

Comparative Analysis of PID and Fuzzy PID Controller Performance for Continuous Stirred Tank Heater

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Abstract

Paper presents a parallel analysis of performance of PID (Proportional Integral and Derivative) controller and Fuzzy PID controller for control of CSTD (Continuous Stirred Tank Heater) Process. CSTD is one of the type of heater which operates at constant temperature. In Heater, a hot fluid is circulated through the jacket. Mass balance and Energy balance equations are used on the process, to develop a mathematical model. A mathematical description of Tank Heater is established and various types of controllers are implemented to it. To revise the performance of different control algorithms, using MATLAB, a simulation work is conceded to control the given process system. To analyze the response of diverse controllers, time domain specifications are used. To determine the entire knowledge about the process, A small literature survey about Tank Heater is premeditated.

Keywords: CSTD, Fuzzy Logic PID, Good Gain Method, PID Control

1. Introduction

A Chemical Industry contains processing units like reactors, heat exchangers, tanks, pumps etc. The goal of process plant is to renovate basis materials into preferred products. All the Equipments in chemical plant are intended for periodic inspection of working condition and disturbance to assure the fulfillment of aspiration. A Mathematical way of analysis of process is vital one to revise the act of various control algorithms. For modeling process, the basic amenities are Energy balance and Material balance principles.

PID controller can be used to control the Laboratory Scale CSTD¹. Methods used for designing PID controllers are Trial and Error and heuristic method. In², for control of CSTD, Model Predictive controller and PID based

controllers are used. Finally the performance has been compared to maintain the tank temperature. In³, MRAC (Model Reference Adaptive control) has implemented for CSTD process using MIT Rule and Neural Networks. These two techniques are compared to produce desired output.

In⁴, Different methods are put into action to control CSTD. Such as PID, adaptive and IMC based PID. To compare the performance of above three controllers, time integral performance criteria are calculated and analyzed. Implementation of Nonlinear model Predictive control using a MNN multiple neural networks is suggested for CSTD process. To construct MNNs, few methods were presented⁵. Simulation of an experimental CSTD pilot plant has been labeled⁶. The heat and volumetric balance principles are realized to design the

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model and various process nonlinearities have been considered. Paper⁷ presents the comparative analysis of the performance of PID, PID with disturbance and delay and Fuzzy Logic controller for control of product concentration in CSTR.

1.1 Mathematical Modeling

A fraction of equations which describes the process is called Mathematical Modeling. The Mathematical Model of a process institutes a set of differential equations. The output of the system will be obtained by solving the differential equations to various input conditions. Thus a Mathematical model defines the reaction of process for a variety of inputs.

2. Continuous Stirred Tank Heater

Generally, heat input to fluid inside the tank is applied using either a jacket surrounding the vessel or a coil. For CSTH, the fluid is heated by jacket. Here the tank is surrounded by jacket. To supply the heat to fluid in tank, a hot fluid is distributed inside the jacket. Consider constant phase exists in tank fluid. The function of CSTH is to maintain the desired temperature of tank fluid.

The CSTH is fine stirred to keep equal temperature in input side as well as output side.

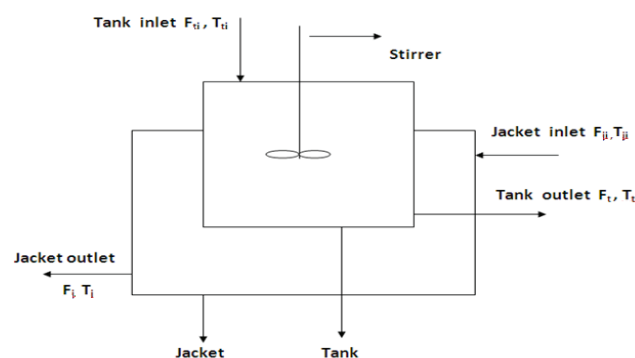


Figure 1. The Continuous Stirred Tank Heater.

2.1 Mathematical Model of CSTH

The input variables to the system are

- Rate of inlet flow for tank
- Rate of inlet flow for jacket
- Inlet temperature for tank
- Inlet temperature for jacket

The output variables of the system are

- Temperature of the liquid (manipulated variable)
- Level or volume of liquid (manipulated variable)
- Flow rate of liquid (manipulated variable)

Heat transmission rate from jacket to tank is written by relation

$$Q = U.A (T_j - T_t),$$

Where, U = Coefficient of heat transmission rate
 A = heat transfer area

T_t and T_j = tank and jacket temperature respectively.

The goal of Tank heater is to maintain the tank fluid temperature at desired value.

Table 1. Parameters used in the process

Terms	Description
V	Volume
Q	Heat transmission rate
Cp	Capacity of Specific heat
A	Area of heat transfer
ref	Reference value
j	Jacket
i	Inlet
F	Rate of change of volume
s	Steady value
ji	Jacket inlet
U	Coefficient of Heat Transfer
ρ	Density (mass/volume)
t	Time
T	Temperature in deg

The Basic principles used for developing the mathematical model are:

- Material balance around tank and jacket
- Energy balance around tank and jacket

Material balance around the tank:

Rate of accumulation = Input flow rate - Output flow rate

$$\frac{d(V\rho)}{dt} = F_i\rho - F_o\rho$$

Material Balance around the jacket:

$$\frac{d(V_j\rho)}{dt} = F_{ji}\rho - F_{jo}\rho$$

Energy balance around the tank:

Rate of Energy accumulation= Heat in- Heat out+
Heat flow rate

$$V_t \rho_t C_{p_t} \frac{dT_t}{dt} = \rho_t F_t C_{p_t} T_i - \rho_t F_t C_{p_t} T_t + Q$$

Energy balance around the jacket:

$$V_j \rho_j C_{p_j} \frac{dT_j}{dt} = \rho_j F_j C_{p_j} T_{ji} - \rho_j F_j C_{p_j} T_j - Q$$

The mathematical equations describing stirred tank heater is written using some conditions and assumptions are:

$$\frac{dT_t}{dt} = \frac{F_t}{V_t} (T_j - T_t) + U.A. \frac{(T_j - T_t)}{V_t \rho_t C_{p_t}} \quad (1)$$

$$\frac{dT_j}{dt} = \frac{F_j}{V_j} (T_{ji} - T_j) - U.A. \frac{(T_j - T_t)}{V_j \rho_j C_{p_j}} \quad (2)$$

2.2 Steady State Conditions

At steady state condition, the state variables are determined by solving equations (1 and 2) for =0

Table 2. Steady state parameters

$F_s = 1 \text{ ft}^3/\text{min}$
$\rho C_p = 61.3 \text{ Btu}/^\circ\text{F} \cdot \text{ft}^3$
$\rho_j C_{p_j} = 61.3 \text{ Btu}/^\circ\text{F} \cdot \text{ft}^3, T_{is} = 50^\circ\text{F}$
$T_s = 125^\circ\text{F}, V = 10 \text{ ft}^3$
$T_{js} = 150^\circ\text{F}, V_j = 1 \text{ ft}^3$

2.3 State Space Model

To find the state space form, modelling equations are converted to linearized form by Taylor Series. Here, only linearized equations are considered and other equations are neglected.

$$\frac{dT_t}{dt} = f_1(T_t, T_j, F_t, F_j, T_{is}, T_{js}) = \frac{F_t}{V_t} (T_j - T_t) + U.A. \frac{(T_j - T_t)}{V_t \rho_t C_{p_t}} \quad (3)$$

$$\frac{dT_j}{dt} = f_2(T_t, T_j, F_t, F_j, T_{is}, T_{js}) = \frac{F_j}{V_j} (T_{ji} - T_j) - U.A. \frac{(T_j - T_t)}{V_j \rho_j C_{p_j}} \quad (4)$$

By substituting the steady state values for above equations we get

$$\dot{T}_t = 0 + [0.4]T_t + [0.3]T_j + [-7.5]F_t + [0]F_j + [0.1]T_{ti} + [0]T_{ji} \quad (5)$$

$$\dot{T}_j = 0 + [3]T_t + [-4.5]T_j + [0]F_t + [50]F_j + [0]T_{ti} + [1.5]T_{ji} \quad (6)$$

The state variable equation is represented by

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

Where the state vector, input vector, and output vectors are, in deviation form

$$x = \begin{bmatrix} T - T_s \\ T_j - T_{js} \end{bmatrix}$$

state variables

$$u = \begin{bmatrix} F_j - F_{js} \\ F - F_s \\ T_i - T_{is} \\ T_{jin} - T_{jins} \end{bmatrix}$$

input variables

$$y = \begin{bmatrix} T - T_s \\ T_j - T_{js} \end{bmatrix}$$

output variables

A=state matrix B=input matrix, C=output matrix, D=translational matrix

$$A = \begin{bmatrix} -0.4 & 0.3 \\ 3 & -4.5 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -7.5 & 0.1 & 0 \\ 50 & 0 & 0 & 1.5 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Convert state space to transfer function. Transfer functions for Tank temperature for various input parameters are:

$$\frac{T_t(s)}{F_t(s)} = \frac{-7.5 - 33.75}{s^2 + 4.9s + 0.9}, \quad \frac{T_t(s)}{F_j(s)} = \frac{15}{s^2 + 4.9s + 0.9}$$

$$\frac{T_t(s)}{T_{ti}(s)} = \frac{0.1s + 0.45}{s^2 + 4.9s + 0.9}, \quad \frac{T_t(s)}{T_{ji}(s)} = \frac{0.45}{s^2 + 4.9s + 0.9}$$

$$\frac{T_j(s)}{F_t(s)} = \frac{-22.5}{s^2 + 4.9s + 0.9}, \frac{T_j(s)}{T_{ti}(s)} = \frac{0.3}{s^2 + 4.9s + 0.9}$$

$$\frac{T_j(s)}{F_j(s)} = \frac{50s + 20}{s^2 + 4.9s + 0.9}, \frac{T_{ji}(s)}{T_{ji}(s)} = \frac{1.5s + 0.6}{s^2 + 4.9s + 0.9}$$

Transfer function related to Jacket for various input parameters are:

The leading transfer function relates the tank temperature to the jacket in flow rate

$$\frac{T_t(s)}{F_j(s)} = \frac{15}{s^2 + 4.9s + 0.9}$$

3. PID Controller Design

A General Model of PID controller is

$$u(t) = K[e(t) + \frac{1}{T_i} \int_0^t e(t) dt + T_d \frac{de(t)}{dt}]$$

Where u = control variable

e = control error $e = y_{sp} - y$.

The parameters of PID controller can be calculated by Good Gain method.

3.1 Good Gain Method

There are many tuning methods available. Most industrial process uses Ziegler-Nichols method, Cohen Coon method, ITAE methods etc. But the above mentioned conventional techniques do not give an acceptable performance. A.S. Rajagopalen has proposed Ziegler Nichols technique to determine the PID controller parameters for CSTD process⁴. The Good Gain method gives the control loop with good stability compared to Ziegler Nichols method. Further, No need of control loop oscillations while tuning for good gain method.

This method is a simple closed loop tuning technique. Increase the value of K_p until the response becomes slight overshoot but a well damped response as shown in Figure 2.

From Figure 2, the integral time can be obtained by

$$T_i = 1.5 T_{ou}$$

$$\text{Setting Derivative time (Td) as, } T_d = \frac{T_i}{4}$$

PID Control for CSTD

The values of controller gains K_p , K_i and K_d are 2.29, 0.5535 and 0.138 respectively. The model of the closed

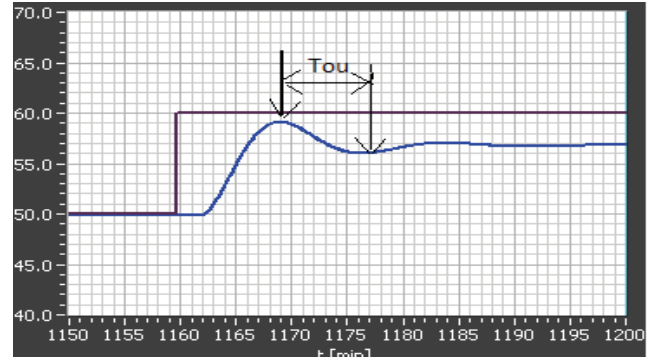


Figure 2. Time duration between the first peak and the first undershoot of the step response with P controller.

loop control of CSTD with classical PID controller is designed using MATLAB-Simulink tools as shown in Figure 3.

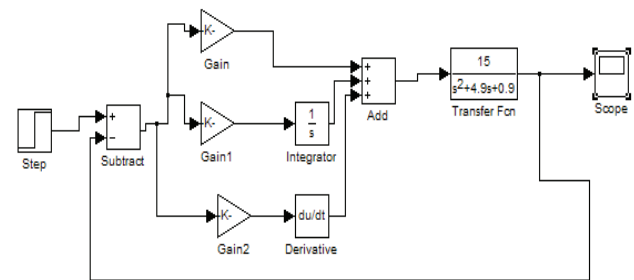


Figure 3. Simulink Model of PID controller for CSTD process.

4. Design of Fuzzy PID Controller

Fuzzy control always employs a set of rules than complex mathematical expressions. These rules are designed by manmade throughout the plant mathematical model. The general form of rules used in FLC (Fuzzy Logic Controller) is of IF-THEN type.

The input variables to fuzzy logic controller are the error and derivative of error. The error is the difference between actual temperature and standard temperature.

4.1 Fuzzy Logic Controller

FLC contains fuzzification, Rule base and Defuzzification stages as shown in Figure 4. Fuzzification is the process of translating crisp sets into fuzzy. The translation of fuzzy values is characterized by the membership functions. To construct the membership function, FLC make use of

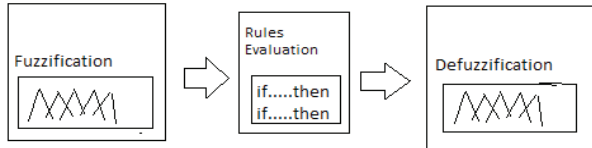


Figure 4. Block Diagram of Fuzzy system.

membership function editor for obtaining input and output variables.

The number of membership function and type of membership function is chosen such as trapezoidal, triangular and Gaussian in the direction of process parameter. Here it is right to select triangular and trapezoidal. For this paper, the number of membership function for each input variable is designated in seven stages. The standard output is distinguished by Fuzzy controller output, which has 49 membership functions.

4.2 Fuzzy-PID Controller

The design of classical PID controller is customized by a new hybrid Fuzzy PID controller. The summation effect in parallel PID controller is replaced by mamdani based Fuzzy inference system. The inputs to the FIS are error and derivative of error.

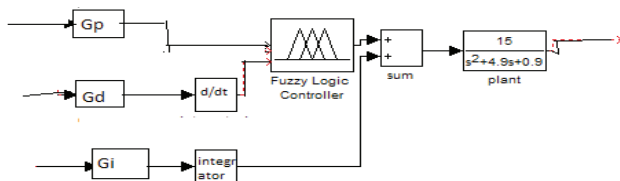


Figure 5. Structure of Fuzzy PID controller.

Among the proportional, integral and Derivative term of PID controller, the Fuzzy inference system is employed in between proportional and derivative term. Third term integral is added to the output of FIS. The values of G_p ,

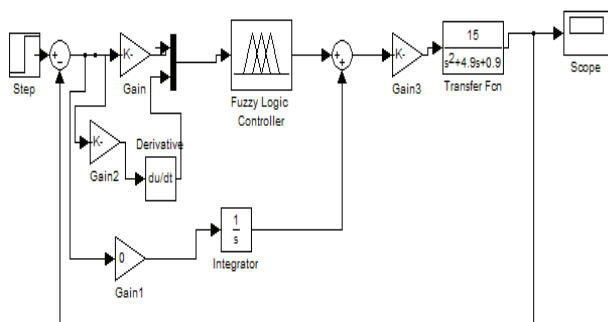
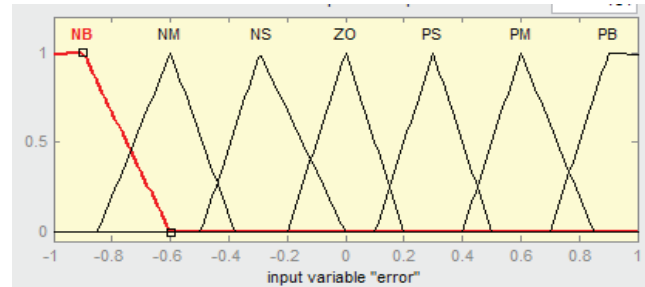
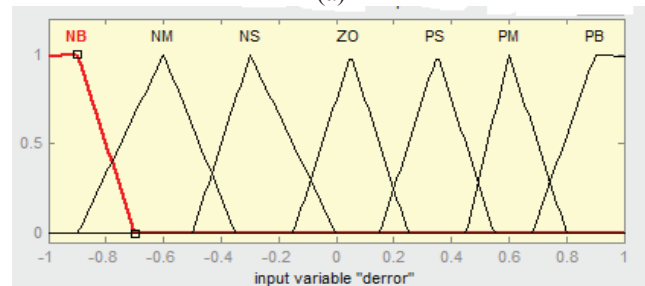


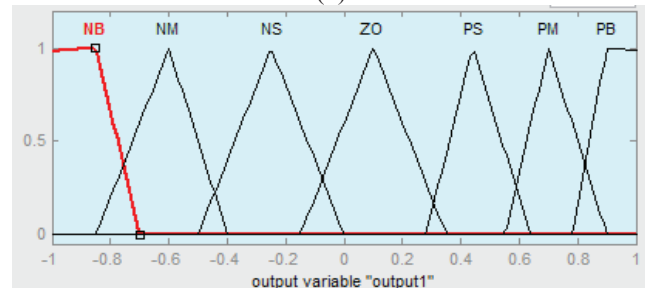
Figure 6. Simulink Diagram of Fuzzy PID controller.



(a)



(b)



(c)

Figure 7 (a), (b) and (c). Membership function for error, derivative of error and output.

G_d and G_i are tuned by trial and error method and are taken very small.

Each input variable and output variable are fuzzified by seven fuzzy sets namely, "Negative Big"(NB),"Negative Medium"(NM), "Negative small"(NS), "Zero"(ZO),

Table 3. Fuzzy Rule base for FIS

u(t)	e(t)							
		NB	NM	NS	ZO	PS	PM	PB
de(t)	NB	NB	NB	NB	NB	NM	NS	ZO
	NM	NB	NB	NB	NM	NS	ZO	PS
	NS	NB	NB	NM	NS	NS	PS	PS
	ZO	NB	NM	NS	ZO	ZO	PM	PM
	PS	NM	NS	ZO	PS	PS	PB	PB
	PM	NS	ZO	PS	PM	PM	PB	PB
	PB	ZO	PS	PM	PB	PB	PB	PB

”Positive Small”(PS), ”Positive Medium”(PM) and ”Positive Big”(PB) respectively.

The input variables (error and derivative of error) and output variable are converted into fuzzy variables by using the membership function as shown in Figure 7 (a), (b) and (c).

After finishing the process of translating the two inputs and one output variable, we sustained to write the rule matrix in rule base.

5. Simulation Results

Figure 8 and 9 shows the performance of conventional PID and hybrid Fuzzy PID controller to the step input. We found that the optimization of linguistic terms change the input variables membership function, can improve the performance of Fuzzy control system. From the Figure, result shows that Fuzzy logic PID controller has small overshoot and settling time as compared to conventional PID controller.

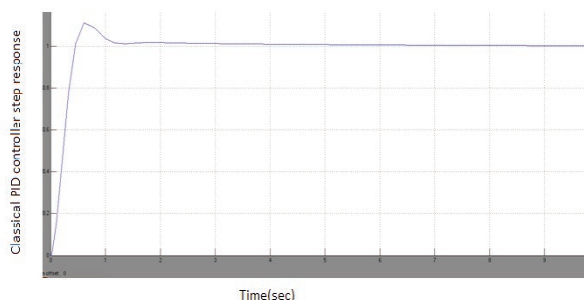


Figure 8. Response of PID controller for CSTD process.

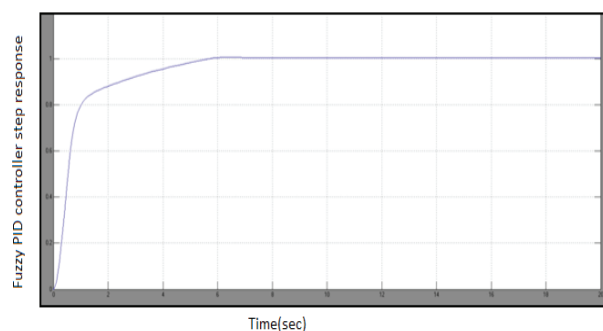


Figure 9. Response of Fuzzy PID controller for CSTD process.

6. Results and Discussion

This Section calculates the controller act in terms of transient response performance. Table 4 presents the

comparative analysis of conventional PID controller and Fuzzy PID controller.

Table 4. Transient Response Analysis

Controller Type	Peak Overshoot (%)	Rise time (sec)	Settling time (sec)	Delay time (sec)
Conventional PID	12	0.46	6.5	0.25
Fuzzy PID Type	0	5.9	6	0.2

7. Conclusion

PID and Fuzzy PID controllers are designed to control the temperature in Continuous Stirred Tank Heater. Both PID and Fuzzy PID controllers are worked in the direction of settling the process variable to a desired set value. The output of Fuzzy PID controller has no overshoot and low settling time compared to classical PID controller. Hybrid Fuzzy PID controller provides a contentment controller performance compared to conventional PID controller. The Response of FLC is free from oscillations in the transient period and it has fast response. Hence the proposed hybrid Fuzzy PID controller is superior to conventional PID.

8. References

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