, have a tendency to initiate passive Q-switching instability and run the laser in to QML mode. In addition, some parameters of the saturable absorption characteristics of the mode-locking mechanism are also responsible for Q-switching instability.⁴³⁻⁴⁵ The QML mode of operation is useful due to enhanced peak power, however, undesirable in applications where pulses of constant energy and high repetition rate are required. It is therefore necessary to manipulate the saturable absorption characteristics of the mode-locking mechanism to eliminate the passive Q-switching instability and thereby to get stable cw mode-locking. It has been shown that one has to achieve at least a critical value of the intra-cavity pulse energy to avoid passive Q-switching instability. The value of the critical pulse energy depends on the response time of the saturable absorber behaviour. For a fast saturable absorber like NLM one has to achieve critical intra-cavity pulse energy $E_c \geq \sqrt{2\Delta R P_A \tau_p E_L}$ for stable cw mode-locking.⁴⁶ Here τ_p is the pulse width and E_L is the saturation

energy for the laser crystal. ΔR , the depth of loss modulation of the saturable absorber is the maximum change in the nonlinear reflectivity or saturable loss and P_A , the saturation power of the saturable absorber is the power for which the nonlinear loss decrease to its half of the maximum value. The value of the critical energy as calculated for the laser described in section 3 is $\sim 54 \mu\text{J}$, where as the maximum intra-cavity pulse energy that was achievable for the same laser is $\sim 70\text{nJ}$. This orders of magnitude difference in the required and practically achievable intra-cavity pulse energy is the reason for passive Q-switching instability in SHG NLM. In practice a very small value of the depth of nonlinear loss modulation or the maximum change of the nonlinear loss is required to self start the mode-locking process. In SHG NLM, however, the value of is very high and it is evident from the nonlinear loss saturation characteristics as shown in Figure 2. In addition, the saturation power of the NLM saturable absorber is also higher. In fact the value of ΔR for SHG NLM is $\sim 20\%$ which is orders of magnitude higher than that is required for self starting stable mode-locking. In order to reduce ΔR and P_A , a very convenient way is to go for an inverse saturable NLM, where the nonlinear loss initially decreases with the peak intensity and then as the peak intensity tries to grow more, the nonlinear loss would increase.^{47,48} This kind of inverse saturable behaviour can be added by adding an additional intensity dependent negative feedback to the laser cavity. The intensity dependent negative feedback can be realised by incorporation of some third order nonlinear optical phenomena e.g two photon absorption, in the laser cavity.⁴⁶⁻⁴⁸ Our group for the first time demonstrated inverse saturable NLM by introducing Third Harmonic (TH) generation and was able to get stable cw mode-locking.⁴⁹ The mechanism of inverse saturable NLM is shown in Figure 6(a). For gain medium like Nd:YVO₄ the laser radiation is vertically polarized. Our group used type -I phase matching in a 15 mm long LBO ($\theta = 90^\circ$, $\varphi = 11.6^\circ$) for SHG so that the SH radiation had horizontal polarization. These orthogonally polarized FW and SH is mixed in another 12 mm long LBO ($\theta = 42.2^\circ$, $\varphi = 90^\circ$) in type-II phase matching condition for Third Harmonic Generation (THG).

The generated TH gives an intensity dependent loss to realise inverse loss saturation. The calculated values of nonlinear loss modulation for this modified SHG-THG NLM comprising of two above mentioned LBO crystals and a dichroic mirror is shown in Figure 6(b). The

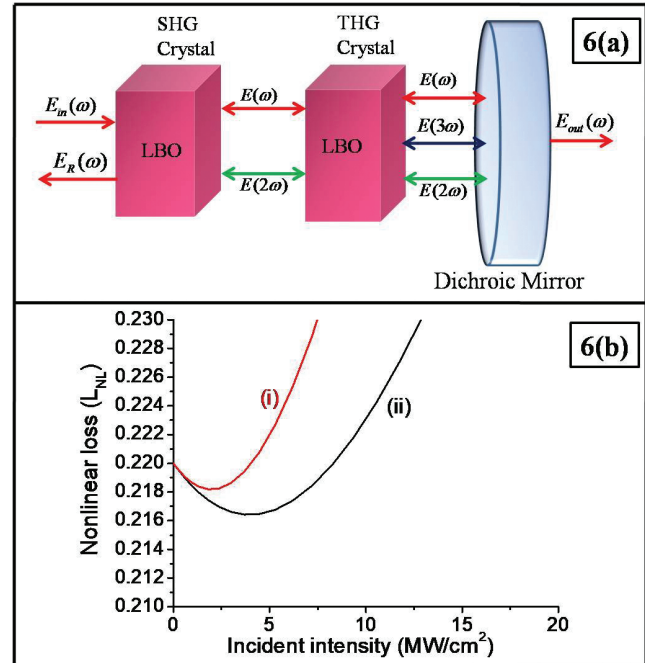


Figure 6(a). Inverse saturable NLM based on SHG and THG, (b) Inverse loss saturation of SHG-THG NLM

dichroic mirror that is used for the calculation has reflectivity for third harmonic (355nm) is 82 % for curve (i) and 0 % for curve (ii). The reflectivity of for the FW (1064nm) and SH (532 nm) are taken as 78% and 100% respectively. The inverse saturable loss modulation of modified NLM as shown in Figure 6(b) indicates clearly orders of magnitude reduction in depth of modulation as well as the saturation power. The critical pulse energy required to avoid passive Q-switching instability for this SHG-THG NLM can be shown to have form

$$E_C = \sqrt{\frac{\Delta R P_A \tau_P}{\tilde{\beta}_{THG} + \frac{1}{2E_L}}}$$

where, $\tilde{\beta}_{THG} = \beta l / 3A_{A,eff} \tau$ is the inverse saturable absorption coefficient related to THG.⁴⁶ Here, l is the length of the THG crystal and $A_{A,eff}$ is the effective beam area in the crystal. The parameter β is related to the efficiency of THG in LBO2 and is given by

$$\beta = 8\pi^2 d_{eff}^2 l / \epsilon_0 n_\omega n_{2\omega} n_{3\omega} c \lambda_{3\omega}^2$$

where d_{eff} is the effective nonlinear coefficient for SFG, n_ω , $n_{2\omega}$, $n_{3\omega}$ are the refractive indices of LBO at FW, SH and TH radiation respectively. The value of critical pulse energy is

now calculated for SHG-THG NLM is now reduced to $\sim 1\text{ nJ}$. Here all the laser parameter kept same as described in Section 3 except the fact that a THG 12 mm long LBO ($\theta = 42.2^\circ$, $\varphi = 90^\circ$) crystal is incorporated and the reflectivity of the dichroic output coupler are taken as 78% for the FW, 100% for the SH and 82% for the TH. The depth of modulation and the saturation power is taken from curve (i) of Figure 6(b). The pulse width of the pulses is taken as 29 ps and the reason will be clear soon. However, as it is evident now that the critical pulse energy for SHG-THG NLM is reduced by orders of magnitude and well below the maximum achievable intra-cavity pulse energy ($\sim 70\text{ nJ}$).

The inverse saturable SHG-THG nonlinear mirror mode-locked laser was realized by our group in same laser as described in section 3, but instead of using simple SHG NLM, now SHG-THG NLM comprising of two LBO crystal, as described in Figure 6(a) and also for calculating curve (i) of Figure 6(b) was used. The laser generated self-starting and self-sustained stable cw mode-locked pulse train with pulse repetition frequency 170 MHz. The stable cw mode-locked pulse train as recorded in millisecond time scale by a 500 MHz oscilloscope is shown in Figure 7.

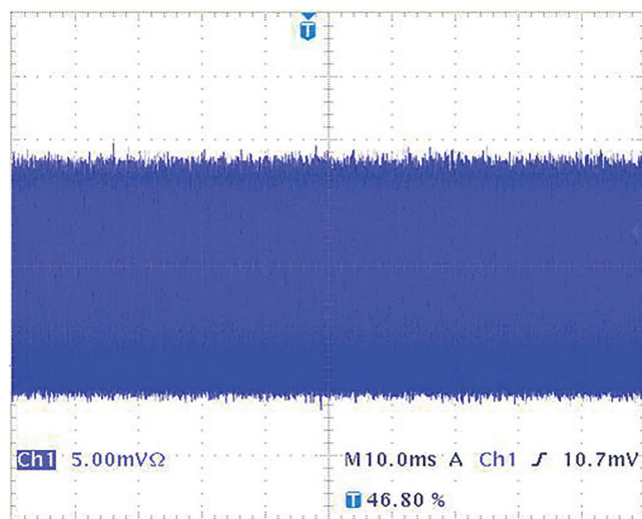


Figure 7. Stable cw mode-locked pulse train in ms time scale from inverse saturable NLM

It is evident from the figure that cw mode-locked pulse train is stable even in long time scale and free from passive Q-switching instability. The maximum mode-locked power achieved was 4.53 W for a pump power of 12.1 W. The mode-locked pulse width, as measured by second order SHG non-collinear intensity autocorrelation was

29 ps. The inverse saturable loss modulation behaviour of modified SHG-THG NLM efficiently stabilized the mode-locking by the way of eliminating passive Q-switching instabilities but the pulse width was increased far beyond the bandwidth limited value. Inverse saturation has an inherent property of increasing the pulse width, however, the Group Velocity Mismatch (GVM) between the FW, SH and the TH in the two LBO crystals plays major contributing role for broadening the pulse. The SH pulse gets delayed from FW the by ≈ 1.3 ps during forward and return pass through SHG LBO. However, the GVM parameter becomes much higher for THG in the second LBO ($\delta_c = 260$ fs/mm). It produces a delay of ≈ 5.3 ps to the SH pulse with respect to the FW during the forward and the return passes. The GVM has significant effect on the steady state pulse width because it leads to incomplete back conversion during the return pass through the nonlinear crystal.

It remains a challenge to get bandwidth limited pulse width from a SHG-THG NLM. Proper GVM compensation mechanism is to be incorporated in the cavity to get significant reduction in the pulse width. A good choice would be to use a birefringent crystal with faces cut parallel to optic axis between the two LBO crystals. The birefringent crystal is to be oriented in such a fashion that fundamental and the second harmonic beam should propagate as ordinary and extraordinary (or extraordinary and ordinary) respectively, depending on whether the crystal is positively (or negatively) birefringent.^{50,51} The GVM incurred between the fundamental and the SH in nonlinear crystals can be compensated during the forward and the return pass through the birefringent crystal. In incorporation of the birefringent crystal not only help compensate the GVM, in addition it will enhance the THG efficiency. Thus a very short THG crystal could be used to get sufficient inverse saturation and this in turn would reduce the GVM in the THG crystal to very small value.

A popular nonlinear crystal 'Beta Barium Borate (BBO)' could be a good choice to be used as GVM compensating plate. BBO crystal with faces cut parallel to the optic axis has large GVM parameter, $\delta_{\text{BBO}} \approx -280$ fs/mm around $1\text{ }\mu\text{m}$ radiation. However, a detailed numerical simulation for SHG-THG NLM, including a GVM compensation plate in between the SHG and the THG crystal can help design efficient nonlinear mirror and thereby achieving bandwidth limited pulse width.

6. Conclusion

This article gives chronological development in the field of nonlinear mirror mode-locked laser for stable picosecond optical pulse generation. NLM comprising of a SHG crystal and a dichroic output coupler was first demonstrated for mode-locking flash lamp pumped, linear cavity Nd:YAG laser in late eighties and got immediate attention for its potential to generate stable mode-locked pulse train in picoseconds regime. There after different laser gain medium having wide fluorescence bandwidth and higher quantum efficiency were employed to get efficient mode-locked lasers with shorter pulse duration. Different Nonlinear crystals and different cavity configurations were employed to get near bandwidth limited pulse width. NLM mode-locked lasers became more efficient with the availability of high power laser diodes as pump source. Diode pumped all solid state NLM mode-locked lasers were really efficient and compact source of picosecond optical pulses. However, due to long upper state lifetime of the solid state laser material and large value depth of nonlinear loss modulation as well as saturation power, passive Q-switching instability was a usual feature in NLM mode-locking. Passive Q-switching instability gives significant enhancement in the peak power but the rapid fluctuation of pulse amplitude and the pulse repetition rate put forward a limitation for scientific applications. Introduction of a active Q-switch in the laser cavity of a NLM laser could stabilize the Q-switch pulse envelope and help utilize enhanced peak power in QML mode of operation. The picoseconds mode-locked pulses underneath of a nanosecond Q-switch envelope could achieve nearly 300 times enhancement in the peak power compared to the cw mode-locking. This actively Q-switched and passively mode-locked became very popular and the principle was extended in semiconductor saturable absorber mode-locking and also mode-locked fiber lasers. However, it was always a challenge to extract pure cw mode-locked pulse train from a NLM mode-locked laser for which parameters controlling the mode-locking dynamics were needed to be tailored. Introduction of a THG crystal, in addition to the SHG crystal significantly changed the fast nonlinear loss modulation characteristic by generating an intensity dependent negative feedback to the laser. It behaved as inverse saturable absorber and reduced significantly the depth of modulation as well as the saturation power. SHG-THG NLM could reduce the required critical intra-cavity pulse energy for stable cw

mode-locking by three orders of magnitude and finally could produce stable cw mode-locked pulse train, not affected by the passive Q-switching instability. However, inverse saturable using two nonlinear crystal increased the mod-locked pulse width far beyond the transform limited value and the reason for it is believed to be the GVM between the FW, SH and the third harmonic in the two nonlinear crystal. It thus remains a challenge and scope for future to design efficient, GVM compensated NLM which would produce stable cw mode-locked pulse train having transform limited pulse width.

7. Acknowledgement

Department of Physics, IIT Kharagpur and Department of Higher Education, Government of West Bengal are gratefully acknowledged.

8. References

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