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On Estimation of Fractal Dimension of Noisy Images

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Abstract

Objectives: This paper studies the noise effect on the estimation of Fractal Dimension (FD) of gray scale images and finds out which filter is best for estimating FD accurately for noised images. **Methods/Statistical Analysis**: Noise can lead to inaccurate estimation of FD, for this experimental analysis we have taken various types of noise factors to generate noisy images. The FD of original and noisy images has been estimated by using improved differential box-counting (IDBC) and compared. Further, three standard noise filtering techniques are used to remove the noise, and then it estimate the variation in fractal dimension of the original and de-noised image. **Findings:** As roughness of image is concerned, it will increase the addition of noise so FD also increased accordingly. In order to accurately estimate the FD of noised images, we have taken various standard filters to remove noise and to finding out which filter is best for estimating FD accurately for noised images. So in this regard, we have taken average FD variation for each image with each filter and found the non-texture images, mean filter has minimum FD variation even if it has a slightly more mean square error than other filters. **Application/Improvements:** it is easier to estimate the accurate fractal dimension of noisy textured images as compared to non-textured images. Further, other techniques are to be explored for estimating the accurate fractal dimension of noisy images.

Keywords: Gaussian, IDBC, MSE, Poisson, Salt and Pepper, Speckle

1. Introduction

The concept of the fractal theory was firstly invented by[1] to explain self-similar sets called fractals. Fractal geometry comes into play where traditional Euclidean geometry fails to express the usual or spited set of natural features as well as complex objects described in[2]. It yields a numerical representation for various complex real world natural objects. One of the major properties of the fractal theory called Self-similarity and it also useful for estimation of fractal dimension (FD). In this regard, many methods have been developed but most have their practical and theoretical limitations. Fractal dimension was broadly used in various applications like segmentation, analysis of texture, image analysis and classification described in[3-7]. The concept of smoothness vs. roughness of image surface with FD as 2 in case of the smooth image surface and FD as 3 for the maximum rough surface image called salt and pepper surface described in[2].

Many researchers contributed their effort on estimating fractal dimension in the field of fractal geometry. Thus, several concepts have been proposed in this regard, reticular cell-counting[8] was most popular, and which enhanced upon by[9] by leading probability theory. Afterward, the probability method was improved and gave additional enhancement by a way of linear interpolation presented in[10,11] and finally Voss summarized and divided these concepts into three key categories called, box-counting, variance and spectral methods described in[9]. Out of these three methods box counting method is most popular because of its simplicity and automatic computability described in[13]. In this regard many efficient box-counting methods were proposed to estimate FD described in[8,12,14-18]. Recently fractal dimension is applied for prediction purpose described in[19] and segmentation of noise and firefly algorithm are discussed in[20].

This article is prepared as follows. In section 2, the basic idea of fractal theory and improved differential

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box-counting method for estimating fractal dimension. Section 3 discusses about different noise models. Section 4 describes different filtering technique. Section 5 representing proposed method. Section 6 shows the experimental results. Section 7 represents concluding remarks.

2. Estimation of Fractal Dimension

The fractal dimension is an essential characteristic of fractal theory for the reason that it has got information regarding in geometric structure. Fractal dimensions of the entire images is used to spreading of pixels, more purposely. The basic idea for estimation of the fractal dimension of an entire image is based on the principle of self-similarity. From the property of self-similarity it can say that fractal is normally an irregular or inexact geometric shape that can be broken down into smaller pieces; each is related to the original.

Fractal dimension D of a set X is defined by equation (1).

$$D = \log(N)/\log(1/r) \qquad (1)$$

Where N is the entire number of dissimilar copies related to X and X is scaled down by a fraction of 1/r.

Fractal dimension usually evaluates the surface roughness of images and accordingly it provides the variation among different grey levels that are found in the image, from the above equation (1) the value of N is the total no of boxes and has to be evaluated by means of technique called box-counting and the fractal dimension is evaluated accordingly. In this regard, several box-counting measures are developed. However, we have considering the improved version of differential box-counting method described in[18], since it removes some demerits of differential box-counting method[14] that is:

- Over counting the amount of boxes covering the image intensity surface.
- Under counting the amount of boxes may occur at the boundary of the neighboring box blocks.

2.1 Improved Differential Box-Counting Method (IDBC)

The improved differential box-counting method[18] was presented for evaluating FD of gray scale image. Image of dimension $M \times M$ in three-dimensional surface plane, where (x, y) plane represents the position of the pixel in an image plane, and the third coordinate z representing gray level, presented at Figure 1. Then the entire no of pixels has been scaled down into block of size of $l \times l$ where l are lies among1 to $M/2$ .Subsequently they evaluate the reduction factor $r = 1/M$. For each and every scaled down block, there is a stake of boxes of size $S \times S \times S'$, where S' indicates height of each box, another assumption they took $G/S' = M/S$, where G represents gray-level. Suppose the minimum and maximum gray-levels are $I_{\min}$ and $I_{\max}$ respectively in the $(i, j)^h$ block. Then the total number of boxes needed to cover the block in z direction is n_{old} and after shifting the δ positions, they estimate n_{new}. Finally $n\,(i, j)$ is calculated by taking maximum contribution from both n_{old} and n_{new}. n_{old} and n_{new} is calculated as follow:

$$\begin{cases} ceil\left(\dfrac{I\max - I\min + 1}{s'}\right) I\max \neq I\min \\ 1.........otherwise \end{cases} \qquad (2)$$

Finally N_r is calculated as follows

$$N_r = \sum n_r(i, j) \qquad (3)$$

FD is calculated using equation (1).

3. Noise Models

Noise represents surplus information which deteriorates image quality. During any processing phase of the digital image such as image acquisition, image transmission etc the noise was present in images. Many aspects are accountable that leads to noise in an image. During image acquisition by camera, there are many factors are like sensor temperature, illumination level, dust particle is responsible for creating noise in the image. The Subsequently variety of noise models available so far, despite in this research we have considered only four models, that is salt and pepper noise, Gaussian noise, speckle noise and Poisson noise. In this case, we form a noisy image as below equation.

$$ni(a,b) = o(a,b) + n(a,b) \qquad (4)$$

Where $o(a,b)$ original image intensity is value and $n(a,b)$ is the noise present in an image and $\dot{n}\,(a,b)$ is the resultant image

3.1 Salt and Pepper Noise

These types of noise are otherwise called Impulse noise or bipolar noise presented in[21,22]. Mainly these types of noised are produced due to the rapid and sudden disturbance in an image signal, these effects can scatter in terms of white and black dot pixel above the image. Due to this nature, the original intensity values are restored by infected intensity values either by maximum or minimum intensity value for gray scale image i.e., 255 or 0 respectively. The salt and pepper noise is usually caused by either malfunctioning camera's sensor, or malfunction of the memory cell or by error due to synchronization in the image digitizing.

$$p(z) = \begin{cases} p_a \text{ if}, z = a \\ p_b \text{ if}, z = b \\ 0 \text{ , otherwise} \end{cases} \tag{5}$$

Where a and b as minimum and maximum intensity level.

3.2 Gaussian Noise

Gaussian noise also called amplifier noise as it arises in amplifiers or detectors. Random fluctuations in the signal can cause Gaussian noise to the images. It is a kind of additive noise caused by the addition of random values to the pixel values. It follows the normal distribution. Its probability density function described in[21-23] and is given

$$p(z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{z-\mu}{2\sigma^2}\right) \tag{6}$$

3.3 Speckle Noise

These types of noise are multiplicative in nature, Hence this can be viewed as arbitrary no multiplication with an intensity value of an image. This can be represented as:

N=Img + ns*Img

Where, Img is the original image, ns is the multiplicative uniform noise added in terms two-factor called mean and variance and N are the resultant speckle noised image. This noise is common incoherent light imaging systems like radars and lasers. Its probability density function is given as:

$$p(z) = \begin{cases} \frac{1}{b-a} if a \leq z \leq b \\ 0 \quad otherwise \end{cases} \tag{7}$$

Where a and b are two real numbers.

3.4 Poisson Noise

This noise also called photo noise. When sensors cannot detect the sufficient statistical information because of errors in photon sensing, this type of noise arises presented in[21]. Poisson noises are represented as follows:

$$p(X = x) = \frac{\lambda^k e^{-\lambda}}{k!} \tag{8}$$

4. Image De-Noising Techniques

It is the mechanism of altering the pixel intensity values of noisy images to reveal certain image characteristics like enhancement, smoothing, pattern matching. These techniques are categorized into two groups called the linear method and non-linear method. Generally, linear methods are fast as compared to non-linear but the major difference is that the linear de-noising technique does not protect the information of the images and non-linier de-noising technique protects the information. Three categories of filters called mean, median and wiener filter from both two methods are taken into consideration in this research.

4.1 Mean Filter

This filter is otherwise called an average filter. The main concept behind this filter is to replaces the pixel value at centre of each image by the average value of all the pixels of the image. These images are named as 1, 2, 3 up to 10 respectively and shown in Figure 4. The details mechanisms of mean filter are represented in Figure 2. Average=((4+3+6+4+1+7+3+8+9)/9)=5

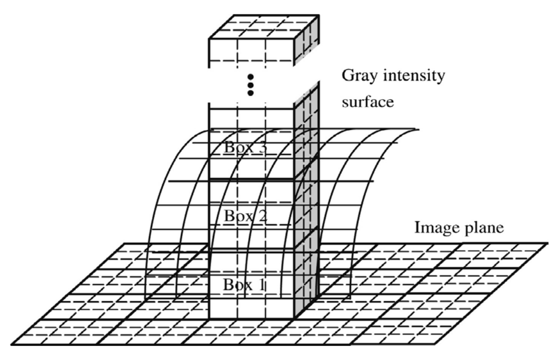

Figure 1. n_r calculation by IDBC.

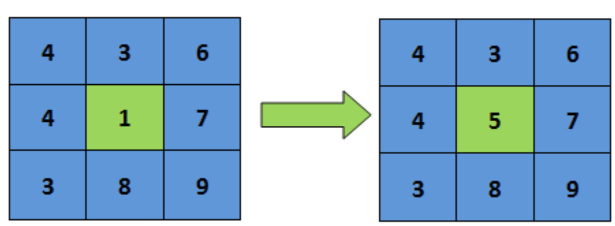

Figure 2. Mean filtering mechanism.

4.2 Median Filter

The median filter is an order static represented in[24], basically, these filters belong to a non-linear category and it replaces the intensity value at the center of each window in terms of the median value of every intensity value of the window. This median value is evaluated by an arrangement of every intensity values either by ascending or descending order and then replaces intensity value being considered with the center intensity value. It is suitable for smoothing images. Generally, it is used for decreasing the pixel deviation between two pixels. These types of filters are best suitable when the percentage of impulse noise is less than 0.1. Detail diagrammatical representation as shown in Figure 3.

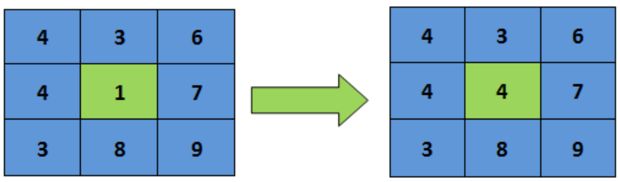

Figure 3. Median filtering mechanism.

4.3 Wiener Filter

The main aim of this filter is used to decrease mean square error as much as feasible and it works in terms of statistical approach. However, we preferred linear time invariant filter when we should information about original signal and spectral property of the noise because it provides the similar output as original signal described in[24]. In this case, mean square error is broadly used to measure the performance.

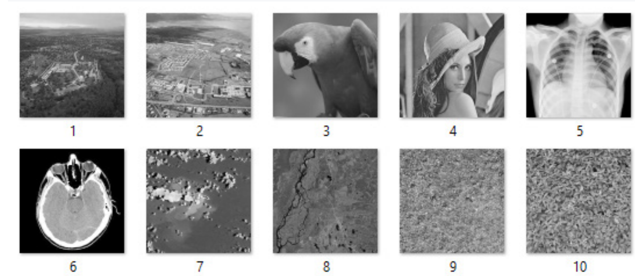

Figure 4. Original images.

Table 1. Computational FD of original images

Image name	FD
1	2.44
2	2.53
3	2.21
4	2.42
5	2.29
6	2.45
7	2.46
8	2.46
9	2.72
10	2.68

5. Proposed Method

In this research, it have estimated fractal dimension of 10 different types of gray scale images (real, textured, medical, aerial, remote sensing images) of size 256x256 shown in Figure 4 and checked the noise effect on FD of above mentioned images are tested, for this purpose we examined with a variety of noise factors are deliberated

Table 2. Computational FD of noised images

Image Name	FD of Noisy images									
	salt & pepper			Gaussian			Speckle			Poisson
	10%	20%	30%	10%	20%	30%	10%	20%	30%	
1	2.8	2.89	2.93	2.91	2.95	2.97	2.71	2.78	2.82	2.53
2	2.82	2.9	2.93	2.93	2.96	2.97	2.8	2.87	2.91	2.6
3	2.78	2.88	2.93	2.91	2.95	2.97	2.69	2.77	2.82	2.46
4	2.8	2.89	2.93	2.92	2.96	2.97	2.77	2.85	2.89	2.55
5	2.79	2.89	2.93	2.89	2.94	2.96	2.78	2.85	2.89	2.51
6	2.81	2.89	2.94	2.87	2.92	2.95	2.66	2.72	2.75	2.5
7	2.81	2.89	2.93	2.92	2.96	2.97	2.72	2.79	2.83	2.56
8	2.81	2.89	2.93	2.92	2.96	2.97	2.71	2.78	2.83	2.55
9	2.87	2.92	2.94	2.94	2.97	2.98	2.85	2.9	2.93	2.74
10	2.85	2.91	2.94	2.94	2.96	2.97	2.83	2.89	2.92	2.7

in section III. The FD of original images and noisy images has been estimated by using Improved Differential Box Counting method and compared. For further validation different de-noising techniques discussed in section IV to eliminate the noise depending on kind of noise present in the image then applied IDBC to find out the variation of fractal dimension. For checking the performance of different filters in terms of variation of fractal dimension and Mean Square Error are estimated.

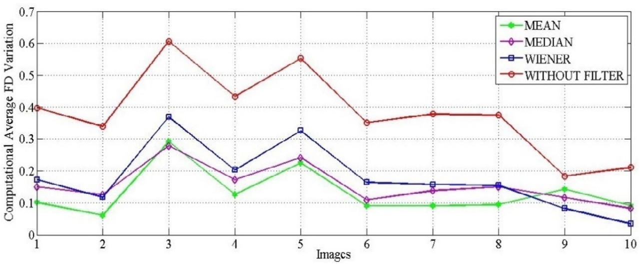

Figure 5. Computational Average FD variation.

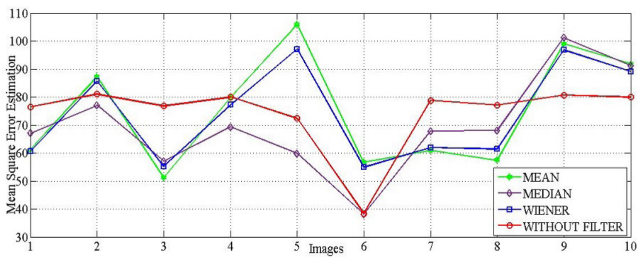

Figure 6. Computational Mean Square Error.

5.1 Methodology

The improved differential box counting method were used in this case for estimating fractal dimension of both noise and de-noised images. First it needs to find out the FD in all the original images. Then it generates noisy images from original images by using different noise models like salt and pepper, Gaussian, speckle and Poisson noise with different noise factors of 0.1, 0.2 and 0.3 and generated de-noised images using Mean filter, Median filter and Wiener filter with above mentioned noise factors and estimated the FD's of all original images, noisy images and de-noised images using IDBC method. The filter has less average FD variation, for more reasonable FD estimation by IDBC of noisy images.

6. Experimental Results

The Proposed method is implemented on matlab12 in windows 8 64 bit operating system, Intel (R) i7-4770 CPU @ 3.40 GHz, for this experimental analysis, we have taken 10 different types of gray scale images of size 256x256. These images are named as 1, 2, 3 up to 10 respectively and shown in (Figure 4). In the first step, we are estimating FD of 10 different images shown in (Figure 4) and the result were represented in Table 1. In the second step, using the noise models of Section 3, noise is added to each image with the different noise factor of 0.1, 0.2 and 0.3. Estimation of fractal dimension values is done in noisy image again using improved differential box-counting mechanism described in section 2 and result were represented in Table 2. In the third step, we are using different de-noising technique discuss in section 4 to eliminate noise factor, then we are estimating FD using improved differential box-counting method, after estimating FD of de-noised images, than we estimate average FD variation and mean square error shown in (Figure 5, Figure 6) respectively using with and without filtering, and shows which filter are best suitable for measuring roughness of noise images. Since noise creates the pixels to be distorted and in general affects the pixel orientation when considered in 3D space; the fractal dimension of the noisy image comes to be inaccurate as compared to non-noisy images. In order to feasible this issue we used different filters like Mean, Median, Wiener described in section 4 and find out mean filter is best suitable for estimating roughness of noisy image based on average FD variation and mean square error represented in Figure 5, Figure 6 respectively.

7. Conclusion

In this research we have done a comparative study of noise and de-noising effect on estimation of fractal dimension by using Improved Differential Box Counting method. As roughness of image is concerned, it will be increased on addition of noise so FD will be increased accordingly. In order to accurately estimate the FD of noised images, wehave taken various standard filters to remove noise and to finding out which filter is best for estimating FD accurately for noised images. So in this regard, average FD variation for each image with each filter and we found for non-texture images mean filter has minimum FD variation even if it has slightly more mean square error than other filters. Therefore, it is easier to estimate accurate fractal dimension of noisy textured images as compared to non-textured images. Further, other techniques are to be explored for estimating accurate fractal dimension of noisy images.

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