

Analysis of Elliptical Contact area of Rolling Element Bearing 6207 using Artificial Neural Network

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Abstract

Objective: In rolling bearing the contact area between the ball and race plays a vital role for the formation of the lubricant film. Objective of present work is calculation of major and minor axis and further the elliptic contact area developed between the ball & races of the rolling element bearing. **Methods/Statistical Analysis:** For this bearing 6207 is selected and analytically the contact area is calculated at different load and how the load affects the contact area is analyzed. In addition to analytical method elliptical contact area is also calculated using Artificial Neural Network (ANN). **Findings:** Using the ANN the error in the calculated contact area is found nearly 1%. So ANN is found to be a successful tool for the prediction of the elliptic contact area. **Application/improvement:** This eliminates the cumbersome of calculation.

Keywords: Artificial Neural Network (ANN), Elliptical Contact Area, Elliptic Parameter, Elliptic Integral, Lubricant Film Thickness, Rolling Element Bearing

1. Introduction

Contact area between the two contacting elastic body is first analyzed by Hertz. Contact stress between wheel rail contact has been analyzed by¹ using the FEA model and concluded that the result has good agreement with the Hertz stress² derived an analytical model for concentrated contact between two isotropic, homogeneous, linear elastic solids with smooth surfaces. When the solids are pressed together with a force Q directed normal to the surfaces, an approximately elliptic or circular contact area is formed which is shown in Figure 1 & 2.² derived an elliptic parameter k , the ratio of semi major axis to semi minor axis also given the formula for minor axis, major axis & equivalent modulus of elasticity and is shown as Eq. (1).

$$k = \frac{D_y}{D_x}, \quad D_x = 2 \left[\frac{6\epsilon QR}{\pi k E'} \right]^{\frac{1}{3}},$$

$$D_y = 2 \left[\frac{6k^2 \epsilon QR}{\pi E'} \right]^{\frac{1}{3}}, \quad \frac{2}{E'} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \quad (1)$$

^{3,4} has given the elliptical parameter k in the form of transcendental equation [Eq.(2)] relating the curvature difference [Eq. (5)] and the elliptic integrals of the first \mathcal{F} [Eq.(3)] and second \mathcal{E} [Eq.(4)] kinds as.

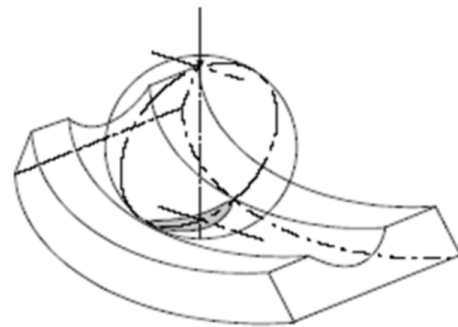


Figure 1. Elliptical Contact Area³

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$$k = \left(\frac{2\mathcal{F} - \varepsilon(1 + R_d)}{\varepsilon(1 - R_d)} \right)^{\frac{1}{2}} \quad (2)$$

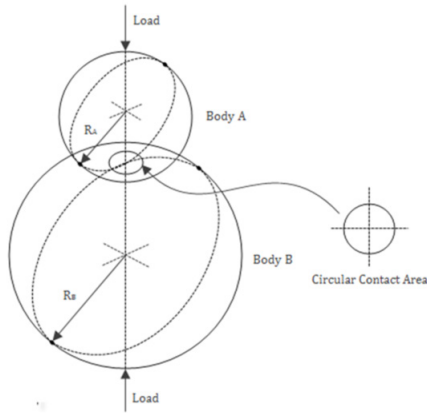


Figure 2. Circular Contact Area².

where,

$$\mathcal{F} = \int_0^{\frac{\pi}{2}} \left[1 - \left(1 - \frac{1}{k^2} \right) \sin^2 \phi \right]^{-\frac{1}{2}} d\phi \quad (3)$$

$$\varepsilon = \int_0^{\frac{\pi}{2}} \left[1 - \left(1 - \frac{1}{k^2} \right) \sin^2 \phi \right]^{\frac{1}{2}} d\phi \quad (4)$$

$$R_d = R \left(\frac{1}{R_x} - \frac{1}{R_y} \right) \quad (5)$$

Hamrock and Anderson, has solved the k , \mathcal{F} & ε by iterative numerical procedure^{5,6} used a linear regression by the method of least squares to power fit the set of pairs of data $[(k_i, \alpha_i), i = 1, 2, \dots, 26]$ and obtained the following simplified solution.

$$k = (\alpha)^{\frac{2}{\pi}} \quad (6)$$

As a result, both inverse and logarithmic curve fits were tried for \mathcal{F} and ε , respectively. Hamrock and Brewe⁵ obtained the following:

$$\varepsilon = 1 + \left(\frac{q_a}{\alpha} \right) \text{ for } \alpha \geq 1 \quad (7)$$

where

$$q_a = \left(\frac{\pi}{2} \right) - 1 \quad (8)$$

$$F = \frac{\pi}{2} + q_a \ln \alpha \text{ for } \alpha \geq 1 \quad (9)$$

^{7,8} compares their 'Effective radius' approximate method with approximate methods of ⁹, and ⁶ of finding the area of contact, the contact pressure and the deformation in elliptical Hertzian contacts. He concluded for $1 \leq (B/A) \leq 5$ the Effective Radius method gives the best value for the Hertz pressure.

In the present work bearing 6207 is selected for calculation of k , \mathcal{F} & ε using the simplified solution given by ⁶ and further it is used to calculate elliptical contact area. This area is used by the author for the calculation of Resistive lubricant film thickness (RFT), which is not the scope of this paper. Similarly The effects of hydrodynamic and boundary lubrication occurring in the plunger-bushing friction pair of the high-pressure fuel injection pump in marine combustion engines have been studied¹⁰.

2. Stribeck's Equation

Figure 3. shows the heaviest loaded ball in the bearing when a radial load of Q_r is applied¹¹ derived the formula for the load on the most heavily loaded ball and is given by equation (10)

$$Q_r = MQ = \frac{zQ}{C} \quad (10)$$

Where

$$M = \left[1 + 2(\cos \beta)^{\frac{5}{2}} + 2(\cos \beta)^{\frac{5}{2}} + \dots \right] = \frac{z}{c}$$

and

$$\beta = \frac{360}{z}$$

In the present work Ball Bearing 6207 is selected for calculation. For this, $Z=9$

So,

$$\beta = 40, M = 2.052 \text{ \& } c=4.386$$

$$\therefore Q_r = \frac{9Q}{4.386} \text{ or } Q=0.487Q_r \quad (11)$$

3. Calculation

The dimensions of the bearing 6207 and related calculated parameters are shown in Table 1. These are used for the calculation of the elliptical contact area.

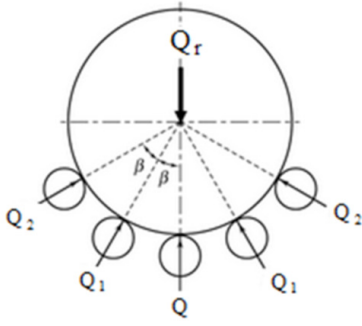


Figure 3. Heaviest Loaded ball in the ball bearing by Stribeck¹¹.

Table 1. Parameter for Bearing 6207 [10]

Parameter	
r_{ax} (mm)	5.75
r_{ay} (mm)	5.75
r_{bx} (mm)	21.00
r_{by} (mm)	-5.98
$1/R_x$	0.22
$1/R_y$	0.0067
R_x	4.51
R_y	149.50
$1/R = 1/R_x + 1/R_y$	0.2282
R	4.38
$\alpha = R_y / R_x$	33.12
$k = (\alpha)^{(2/\pi)}$	9.28
$q_a = (\pi/2) - 1$	0.57
$\mathcal{E} = 1 + (q_a / \alpha)$	1.02
$'$ (N/mm ²)	227363

The elliptical contact area at inner race is calculated by given formula:

$$a = \pi \times a_i \times b_i$$

3.1 Sample Calculation

The sample calculation of minor axis, major axis & contact area of ellipse is for the radial load of 40 kg is shown below. As the static load capacity of the bearing 6207 is 1370 kg¹², so the calculation is done from 10 kg to 1300 kg in the interval of 10 kg but not shown in the present paper.

$$\text{For } Q_r = 40 \text{ kg } Q = 0.487 \times 40 \times 9.81 = 191.1 \text{ N}$$

$$a_i = \frac{D_x}{2} = \left[\frac{6 \times 1.0_2 \times 191.1 \times 4.3_8}{\pi \times 9.2_8 \times 22736_3} \right]^{\frac{1}{3}} = 0.0917 \text{ mm}$$

$$b_i = \frac{D_y}{2} = \left[\frac{6 \times 9.2_8 \times 1.0_2 \times 191.1 \times 4.3_8}{\pi \times 22736_3} \right]^{\frac{1}{3}} = 0.8516 \text{ mm}$$

$$a = \pi \times 0.0917 \times 0.8516 = 0.2453 \text{ mm}^2$$

Table 2. Parameters for nntool of MATLAB

Parameters for nntool				
Network type	-	Feed-forward backprop		
Training function	-	TRAINLAM		
Adaption learning function	-	LEARNGDM		
Performance function	-	MSE		
No. of layer	-	4		
Layer	1	2	3	4
No. of Neurons	10	10	10	-
Transfer function	TANSIG	TANSIG	TANSIG	PURELIN

4. Artificial Neural Network (ANN)

ANN is a tool used widely in the research work. The performance of LDA, QDA and ANN with LPC and MFCC is analyzed. It is evident that ANN in combination with MFCC gives the best result and showing efficiency about 90%¹³. It is found that traditionally the PI, Fuzzy and Neural Network control techniques have been used for

Table 3. Calculated and Predicted value of major axis, minor axis & contact area of ellipse

Load (kg)	a_i (mm)			b_i (mm)			a (mm ²)		
	Calculated	Predicted	% Error	Calculated	Predicted	% Error	Calculated	Predicted	% Error
40	0.0917	0.0903	1.5487	0.8516	0.8532	-0.2347	0.2452	0.2414	1.5290
60	0.1050	0.1041	0.8077	0.9743	0.9819	-0.7780	0.3213	0.3209	0.1250
80	0.1155	0.1150	0.4442	1.0724	1.0792	-0.6333	0.3892	0.3904	-0.3135
100	0.1244	0.1241	0.2847	1.1552	1.1586	-0.2964	0.4516	0.4528	-0.2646
% Ave. Error	0.7713			0.4856			0.5580		

the four control units of the UPFC¹⁴ presents the design and development steps of a novel control scheme for the UPFC. The proposed technique believes that a single combined ANN based controller can be trained appropriately to handle all the mappings associated with the inputs and the corresponding outputs associated with all the controllers of the UPFC¹⁴. So it can be seen that the ANN is used in all field of engineering. An attempt is made to apply in mechanical engineering. In the present work artificial neural network is applied to predict the value of major axis, minor axis and area of the contact ellipse using the nntool of the artificial neural network in the MATLAB software. Different parameter to create the neural network is shown in Table 2. The calculation is done for the radial load of 10 kg to 1300 kg in the interval of 10 kg and these are taken as the data for training of the neural network. Once the network is trained, the values of a_i , b_i & a are predicted for radial load of 40 kg, 60 kg, 80 kg & 100 kg and are shown in Table 3. It is found that the predicted value of a_i , b_i & a having the error of 0.7713%, 0.4856% & 0.558% respectively. Only 40 kg, 60 kg, 80 kg & 100 kg loads are taken into interest, because these values are used for calculation of resistive lubricant film thickness in a separate experiment. As the experiment is not the scope of the present paper, so it is not discussed here.

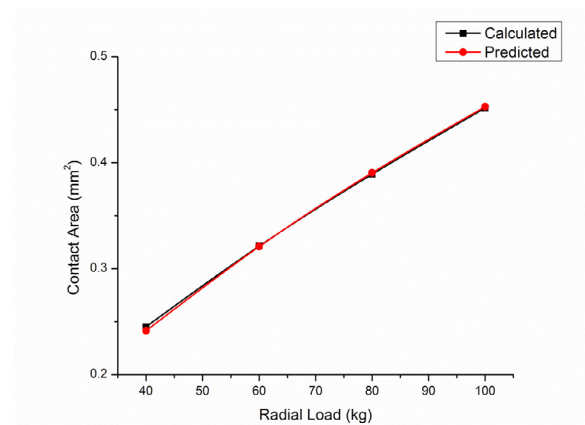
5. Graphical Analysis

It is seen in the Figure 4. that the elliptical contact area increases with the increase of the radial load, which is in-line with the Hertz contact theory. Also the predicted value by ANN has the good agreement with calculated value.

6. Conclusion

Elliptical contact area calculated here is further used for the calculation of the resistive lubricant film thickness.

As the load increases the contact area also increases with increase in major & minor axis in same proportionate so as to maintain the constant elliptic parameter k . Artificial neural network is found as an alternative tool for the calculation of the contact area. This gave the error below the 1%, so it can be applied successfully for the calculations. As this tool is not used by the any researcher of this field, the aim of this paper was to introduce this tool to make the calculation in the different angle.

**Figure 4.** Variation of Elliptical Contact area with Radial Load.

7. References

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Nomenclature

k - Elliptical parameter

D_x	- Diameter of contact ellipse in x-axis (mm)
D_y	- Diameter of contact ellipse in y-axis (mm)
a_i	- Radius of contact ellipse in x-axis (mm)
b_i	- Radius of contact ellipse in y-axis (mm)
a	- Elliptical contact area (mm ²)
F	- Elliptic integral of first kind
E	- Elliptic integral of second kind
Q_r	- Radial load on bearing (kg)
Q	- Load on heaviest loaded ball (N)
$1/R$	- Curvature sum
E'	- Equivalent modulus of elasticity (N/mm ²)
E_1, E_2	- modulus of elasticity (N/mm ²)
Φ	- Auxiliary angle (deg)
r_{ax}, r_{bx}	- Principal radii of solid a (mm)
r_{by}, r_{bx}	- Principal radii of solid b (mm)
R_x	- Effective radius of curvature in x-direction (mm)
R_y	- Effective radius of curvature in y-direction (mm)
R_d	- Curvature difference
α	- Radius ratio
q_a	- Constant