

Image Compression in Spatial Domain: Weighted C-Mean Method

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Abstract

Objective: For reducing the data redundancy to save more hardware space and transmission bandwidth, data compression is used. **Methods:** For image compression, we have proposed a new method based on c-mean and absolute moment Block Truncation Coding (BTC) techniques. In this technique an image is segmented into several non-overlapping blocks. For each block, calculate the weighted c-mean by means of linear combination of its arithmetic mean and c-mean. The c-mean is evaluated by expressing every pixel into a sum of largest perfect square of non-negative integer. For each image block, the pixels are classified into two range of values based on weighted c-mean. The gray values of the image block which are greater than block weighted c-mean are considered as upper range and otherwise lower range. The weighted c-mean of upper range and lower range are termed as higher c-mean and lower c-mean respectively. Then construct the binary block matrix (0/1) based on weighted c-mean as a quantization level. In the decoder part, an image block is reconstructed by replacing the 1's with upper c-mean and 0's by lower c-mean of block matrix. **Findings:** The newly designed method is tested some 8-bit standard images. It is also compared with related works in term of Peak Signal to Noise Ratio, Bit Rate, and Structural similarity index. It shows the effectiveness of the proposed method. **Applications:** Image storage is required for several purposes like document, medical images, magnetic resonance imaging and radiology, motion pictures etc. All such applications are based on image compression.

Keywords: AMBTC, BTC, Image Compression, PSNR

1. Introduction

A process by which data are stored with fewer bits than original bits of data in a computer or the same data is transmitted by using a smaller number of bits are known as data compression¹. Now a day's various multimedia items are processed in digital form. These items may be text, image, audio and video or combination of some or all. The quantity of such items is huge and as a result a large amount of storage space is required to store in digital computer. It is also transmitted via limited bandwidth channel and hence time consume. The quantity of such data transmitted through Internet grows heavily every year, and a huge portion of this data is image. The usefulness of digital images in communicating information is

well appreciated; however, to store the images is required large quantity of storage space and to transmit the images is time consuming. In response to the demand, the capacity of the storage devices as well as the Internet bandwidth is increased. Still, it is not sufficient to meet the current requirement. Under such circumstances, image compression technique has come up as a most effective solution. In image compression techniques, image is represented in compact way. Therefore, it increases more data to be stored in the storage device.

This is known as data reduction (C). Mathematically the amount of data reduction is formulated²⁻⁴, as follows:

Let $(g = g_0, g_1, g_2, \dots, g_{M-1})^T$ be a vector having M elements. Let each element is coded by 'b' bits. Therefore,

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total bits are required to store 'g' is Mb, as 'g' contains 'M' elements. Now the vector 'g' is transformed to another vector 'G' by some transformation T i.e. $G=Tg$. Let the vector 'G' is represented by k bits. So there are three possible cases:

Case 1: $Mb > k$ Case 2: $Mb = k$, Case 3: $Mb < k$

The compression is efficient in case 1 only by either retaining only M' ($< M$ elements of G) or by encoding the elements of G in such a way that the average number of bits per elements is $b' (< b)$. Therefore $k = M'b$ or $k = Mb'$. The amount of data reduction (C) is given by

$$C = \frac{Mb - k}{Mb} \times 100\%$$

The quality of reconstructed image is application dependent. For example, in medical science and in some other scientific application, the reconstructed image has to be an exact replica of the original. Such type of compression is known as lossless but on the other hand, in applications like motion picture, sound, image etc a certain amount of information loss is acceptable. These types of compression techniques are known as lossy compression. The major steps of compression techniques includes transformation of input image (g) followed by feature selection of transformed image and Encoding the selected features. Then we can get the output image (G'). Here 'g' and 'G' is the input and compressed images respectively of the compression technique. Now we have to reconstruct the digital image ($\bar{g}$) from G'. Hence, there may be some discrepancy between g and $\bar{g}$. This discrepancy is considered as error. Therefore, $\text{Error}(E) = \|g - \bar{g}\|$. So the objective of compression technique is to achieve the maximum amount of data reduction (C) without introducing objectionable error (E). If the amount of error (E) is zero, then it is called error free compression or lossless compression. But on the other hand, if the error (E) is nonzero, then such types of compression is known as lossy compression. The data reduction can be achieved by removing different types of redundancies, such as coding redundancy; psycho visual redundancy, inter-pixel redundancy, and spectral redundancy are present in the images. There are different techniques are available for gray and color image compression; some are listed below. In⁵ describes an image compression technique based on vector quantization method. In⁶ gives an efficient image compression technique based on image folding method.

It gives an efficient technique based on modulus. It is known as Five Modulo Method (FMM)⁵. In⁷ proposed a compression technique depending on utilizing reference points coding and threshold value, and it able to perform both lossy and lossless compression. In⁸ described a technique for image compression based on different Wavelet for image coding and Huffman coding for further compression. In⁹ presents a novel technique for lossless image compression based on multi-wavelet transform coding. It is discussed in reference and tries to implement of IMWT (Integer Multi-wavelet Transform) for lossless compression. Block Truncation Coding (BTC) method is simple, fast, lossy, fixed length image compression technique for gray scale images, it provides high quality reconstructed image, but the compression ratio is low. It is described in¹⁰. Here, an image is decomposed into $n \times n$ blocks and each block is processed separately (in general $n=4$). Each block is quantized by Q level quantize and these quantizer levels are selected such that low order moments are preserved in the quantized output. In the BTC method, first two moments mean and variance are preserved and the blocks are represented by two quantization levels. Higher order moments can be preserved by incorporating some additional constraints. There are some disadvantages of BTC, like bit rate ($=2$) and time requirement to reconstruct the pixel value at each point at the decoder part. The concepts of AMBTC¹¹ is introduced by D' Lema and Robert Mitchell to overcome the time requirement for reconstruction of each pixel of BTC method. Here, an image is separated into non-overlapping blocks. For each block calculate the mean. For each image block, the pixels are classified into two range of values based on block mean. The gray values of the image block which are greater than block mean are considered as upper range and otherwise lower range. The mean of upper range and lower range are termed as higher mean and lower mean respectively. Then construct the binary block matrix (0/1) based on mean as a quantization level. In the decoder part, an image block is reconstructed by replacing the 1's with higher mean and 0's by lower mean. Here PSNR values are improved but bpp is always two. For more improvement of the bit rate the inter pixel correlation property is used in the EBTC method. It is described in¹². The EBTC method is gives the effective result in terms of bit rate and PSNR values compared to conventional BTC method. For more improvement of the PSNR values, here we are proposed a new method based on c- mean¹³, proposed by Chanda.

The present paper is organized as follows: The proposed method based on c- mean and AMBTC is described in section 2. The experimental result and conclusion are drawn in the section 3 and 4 respectively.

2. New Mean or C-Mean

There are different ways are exist to express a real number, some such ways are decimal, binary number system. There are other methods are also exist. The choice of number representation system depends on the purpose for which it will be used. The conventional representation of real numbers with base $b \geq 2$ has been generalized. It represent any non-negative real number x with arbitrary real base $\beta > 1$. The integer numbers with irrational basis have some important roles in some branches of science. Here we do not choose a different base to represent a positive integer. We introduce a new system of representing positive integers and we find that with this system some new features of the statistics of positive integers come to light. This representation is unique also. In the line of arithmetic mean, we shall get new mean or c-mean based on this representation of positive integer. First of all we shall discuss the new representation method of positive integer, and then apply the method to unravel some basic statistical properties of the positive integer. The c- mean can be calculated as follows¹³. In this method, we are able to represent a positive integer as a sum of largest perfect square of non-negative integer. The largest square is calculated from the given integer, and then the same technique is continued from the residual part of the integer successively.

Example: Suppose the values are {209, 168, 98, 96, 105, 202, 146, 92, 103, 107, 190, 115, 95, 95, 109, 0}

$$209 = 14^2 + 3^2 + 2^2 + 0^2 + 0^2 \quad 095 = 9^2 + 3^2 + 2^2 + 1^2 + 0^2$$

$$168 = 12^2 + 4^2 + 2^2 + 2^2 + 0^2 \quad 109 = 10^2 + 3^2 + 0^2 + 0^2 + 0^2$$

$$098 = 9^2 + 4^2 + 1^2 + 0^2 + 0^2 \quad 0 = 0^2 + 0^2 + 0^2 + 0^2 + 0^2$$

$$096 = 9^2 + 3^2 + 2^2 + 1^2 + 1^2$$

$$105 = 10^2 + 2^2 + 1^2 + 0^2 + 0^2$$

$$202 = 14^2 + 2^2 + 1^2 + 1^2 + 0^2$$

$$146 = 12^2 + 1^2 + 1^2 + 0^2 + 0^2$$

$$092 = 9^2 + 3^2 + 1^2 + 1^2 + 0^2$$

$$103 = 10^2 + 1^2 + 1^2 + 1^2 + 0^2$$

$$107 = 10^2 + 2^2 + 1^2 + 1^2 + 1^2$$

$$190 = 13^2 + 4^2 + 2^2 + 1^2 + 0^2$$

$$115 = 10^2 + 3^2 + 2^2 + 1^2 + 1^2$$

Total sum: 1930; No. of elements: 16

Original mean: $1930/16 = 120.625$

Original mean = 120.625;

For 1st component: 100;

For the 2nd component: 8.0089

For the 3rd component: 1.72266;

For the 4th component: 0.47266;

For the 5th component: 0.03516 Therefore

C-mean=110.23938

Weighted mean= $120.625 + 0.267(120.625 - 110.23938) = 123$

2.1 Properties of C-Mean

1. Arithmetic mean is always greater than C-mean for a given data set until and unless the data set is completely non-random.
2. The data set is more random if the difference between c-mean and arithmetic mean is more.
3. Unlike arithmetic mean C- Mean is sensitive to the distribution pattern of the set.

Proof: Let N = Set of natural number.

Let us take $a \in N$ by SSGS system.

'a' can be express as sum of perfect square as below:

$$a = x_1^2 + x_2^2 + \dots + x_n^2$$

Let us take 'm' number of positive integers and express each one as above.

So by averaging all these number we get

$$\begin{aligned} \bar{a} &= \overline{x_1^2} + \overline{x_2^2} + \overline{x_3^2} + \dots + \overline{x_n^2} \\ &= \left[\overline{x_1^2} - (\overline{x_1})^2 \right] + \left[\overline{x_2^2} - (\overline{x_2})^2 \right] + \left[\overline{x_3^2} - (\overline{x_3})^2 \right] + \dots + \left[\overline{x_n^2} - (\overline{x_n})^2 \right] + \\ &\quad \left[(\overline{x_1})^2 + (\overline{x_2})^2 + (\overline{x_3})^2 + \dots + (\overline{x_n})^2 \right] \\ &= \left[\text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(X_3) + \dots + \text{Var}(X_n) \right] \\ &\quad + \left[(\overline{x_1})^2 + (\overline{x_2})^2 + (\overline{x_3})^2 + \dots + (\overline{x_n})^2 \right] \end{aligned}$$

$$= \lambda + \mu_N;$$

where λ is the sum of variances of components and μ_N

= C-Mean

For any random set of natural number we can say that

$$\bar{a} > \mu_N \quad \text{As } \lambda > 0 \quad \text{Hence the statement 1 is}$$

proved.

The remaining properties of C-Mean are articulate from our observations. The concepts of c-mean are applied to image compression based on absolute moment

BTC method for gray and color images. It gives the satisfactory result. This method can be summarized as below:

2.2 Proposed Method

Step 1: The image S (of size $M \times N$) is divided into non-overlapping blocks S_i of the size $m \times m$ (Normally 4×4) i.e. Let $S_i = \{a_k : \forall a_k \in S_i, k=1, \dots, m \times m\}$ such that $S = \cup S_i$, $S_i \cap S_j = \emptyset$ ($\forall i \neq j$) and then each block will process separately.

Step 2: Calculate the C-mean of each blocks of the image. To calculate the C-mean of each block, we pursue the following steps:

Step 2.1: For each block S_i do

Step 2.2: Represent each element of S_i say n into a sum of largest perfect square of positive integer, and arranged them in descending order. Those integer numbers must be belongs to $[0, 15]$.

Step 2.3: For each index chooses the element which has maximum occurrence and if the possibility of occurrence of some elements are same in some particular index then select that element which has the minimum valued integer.

Step 2.4: Calculate the sum of square of each element, which is selected from each index one element. It is termed as c-mean. Goto step 2.2

Step 2.5: Now calculate the weighted c-mean by means of linear combination of arithmetic mean and c-mean for each block.

Step 3: The pixels in the image block (C) are then classified into two ranges of values. The upper range (C_1) is those gray values which are greater than the block weighted c-mean and the remaining brought into the lower range (C_0). The weighted c-mean of upper range and lower range are termed as higher c-mean (X_H) and lower c-mean (X_L) respectively. Where $C = C_0 \cup C_1$ and $C = C_0 \cap C_1 = \phi$. The higher c-mean represented by (X_H) and lower c-mean represented by (X_L) are calculated as follows:

$$X_H = cmean\{f(x_i) : f(x_i) \geq cth; f(x_i) \in C\}$$

$$X_L = cmean\{f(x_i) : f(x_i) < cth; f(x_i) \in C\}$$

$$cth = cmean\{f(x_i) : f(x_i) \in C\}$$

Step 4: Let 'cth' is the threshold value. If the pixel intensity value $f(x_i) < cth$ then pixel values $f(x_i)$ are quantized to zero (0) otherwise quantized to one (1). Then we can construct the binary matrix (B) for each block as follows:

$$B = \begin{cases} 1 & \text{if } f(x_i) \geq cth \\ 0 & \text{if } f(x_i) < cth \end{cases}$$



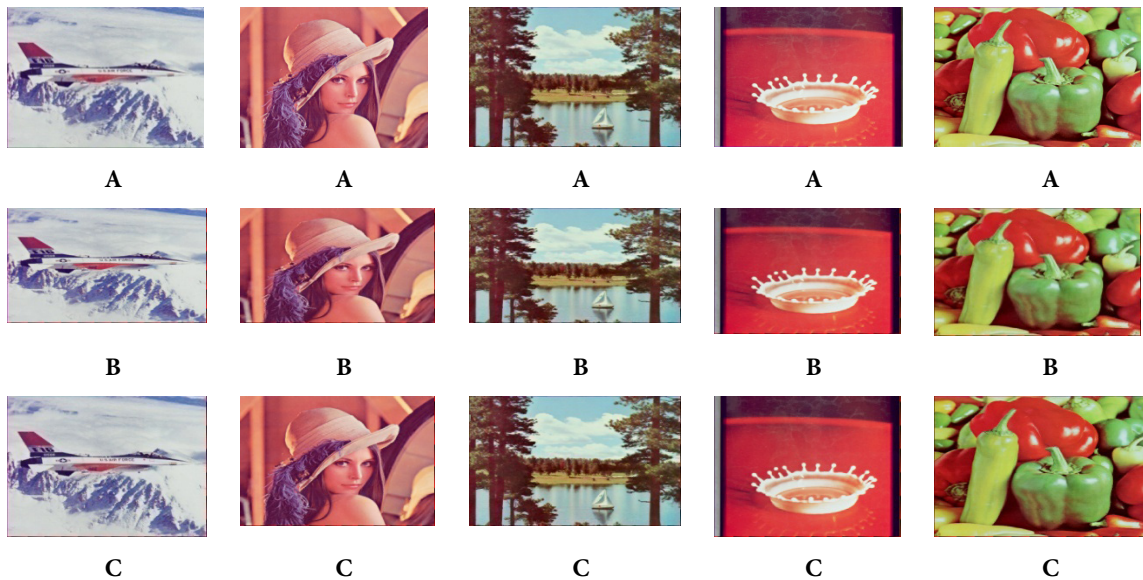
Figure 1. Original image: top (left to right): Lena, airplane, baboon, boat, bridge, couple, girl, down (left to right): house, lake, pepper, Sakai, smart girl, splash, Zelda.



Figure 2. Output of proposed method: top (left to right): Lena, airplane, baboon, boat, bridge, couple, girl, down (left to right): house, lake, pepper, Sakai, smart girl, splash, Zelda.

Table 1. Comparative results of our proposed method with BTC, AMBTC and EBTC

SL. No.	Image name	PSNR of BTC	PSNR of AMBTC	PSNR of EBTC	Bit rate of EBTC	PSNR of Proposed method
1	Lena	24.162603	32.846737	27.494301	0.5287	35.966045
2	Airplane	22.985367	31.088243	26.376217	0.5852	34.281054
3	Baboon	18.485086	26.701086	21.692959	0.6579	29.753044
4	Boat	22.361092	31.143612	25.788376	0.5559	34.263767
5	Bridge	19.934486	28.164015	23.269722	0.5742	31.501816
6	couple	25.197689	33.909733	27.807312	0.5230	37.083973
7	Girl	26.077530	25.231112	30.159161	0.5367	38.057850
8	House	22.012747	29.688639	25.149561	0.5763	33.509109
9	Lake	21.025698	29.307894	24.474900	0.5980	32.403481
10	Peppers	24.108854	32.828274	27.316702	0.5522	35.928539
11	Sakai	22.563337	31.939985	26.409620	0.5476	35.030891
12	Smart girl	26.497623	28.726990	28.668950	0.5279	37.264744
13	Splash	26.012661	36.04607	30.174852	0.5442	41.867596
14	Zelda	24.768511	33.427841	28.264141	0.5260	36.528137

**Figure 3.** Experimental result A) original images (airplane, Lena, sailboat, splash, and pepper) B) output of AMBTC method C) Output of the proposed method.

Step 5: The block matrix (B), X_L and X_H have to send for each block.

Step 6: Further, image block is reconstructed by following rules in the decoder portion

$$X = \begin{cases} X_L & \text{If } B = 0 \\ X_H & \text{If } B = 1 \end{cases}$$

Step 7: Continue above process for all blocks

3. Experimental Results

The proposed method is tested for some standard 8-bit images of different sizes (i.e. 512×512, 256×256). The experimental results are also compared with some well

known existing methods. The quality of output image of newly designed method of image compression can be measured by the parameters bit rate, PSNR, SSIM^{3,14}. The proposed method is also tested for some 24-bit images of different sizes. Almost all the cases it gives the satisfactory results. The experimental result of our newly designed method is displayed in the Table 1 for different gray images along with other well known methods like BTC, AMBTC and EBTC. Here we consider threshold value is 500 for EBTC method. The BTC and AMBTC methods have bit rate is 2 bpp, whereas EBTC method has different bit rate. Actually it depends on the nature of the test images and threshold value. The proposed method gives the satisfactory result in term of PSNR values compared to other well known spatial domain methods like BTC, AMBTC and EBTC with bit rate of 2bpp as shown in Figure 2 corresponding to Figure 1. The newly designed method is also applied on 24-bit images of different sizes. It gives the satisfactory results for almost all images in terms of PSNR and SSIM. The experimental results of proposed method for color images (Airplane, Lena, Sailboat, Splash, and Pepper) are shown in the Figure 3 and it is tabulated in the Table 2 in terms of PSNR and SSIM.

Table 2. Experimental results of the proposed method for 24-bit images in terms of PSNR and SSIM

Images	Parameters	AMBTC	Proposed Method
Airplane	PSNR	31.278437	34.476641
	SSIM	0.986557	0.987114
Lena	PSNR	31.986589	35.073399
	SSIM	0.994937	0.995348
Sailboat	PSNR	28.188711	31.353441
	SSIM	0.987938	0.988573
Splash	PSNR	34.910271	47.779186
	SSIM	0.997406	0.997057
Pepper	PSNR	31.464678	34.607178
	SSIM	0.994786	0.994903

4. Conclusions

We present the gray level image compression method based on weighted c-mean and absolute moment BTC method. Our newly designed technique is unable to give the optimal compression of images although it can give better quality images compared to some well known exist-

ing methods like BTC, AMBTC, and EBTC. The quality of output images of proposed method are measured in terms of PSNR values. The bit rate of the designed technique is same as that of BTC or AMBTC methods. The proposed method is also successfully tested to 24-bit images. It gives the satisfactory results in term of PSNR and SSIM values. The pattern fitting approach can improve the bit rate of our proposed method.

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