

# Modified Gain Bearing-only Extended Kalman Filter for Underwater Target Tracking

N. Priyanka\*, M. Yamini, S. Koteswara Rao and A. Jawahar

Department of ECE, KL University, Greenfields, Vaddeswaram, Guntur - 522502, Andhra Pradesh, India;  
nandigam.priyanka@gmail.com, yammy.maturi@gmail.com, rao.sk9@gmail.com, jawaharaee@outlook.com

## Abstract

**Objectives:** This paper proposes the own ship manoeuvring strategy to estimate the proper target state in the bearings only tracking. **Methods/Statistical Analysis:** The modified gain bearings-only extended Kalman filter is used for analysis, where the gain is modifiable. In the context of Target Motion Analysis, the target state is monitored through nonlinear estimations. The own ship exact manoeuvre is to increase the range observability. **Findings:** The proposed target estimation algorithm is effective in maritime tracking based sonar applications. **Application/Improvements:** The methodology proposed here is very useful for tracking underwater targets and finds immense usage for Indian Navy.

**Keywords:** Estimation, Manoeuvre, Own Ship, Simulation, Sonar, Underwater Tracking

## Notations-

$\dot{x}$  --  $X$  Component to a target velocity, (meters/sec)  
 $\dot{y}$  --  $Y$  Component to a target velocity, (meters/sec)  
 $R_x$  --  $X$  Component to a relative range, (meters)  
 $R_y$  --  $Y$  Component to a relative range, (meters)  
 $x_0$  --  $X$  Component to an observer position, (meters)  
 $y_0$  --  $Y$  Component to an observer position, (meters)  
 $b$  -- Deterministic vector  
 $\phi$  -- Transition matrix  
 $\omega(u)$  -- Plant noise  
 $u$  -- Secondary scaling parameter  
 $\Gamma$  -- Gain matrix for Plant noise  
 $B_m$  -- Bearing measurement, (degrees)  
 $C$  -- Target Course, (degrees)  
 $P$  -- Estimated target state vector Covariance matrix  
 $R$  -- Range of a Target, (meters)

$S$  -- Speed of a Target, (meters/sec)  
 $W$  -- Inverse of the input measurement covariance matrix  
 $X$  -- Target state vector  
 $Y$  -- Predicted measurement vector  
 $h(u+1, X_s(u+1, u))$  -- Bearing using predicted estimate with time index  $(u+1)$   
 $t$  -- Sample time  
 $Q$  -- Covariance matrix of Gaussian white noise

## 1. Introduction

In undersea surroundings, 2D target analysis is vividly utilized. An own ship monitors most of the noisy bearings from an acoustic target and evaluates Target Motion Parameters such as Range, Speed, Course and Bearing. Therefore, essential supposition is that the target moves at nearly steady speed. The own ship movement is unhindered. The target and own ship are taken to be in same X- plane. The determination of path of the target exclusively from bearing observations is called Bearings-

\*Author for correspondence

**Table 1.** Mathematical modelling

1.) The target state vector  $X_s$  is,

$$X_s(u) = [\dot{x}(u) \ \dot{y}(u) \ R_x(u) \ R_y(u)]^T$$

2.) The objective state dynamic mathematical model is,

$$X_s(u+1) = [\phi(u+1, u) * X_s(u)] + b(u+1) + \omega(u)$$

3.) The transition matrix is,

$$\phi(u+1, u) =$$

$$b(u+1) = [0 \quad 0 \quad -(x_0(u+1) - x_0(u)) \quad -(y_0(u+1) - y_0(u))]^T$$

Furthermore,

$$\Gamma = \begin{bmatrix} t & 0 \\ 0 & t \\ \frac{t^2}{2} & 0 \\ 0 & \frac{t^2}{2} \end{bmatrix}$$

5.) The deliberate bearing,  $B_m$  is,

$$B_m(u+1) = \gamma_B(u+1)$$

6.) The estimated matrix is,

$$H(u+1) = \begin{bmatrix} 0 & 0 & \frac{\hat{R}_y(u+1, u)}{\hat{R}^2(u+1, u)} & \frac{\hat{R}_x(u+1, u)}{\hat{R}^2(u+1, u)} \end{bmatrix}$$

7.) The covariance matrix is hence,

$$P(u+1, u) = (\phi(u+1, u)P(u, u)\phi^T(u+1, u)) + \Gamma Q(u+1)\Gamma^T$$

8.) The Kalman gain is,

$$G(u+1) = P(u+1, u)H^T(u+1) [\sigma_B^2 + H(u+1)P(u+1, u)H^T(u+1)]^{-1}$$

9.) The state corrections are taken as,

$$X_s(u+1, u+1) = X_s(u+1, u) + G(u+1) [B_m(u+1) - h(u+1, X_s(u+1, u))]$$

10.) Modified gain function is defined as,

$$g = \begin{bmatrix} 0 & 0 & \left( \frac{\cos B_m}{\hat{R}_x \sin B_m + \hat{R}_y \cos B_m} \right) & \left( \frac{-\sin B_m}{\hat{R}_x \sin B_m + \hat{R}_y \cos B_m} \right) \end{bmatrix}$$

11.) The covariance corrections are taken as,

$$P(u+1, u+1) = [I - G(u+1)g(B_m(u+1), X_s(u+1, u))] P(u+1, u) \\ \times [I - G(u+1)g(B_m(u+1), X_s(u+1, u))]^T + \sigma_B^2 G(u+1)G^T(u+1)$$

**Table 2.** Scenarios taken for medium ATB

| Scenario                   | Range initially(m) | Bearing initially(deg) | Speed of Target (m/s) | Speed of Observer (m/s) | Target course (deg) |
|----------------------------|--------------------|------------------------|-----------------------|-------------------------|---------------------|
| 1.(Submarine to submarine) | 7000               | 0                      | 8                     | 5                       | 132                 |
| 2.(Submarine to ship)      | 12000              | 0                      | 12                    | 5                       | 132                 |
| 3.(Submarine to torpedo)   | 16000              | 0                      | 15                    | 5                       | 132                 |

For Submarine to Submarine (Medium ATB):

**Table 3.** Converged time (sec) for medium ATB scenarios

| Number of Runs | MGEKF(Submarine to Submarine) |               |              |                |
|----------------|-------------------------------|---------------|--------------|----------------|
|                | Err in Range                  | Err in Course | Err in Speed | Total solution |
| 1 run          | 713                           | 806           | 549          | 806            |
| 10 runs        | 813                           | 819           | 303          | 819            |
| 50 runs        | 597                           | 595           | 320          | 597            |

**Table 4.** Converged time (sec) for medium ATB scenarios

| Number of Runs | MGEKF (Submarine to Ship) |               |              |                |
|----------------|---------------------------|---------------|--------------|----------------|
|                | Err in Range              | Err in Course | Err in Speed | Total solution |
| 1 run          | 886                       | 877           | 623          | 886            |
| 10 runs        | 861                       | 855           | 589          | 861            |
| 50 runs        | 844                       | 838           | 584          | 844            |

only Tracking (BOT)<sup>1-5</sup>. Appropriate manoeuvre is done only when the process becomes observable and the bearing values obtained are from single sensor. In practicality, the target trajectory is not known initially, hence the observer manoeuvre can be chosen utilizing the adequate measurements. Own ship adjustments in bearing rate need to be improved which is for the estimation, which is not sufficient for the estimation of speed. The ideas are stretched out to get the whole arrangement, viz., range, course, speed estimation utilizing basic science<sup>3-8</sup>. The technique for picking initial covariance matrix is additionally depicted<sup>7-10</sup>. Mathematical modelling of MGBEKF is shown in Table 1.

## 2. Implementation and Simulation

The own ship moves according to the initial parameters given such as Bearing, Course and Velocity of target and own ship respectively. Initially we have taken the values as given in the Table 2.

The expected value of initial range considered for the scenarios are as follows

i. Submarine to Submarine scenario-8500 m

ii. Submarine to Ship scenario -13500 m

iii. Submarine to Torpedo scenario -21000 m

Implementation of this algorithm is done in MATLAB. The targets are Submarine, Ship and Torpedo depending upon the scenarios. The target is said to be tracked if at all the algorithm converges the parameters like Range, Speed, and Course. In this MGEKF algorithm converged time in seconds are shown in Table 3 and Table 4.

Some samples are taken for calculating errors in Range, Course, and Speed of Medium ATB.

With MGEKF algorithm, in the scenario of Submarine to Submarine, estimate Speed, estimate Range, estimate Course of the target are converged at 549 sec, 713 sec, 806 sec, and hence for single run mode Total solution is converged at 806 sec. Similarly, for 50 runs the Total solution have converged time at 597 sec, which is clearly shown in Table 3.

With MGEKF algorithm, in the scenario of Submarine to Ship, estimate Speed, estimate Course, estimate Range of the target are converged at 623 sec, 877 sec, 886 sec and hence for single run mode Total solution is converged at 886 sec. Similarly, for 50 runs the Total solution have converged time at 844 sec, which is clearly shown in Table 4.

For Submarine to Submarine:

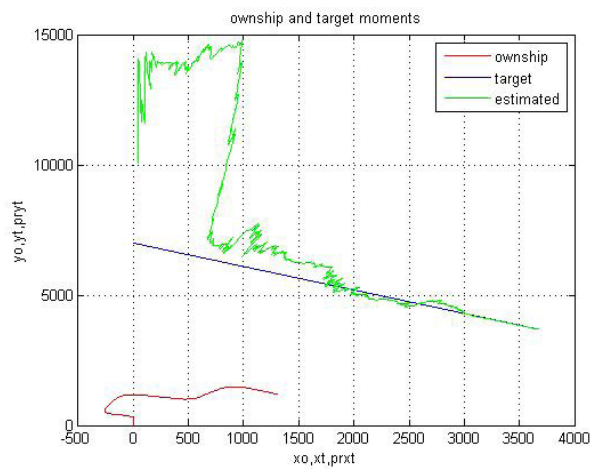


Figure 1. Ownship and target moments.

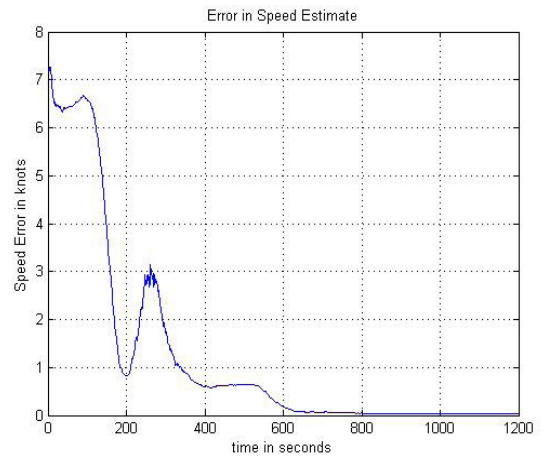


Figure 4. Speed estimate error.

For Submarine to Ship:

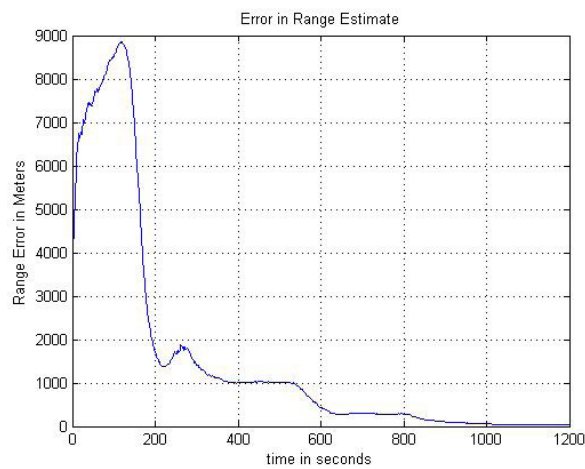


Figure 2. Range estimate error.

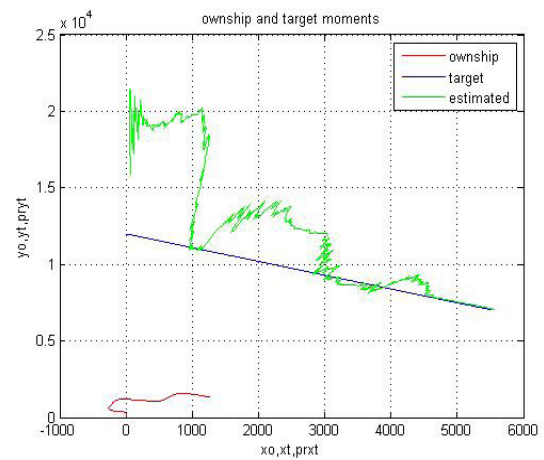


Figure 5. Ownship and target moments.

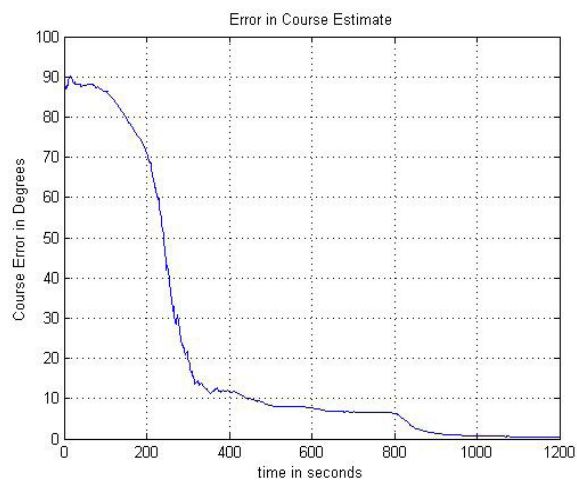


Figure 3. Course estimate error.

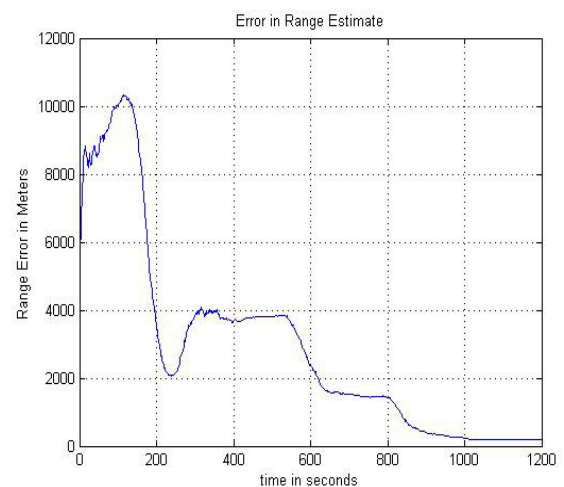


Figure 6. Range estimate error.

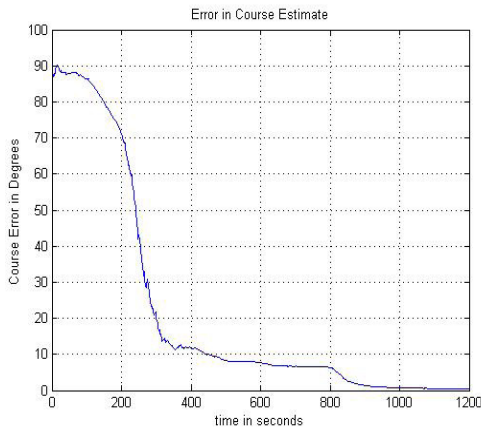


Figure 7. Course estimate error.

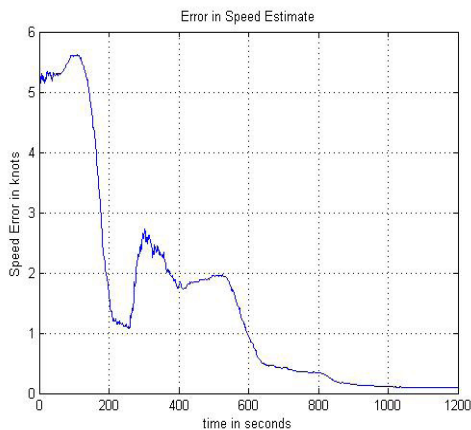


Figure 8. Speed estimate error.

It is not apt to estimate the performance of the algorithm for single run, due to randomness. Monte-Carlo Algorithm with 50 runs is done and result is compared for each runs. Maximum error acceptable in estimated range, course and speeds are 8%, 3° and 1m/s respectively. By this criterion, the converged solution obtained in Monte-Carlo Algorithm simulation for 50 runs is shown in Table 3 and Table 4. The outputs obtained as graphs in these Monte-Carlo simulations for 50 runs are clearly shown in Figure 1 to Figure 8.

With MGEKF algorithm, in the scenario of Submarine to Torpedo scenarios at medium ATBs it is realized that the solution is not converged due to low bearing rate.

### 3. Conclusion

This algorithm created for supervision of target is stretched out to the application that is passive target

tracking in highly noisy environment such as submerged. The execution of the algorithm is displayed for a situation and it is watched that the arrangement with required precision is acquired at 8 minutes that is adequate for underwater tracking of target. The simulation results show that MGBEKF is relevant for bearings-only passive target tracking in submerged area.

### 4. References

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