

A Numerical Study of MHD and Heat Transfer Analysis in a non-Newtonian Eyring-Powell Fluid from an Isothermal Sphere with Thermal Slip

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Abstract

The study of magneto hydrodynamic flow and heat transfer analysis in a non-Newtonian Eyring-Powell fluid fluid past an isothermal sphere with thermal slip. The radiation effects also considered in the energy equation. The governing momentum and energy equations are transformed to nonlinear ordinary differential equations by the use of a non-similarity transformation. These equations are solved by numerically subject to physical appropriate boundary conditions using the second order accurate implicit finite difference Keller-box technique. The effects of magnetic field, Eyring-Powell fluid parameter, Prandtl number, Slip parameter on velocity and temperature profiles as well as on the local skin friction coefficient and the local Nusselt number are calculated.

Keywords: Difference Scheme, Eyring-Powell Fluid, Isothermal Sphere, Kellerbox Finite, Magneto Hydrodynamic

Terminology

a	Sphere of radius
c	Fluid parameter of the sphere
Gr	Grash of number
F	dimensionless stream function
g	Acceleration due to gravity
Pr	Prandtl number
C_f	coefficient of skin friction
Nu	Nusselt number
T	Fluid Temperature
u, v	Dimensionless velocity Components along x-and y- directions, respectively
x	Stream wise coordinate
y	Transverse coordinates
α	Thermal diffusivity
η	Radial co-ordinate dimensionless system
μ	Dynamic viscosity
ν	Kinematic viscosity

θ	The dimensionless parameter
ρ	Density of the fluid
ε	Eyring -Powell fluid parameter
ξ	Tangential coordinate with dimensionless system
ψ	Non dimensional stream function

Subscripts

W	Conditions at the wall
∞	Free stream conditions

1. Introduction

In present days no and polymer melt in the plastic processing industries. Many authors has investigates so many decades¹⁻⁶ and as well as many physical effects is also included. These flow shows up in a wide variety of engineering applications, and moreover, in numerous natural conditions, for example, geothermal extraction, storage, nuclear waste material, oil refining strategies, ground

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water, streamsetc., in this way the expertis exploring to examine its thermo physical properties.

Magneto hydrodynamic flows additionally emerge in rheological wire covering processes⁷, MHD levitation control of the diamagnetic material manufacturer⁸. Each of these studies also have an impact in flow variations. Theheat and mass transfer on Magneto hydrodynamic flows in both non-permeable and permeable regimes have received considerable among engineering scientists.

A non-Newtonian fluid i.e. eyring powell model has very important role in Chemical engineering . The Eyring-Powell fluid model in non-Newtonian fluids is created for the study of Biological engineering structures. This model of rheology has definite advantages over the-non-Newtonian definitions. Powell and Eyring² examined the devices of reduction theory of viscosity. Several communications using Eyring-Powell fluid model has been exhibited in the scientific Research. Islam *et al.*¹⁰ identifying the resulting homotopy perturbation solutions for slider behaviors lubricated with Eyring-Powell fluid model. Patel and Timol¹¹ studied measurably inspected the stream of Eyring-Powell fluids from a two dimensional wedge. Recently Malik *et al.*¹² concentrated on mixed convection flow of MHD Eyring-Powell fluid over a stretching sheet. They found that rate of heat and mass transfer diminishes for all parameters. Abdul Gaffar *et al.*¹³ focused on the MHD free convection flow of Eyring-Powell fluid from vertical surface in permeable media with hall/ion slip present and ohmic dissemination.

The main aim of the present study of MHD and heat transfer analysis in a non-Newtonian Eyring-Powell fluid from an isothermal sphere with thermal slip. Keller-Box finite difference method is used to get Numerical solutions for the velocity and the temperature distribution. The graphs are plotted and examined variations of parameters.

2. Mathematical Analysis

Let's take the two-dimensional, viscous, incompressible, lightness driven convection heat assignment flow from an Isothermal Sphere rooted in an Eyring Powell non-Newtonian fluid. Figure 1demonstrates the flow model and physical coordinate system. Now x is taken along the external of the sphere and y is taken along normal to the surface, correspondingly, and the radiated space (r) is taken from symmetric axes to $r = a \sin(x/a)$ surface. ' a ' is the domain of the radius. The gravity, g acting downwards

and uniform magnetic field B_0 is chosen the circular direction, i.e. Ordinary to the cylinder surface.

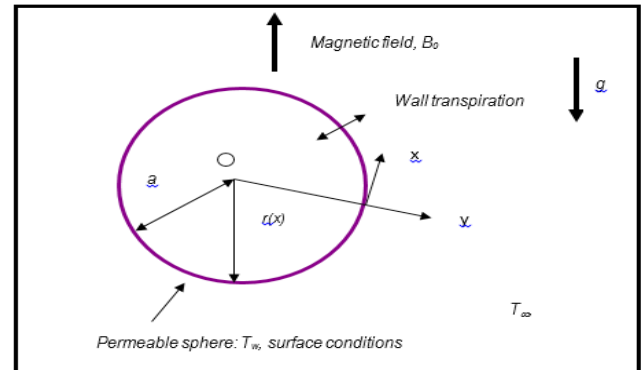


Figure 1. Physical model and coordinate system.

The equivalent velocities in the x and y directions are u and v . The main equations can be given as follows:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left(\nu + \frac{1}{\rho \beta c} \right) \frac{\partial^2 u}{\partial y^2} - \frac{1}{2\rho \beta c^3} \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \sin\left(\frac{x}{a}\right) - \frac{\sigma B_0^2}{\rho} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial^2 q_r}{\partial y^2} \quad (3)$$

Where u and v are the velocity components in x and y directions and all the other terms are defined in the terminology. At the surface and the edge of the boundary layer regime, the boundary conditions are prescribed, the following ways:

$$\text{At } y=0 : \quad u=0, \quad v=0, \quad -K \frac{\partial T}{\partial y} = h_w(T_w - T) \quad (4)$$

$$\text{As } y \rightarrow \infty : \quad u \rightarrow 0 \quad T \rightarrow T_\infty$$

Here T_∞ stands for free stream temperature, k is the warm conductivity, h_w is the convective thermal transfer coefficient, T_w conductive wall temperature of the fluid. We can define the stream function ψ by

$$ru = \frac{\partial(r\psi)}{\partial y}, \quad rv = -\frac{\partial(r\psi)}{\partial x}.$$

Consequently, the equation of continuity is spontaneously fulfilled. The dimensional fewer quantities are presented as,

$$\xi = \frac{x}{a}, \quad \eta = \frac{y}{a} Gr^{1/4}, \quad \psi = \nu \xi Gr^{-1/4} f, \quad \varepsilon = \frac{1}{\mu \beta c} \quad (5)$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad Gr = \frac{g\beta(T_w - T_\infty)a^3}{\nu^3}, \quad Pr = \frac{\nu}{\alpha}, \quad \delta = \frac{\nu^2}{2c^2 a^4} Gr^{3/2}$$

In the vision of the transformations clear in the above equations, the boundary layer eqns. Reduce to the following third order system of dimensions, the momentum, and energy, PDE for the regime:

$$(1+\varepsilon)f''' + (1+\xi \cot \xi)ff'' - f'^2 - \varepsilon\delta\xi^2 f''^2 f''' - Mf' + \frac{\sin \xi}{\xi} \theta = \xi \left(f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} \right) \quad (6)$$

$$\frac{1}{Pr} \left(1 + \frac{4F}{3} \right) \theta'' + (1+\xi \cot \xi) f \theta' = \xi \left(f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right) \quad (7)$$

The converted dimensionless boundary conditions are as follows

$$\begin{aligned} \text{At } \eta = 0, \quad f = 0, \quad f' = 0, \quad \theta = 1 + S_T \theta'(0) \\ \text{As } \eta \rightarrow \infty, \quad f' \rightarrow 0, \quad \theta \rightarrow 0 \end{aligned} \quad (8)$$

Here primes indicate the derivative w. r. to η and

$$S_T = \frac{ah_w}{k} Gr^{\frac{1}{4}} \text{ is the thermal slip parameter. The wall}$$

thermal boundary condition in (8) relates to convective cooling. The coefficient of skin friction and Nusselt number (rate of heat transfer) can be obtained by the conversions depicted in the above with the resulting expressions.

$$Gr^{\frac{3}{4}} C_f = (1+\varepsilon) \xi f''(\xi, 0) - \frac{\delta}{3} \varepsilon \xi^3 (f'''(\xi, 0))^3 \quad (9)$$

$$Gr^{\frac{1}{4}} Nu = -\theta'(\xi, 0) \quad (10)$$

3. Numerical Solution

We have applied the implicit effectual Keller-Box difference method to evaluate the flow model characterized by equations (6) – (7) with the boundary conditions (8), initially developed for low speed flowing boundary layers (produced by Keller¹⁴). This framework was developed by Cebeci and Bradshaw¹⁵. This strategy has been utilized in a various scope of modern physical fluid flow issues. These incorporate Casson slip boundary layer flows^{16,17}. This technique remains among the most effective, adaptable and exact computational finite difference schemes employed in modern viscous fluid dynamics simulations. This method has been utilized broadly and effectively for more than three decades in a large spectrum of nonlinear fluid mechanics problems. Keller's techniques provide unconditional stability and rapid convergence for strongly non-linear flows. It includes four key stages, summarized below.

- Reduction of the N^{th} order partial differential equation frame work with N initial order equations
- Finite difference discretization of reduced equations
- Quasi linearization of non-linear Keller mathematical equations
- Block-tridiagonal elimination of linearized Keller mathematical equations

4. Results and Discussion

Figure 2-5 gives the detailed solution. The mathematical issues include double free variables (ξ, η), two dependent fluid dynamic variables (f, θ) and 4 thermo-physical and body constrained control parameters, namely ε, δ, S_T and M . The effect of Eyring-Powell fluid factor ε , on velocity and temperature as shown in the Figure 2a and 2b and observed that the increase in ε , the boundary layer flow is accelerated with increasing Eyring-Powell fluid constraint and controls and temperature contours are diminished through the boundary layer regime.

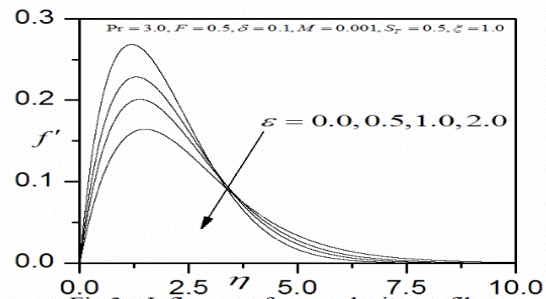


Figure 2a. Influence of ε on velocity profiles.

Figure 3a and 3b depict the effects of radiation parameter F , if it is noticed the velocity and temperature profiles stay converged and close to the boundary layer when increasing the radiation parameter. The flow is accelerated and velocity is increased. The temperature profiles also increase when increasing the radiation parameter. For various values of the magnetic parameter M the velocity and temperature profiles are plotted in Fig.4a and 4b. Increases the magnetic parameter M when opposing the flow decreases and as well as enhanced the deceleration of the flow in this reason the velocity and temperature automatically decrease. Figure 5a and 5b illustrate the influence of the thermal slip S_T on the transient velocity and temperature. As S_T increases the velocity and tem-

perature decreases. The cause the temperature buoyancy effects to decrease yielding a reduction in the fluid velocity. The increasing in the velocity and temperature profiles is accompanied by simultaneous decreases in velocity and temperature of the boundary layer.

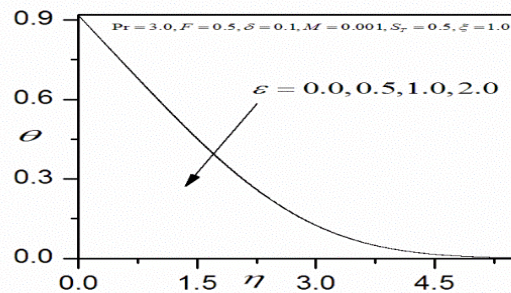


Figure 2b. Influence of ε on temperature profiles.

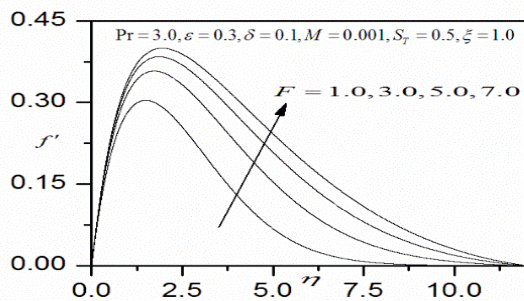


Figure 3a. Influence of F on velocity profiles.

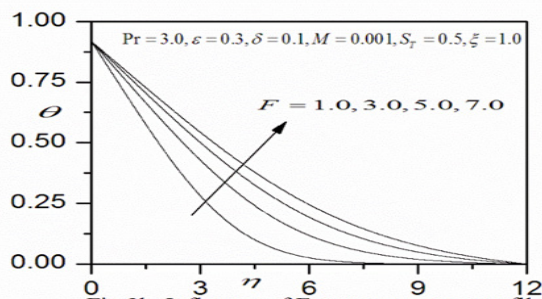


Figure 3b. Influence of F on temperature profiles.

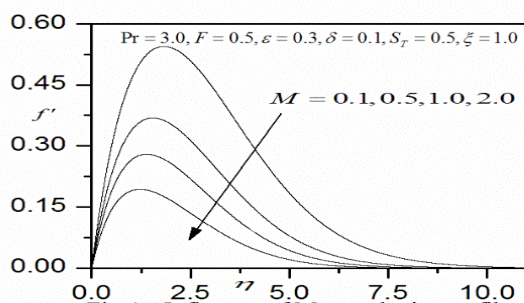


Figure 4a. Influence of M on velocity profiles.

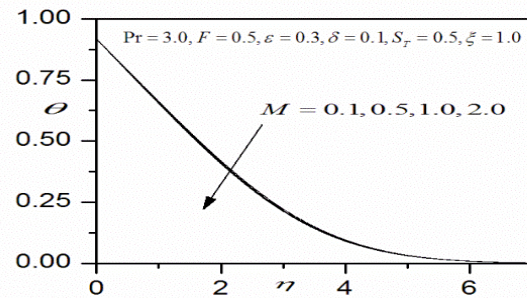


Figure 4b. Influence of M on temperature profiles.

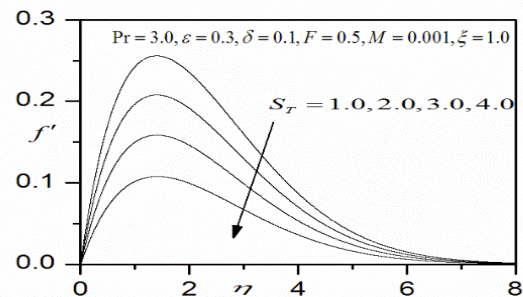


Figure 5a. Influence of S_T on velocity profiles.

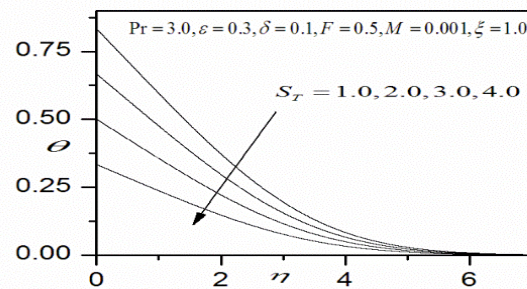


Figure 5b. Influence of S_T on temperature profiles.

5. Conclusion

We examined the steady boundary layer MHD stream and heat transfer analysis in an Eyring Powell fluid from an isothermal circle with thermal slip. Accurate modelling through equations of continuity and motion leads to a non-linear differential equation even after employing the boundary layer assumptions. This study, my knowledge, has not appeared in the technical literature. Mathematical solutions have been presented for the heat transfer of Eyring-Powell flow over an isothermal sphere. Keller box implicit finite difference method is efficiently useful to evaluate the transformed flow characteristics and fluid boundary layer equations. In this paper we have observed the points as shown below:

- (1) Cumulative the Eyring – Powell fluid parameter (ε), reduces the velocity and temperature in the boundary layer.
- (2) When increasing the radiation parameter (F), velocity and temperature are increased.
- (3) Accession the magnetic parameter (M), both the velocity and temperature are decreased.

6. Acknowledgement

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