

# Design and Development of Optimal Control System for Quad Copter UAV

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## Abstract

**Background/Objectives:** Design and development of an optimal control system for a quadcopter unmanned aerial vehicle (UAV). **Methods/Statistical Analysis:** The 6DOF quad copter state-space models was used for Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG) simulations in MATLAB/Simulink. The simulations produced satisfactory results, which have been presented. **Findings:** A comparison between Low Pass Filter (LPF) and Kalman filter is also shown which shows that LQR is useless in presence of noise hence LQG was employed in such a situation. **Application/Improvements:** The optimal control system for quadcopter was successfully developed, which can be practically implemented on an actual quadcopter for stable unmanned flight of the aerial vehicle.

**Keywords:** Control, DOF, GPS, LQR, LQG, LPF, Quad-Copter, UAV

## 1. Introduction

A quad copter is a multi-rotor UAV that uses four rotors to lift itself off the ground. As the name 'quad' suggests. The two pairs of rotors move in opposite directions Clockwise (CW) and Counter-Clockwise (CCW) so that the yawing effect is cancelled. In the recent years the use of such UAVs has increased exponentially. Teal Group predicted that UAV production worldwide will total 93 billion dollars by the end of 2015<sup>1</sup>. Quad copters are among the Vertical Take Off and Landing (VTOL) UAVs and this ability makes them quite useful for reconnaissance and surveillance as they do not require a runway for landing and taking off. These UAVs can perform missions that have quite a high risk factor and hence the life of a pilot as in case of manned aerial vehicles, does not have to be put at risk. These quad copters are also being used a lot in many nonmilitary missions also such as video

shooting by news agencies, local terrain mapping, police, rescue operations, personal photography etc. UAVs are roughly classified into two types. Fixed wing and rotary wing UAVs.

Fixed wing UAVs have more speed and endurance than Rotary wing UAVs but they need runways and other launch frameworks for landing and takeoff. Hence they have quite a limitation in their role when it comes to indoors and when landing and takeoff strips are not available<sup>2</sup>. Rotary wing UAVs are more flexible and versatile in this case that they do not need runways for landing and taking off. They can easily navigate through tight regions and indoors. Hence because of their flexibility Rotary wing UAVs are usually preferred<sup>3</sup>.

Quad copter uses four rotors to lift itself off the ground and to move about in the air. Two pairs of rotors move in opposite directions, one Clockwise (CW) and one Counter-Clockwise (CCW). It has six degrees of

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freedom (6DOF). Unlike helicopters it has fixed pitch and fixed angle rotors. By changing the rpm of the rotors the quad copter moves in all direction. Quad copter tilts toward the direction of slow spinning motor, which enables it to roll and pitch. Roll and pitch angles divides the thrust into two directions due to which linear motion is achieved<sup>4</sup>. The rotors rotate in clockwise- anticlockwise pairs, (Figure 1) to control the yaw produced due to the drag force on propellers. The Center of Gravity (CG) lies almost at the same plane which contains all the rotors. Also all four motors of same class differ in efficiency with

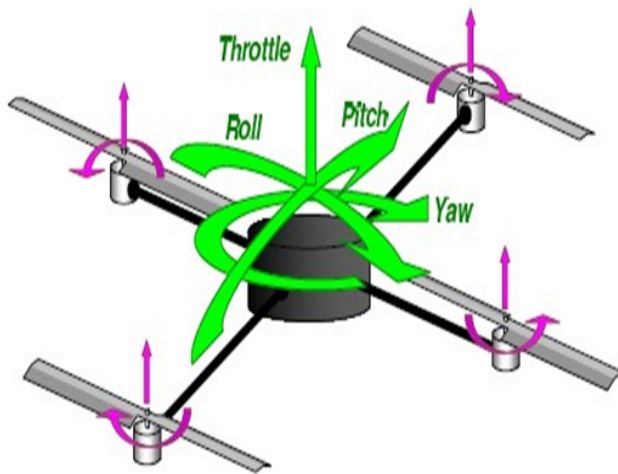


Figure 1. Quad copter schematic.

each other. This differentiates it from helicopters and it is very difficult to stabilize a quad copter by human control. Therefore a sophisticated control is essential for a balance flight of quad copter.

## 2. Simulations and Results

### 2.1 6DOF Open Loop Modeling and Simulation

6DOF setup was modeled in Simulink. Initially, the modeling was done using the 'State-Space' block from the Simulink library (Figure 2).

#### 2.1.1 Open Loop Test 1

The initial state vector given was  $[0 \ 0 \ 4 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$ , i.e. initial height of 4 meters above the reference position. The initial thrust was 1000 gms (constant). It was desired that the quad copter stabilize itself at a reference height of 0 meters. But since no controller was employed, it was expected that the quad copter will not maintain its height at 0 m but will go further downwards as the weight (1200 gms) is greater than the total upwards thrust (1000 gms) and all rotors are spinning at equal speed (i.e. no rolling, pitching or yawing motion). The simulation result is depicted in (Figure 3):

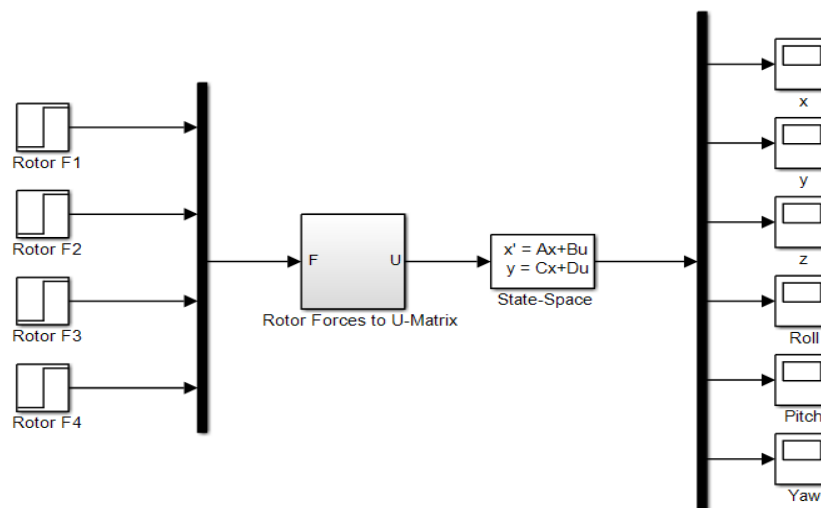
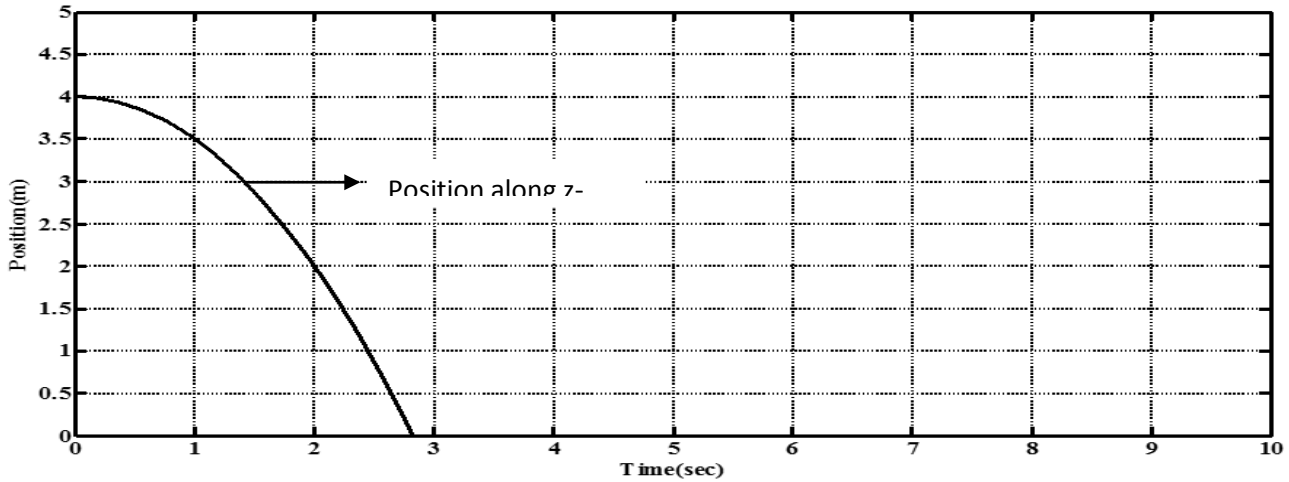
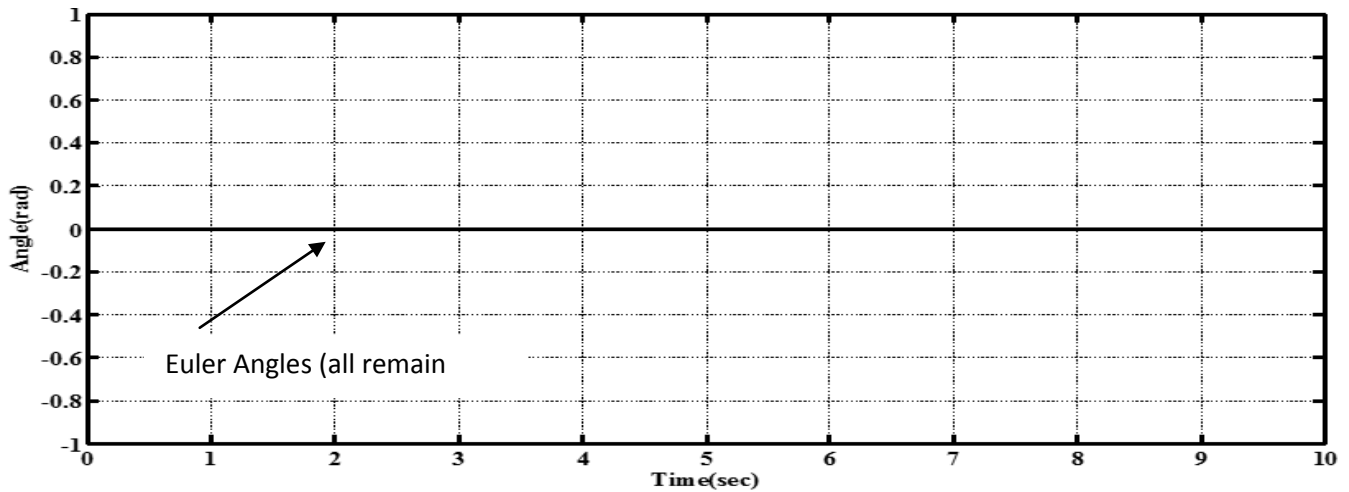


Figure 2. 6DOF Open Loop Quad Copter Model.

The A, B, C & D matrices used were as derived for the 6DOF setting<sup>5</sup>.



**Figure 3.** Vertical Position Vs Time Plot (OL Test 1).



**Figure 4.** Euler Angles Vs Time Plot (OL Test 1).

From Figure 3, it can be observed that, as expected, the height of the quad copter decreases below the desired reference or in simple words, it falls down without control.

It can be observed from the Euler angles plot (Figure 4) that they remain unchanged during the fall. It is expected as all the rotors are spinning at the same speed and no net torque is produced about any axis. From the above results, it becomes obvious that some form of actuating control mechanism is required to stabilize the quad copter at a desired reference position.

### 3.1 6DOF LQR Modeling and Simulation

After the open loop tests, an LQR for the 6DOF quad copter system was modeled in MATLAB Simulink (Figure 5):

#### 2.2.1 LQR Stability Test 1

Initial state vector =  $[0 \ 0 \ 4 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$

The initial conditions are the same as that of the 6DOF Open Loop Test 1. In that case, the quad copter was not able to maintain zero reference position. Now that LQR has been implemented, it is expected that the quad cop-

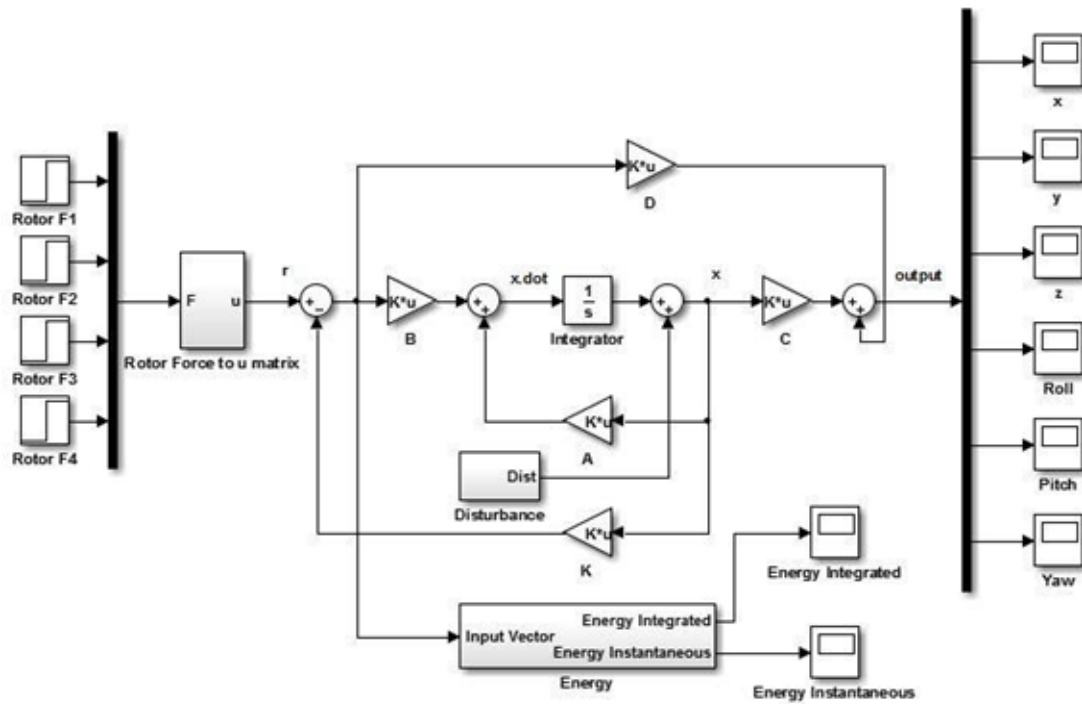


Figure 5. The curve under the condition of (A).

Q & R matrices for the 6DOF LQR with example values are given as follows:

$$Q = \text{diag}([30 \ 30 \ 30 \ 10 \ 10 \ 10 \ 100 \ 100 \ 100 \ 15 \ 15 \ 15])$$

$$R = \text{diag}([0.1 \ 0.1 \ 0.1 \ 0.1])$$

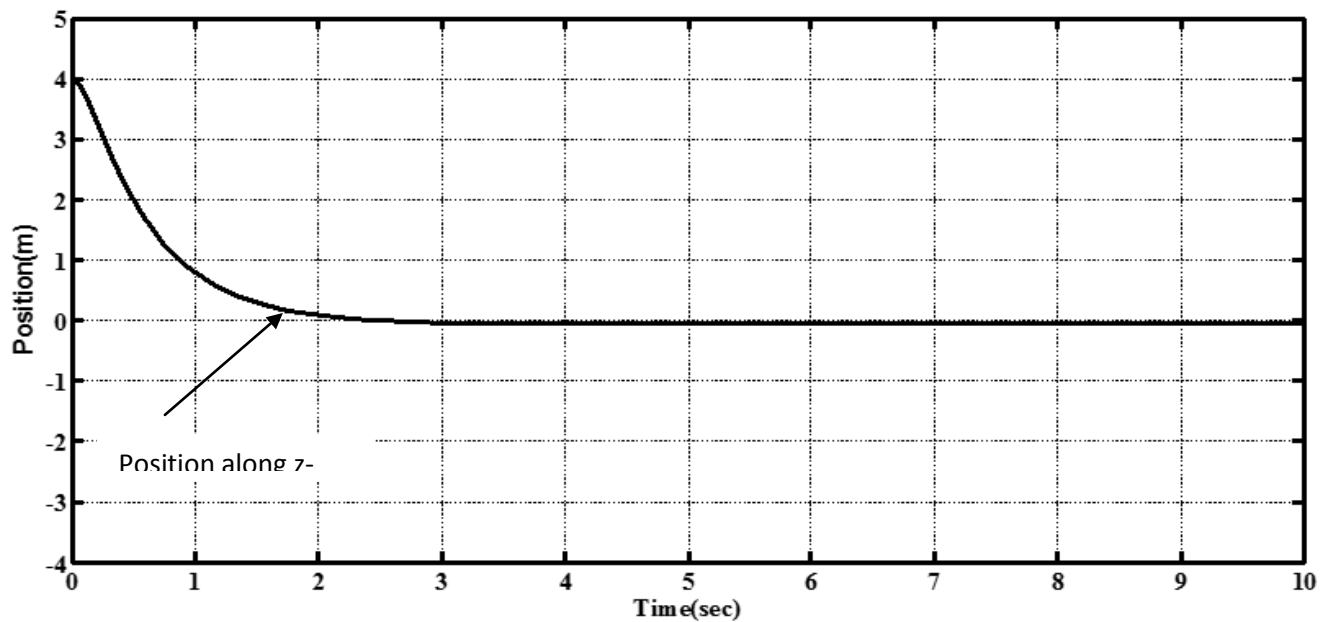
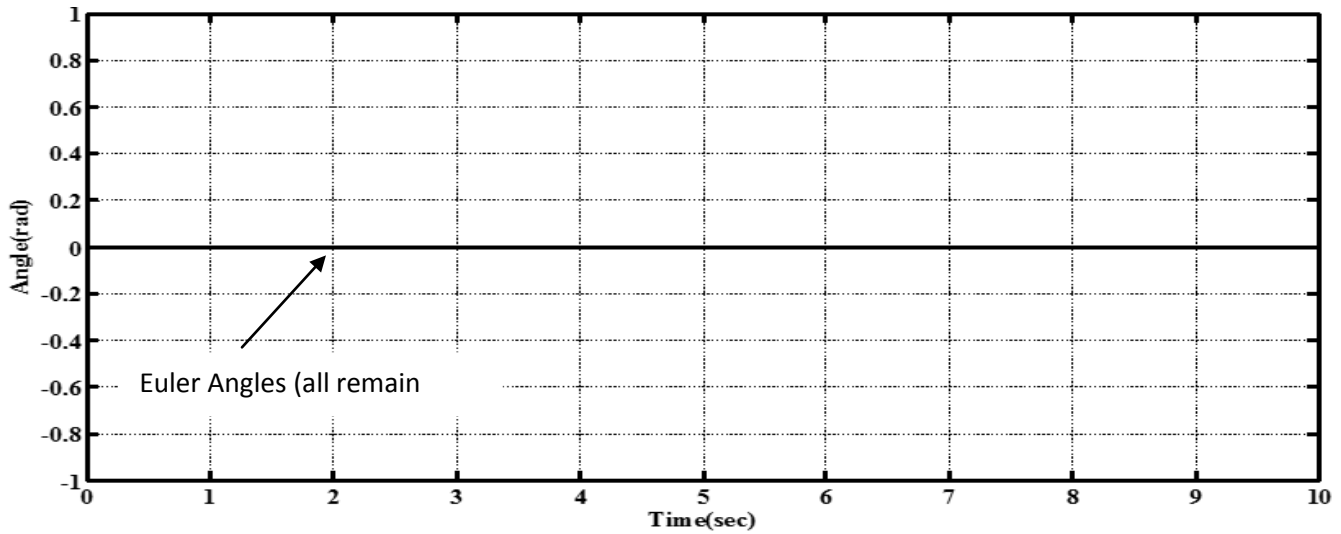


Figure 6. Vertical Position Vs Time Plot (LQR Test 1).



**Figure 7.** Euler Angles Vs Time Plot (LQR Test 1).

ter will stabilize near the zero reference. The simulation results are depicted as follows (Figures 6 & 7):

The curve above represents the vertical position. It can be observed that, as expected, after the displacement, the quad copter stabilizes its altitude at the reference position, and it does so in about 2 seconds.

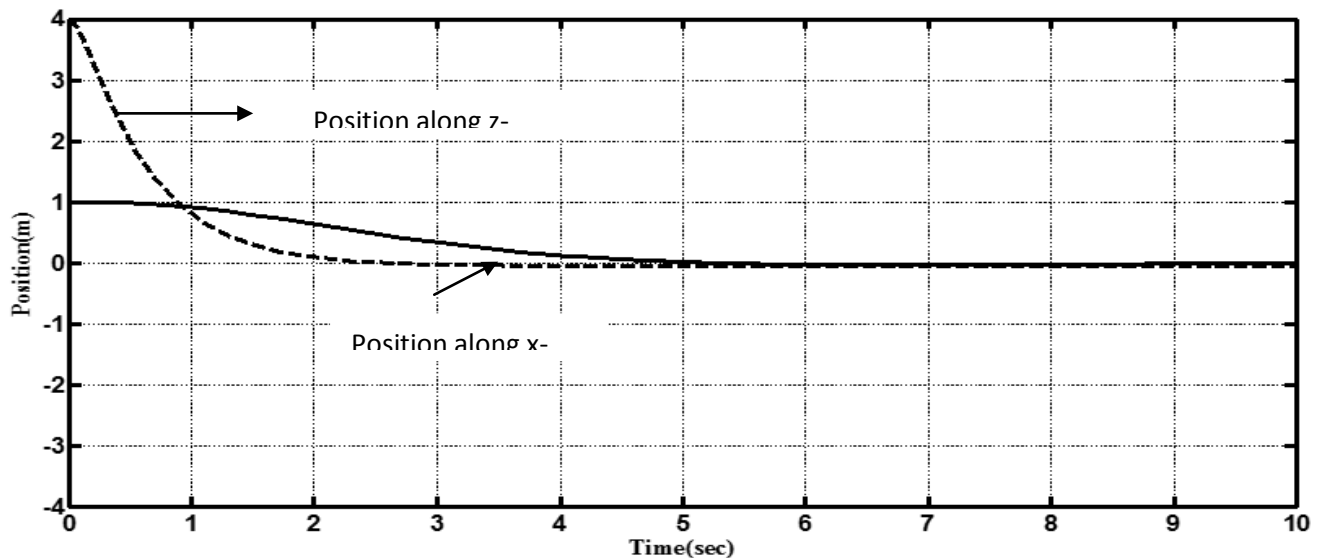
It can be observed from the Euler angles plot (Figure 7) that they remain unchanged during the fall. It is expected as all the rotors are spinning at the same speed and no net torque is produced about any axis.

### 2.2.2 LQR Stability Test 2

Initial state vector =  $[1 \ 0 \ 4 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$

The initial conditions are show that the quad copter has been displaced 1 meter in the x direction, and 4 meters in height. It is expected that the LQR will cause the quad copter stabilize near the zero reference. Position against time graph has been plotted in figure 8 given below:

The curves represent the position along z and x axes. It can be observed that, as expected, after the initial dis-



**Figure 8.** Position Vs Time Plot (LQR Test 2).

placement, the quad copter returns to its zero reference position. It does so in about 5 seconds.

### 2.2.3 LQR Stability Test 3

Initial state vector =  $[0 \ 0 \ 0 \ 0 \ 0 \ 0.75 \ 0.5 \ 0.3 \ 0 \ 0 \ 0]$

The initial conditions are show that the roll, pitch and yaw angles have been displaced by 0.75, 0.5 and 0.3 radians respectively. It is expected that the LQR will cause the quad copter to stabilize the attitude by bringing the angles back to zero in a suitable amount of time (~2-3 sec). In 6DOF, as the quad copter is in free flight, the change in Euler angles will also disturb the position of the aircraft in space as well. But since LQR has been implemented in all 6 degrees of freedom, the quad copter should come back to its original position in space as well.

The results are as expected. In addition to the stabilization of position, it is evident from (Figure 9) that the controller was able to stabilize the quad copter attitude as well. The response is very suitable as the angles as well as position are being stabilized within about 2 seconds, which is very practicable.

## 2.3 Tuning the LQR with simple Weighing Matrices

The response of the quad copter can be optimized to suit our requirements by simply changing the weights of the states or inputs in simple weighing matrices. This is very important in conditions when there is a lot of wind (external disturbance) and we would want a fast, aggressive control over the quad copter attitude to prevent it from crashing, or, (even during the same flight) in wind-less conditions when we want to conserve energy, hence increasing endurance, by avoiding an over aggressive response by the quad copter).

Two tests are shown to verify that the above is possible by simply changing the values in the weighing matrices Q & R.

### 2.3.1 Test 1 (Aggressive control, Less Endurance)

Initial state vector =  $[0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0.5 \ 0 \ 0 \ 0 \ 0]$

$Q = \text{diag}([30 \ 30 \ 30 \ 10 \ 10 \ 10 \ 100 \ 100 \ 100 \ 15 \ 15 \ 15])$ ,  $R = \text{diag}([0.1 \ 0.1 \ 0.1 \ 0.1])$

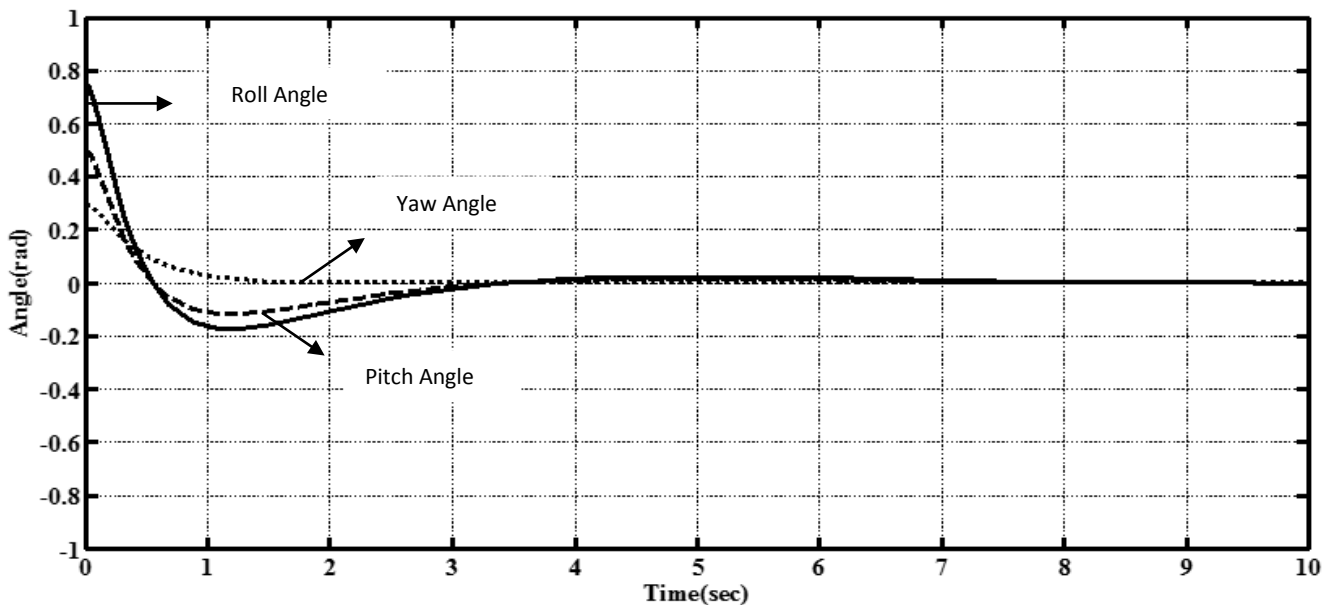
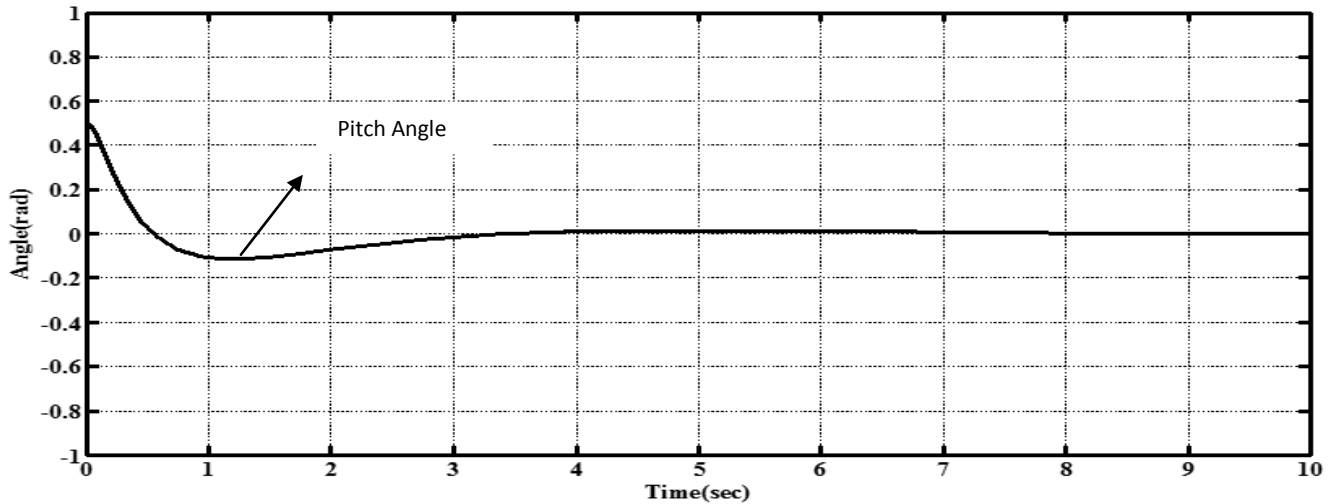
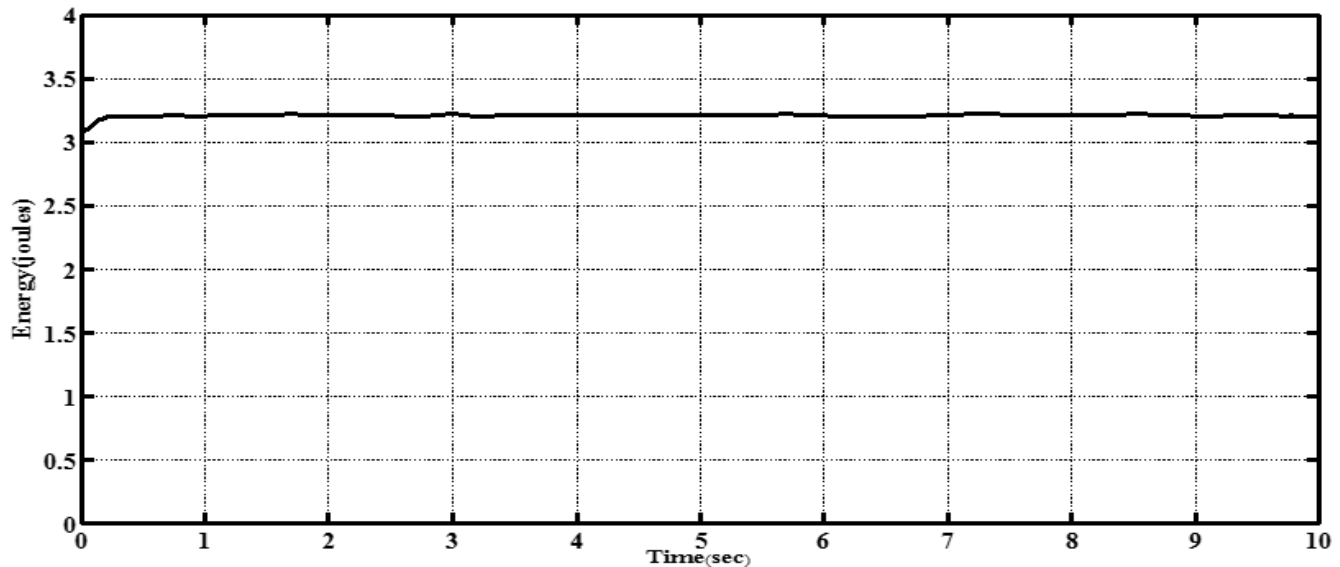


Figure 9. Euler Angles Vs Time Plot (LQR Test 3).



**Figure 10.** Euler Angles Vs Time Plot (LQR Tuning Test 1).



**Figure 11.** Energy Vs Time Plot (LQR Tuning Test 1).

In the Q matrix, the Euler angles have been given the highest weights, as maintenance of attitude is most important in quad copter stability. Next highest weights come for the position control. The position and angular rates have been rated low as they do not directly threaten the quad copter stability. The simulation results have been shown below (Figures 10 & 11):

### 2.3.2 Test 2 (Less Aggressive control, More Endurance)

Initial state vector =  $[0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0.5 \ 0 \ 0 \ 0 \ 0]$

$Q = \text{diag}([30 \ 30 \ 30 \ 10 \ 10 \ 10 \ 10 \ 10 \ 10 \ 15 \ 15 \ 15])$ ,  $R = \text{diag}([1.5 \ 1.5 \ 1.5 \ 1.5])$

Now, the weighing matrices have been edited such that inputs are weighed much higher this time and the Euler angles are weighed lower (Figures 12 & 13):



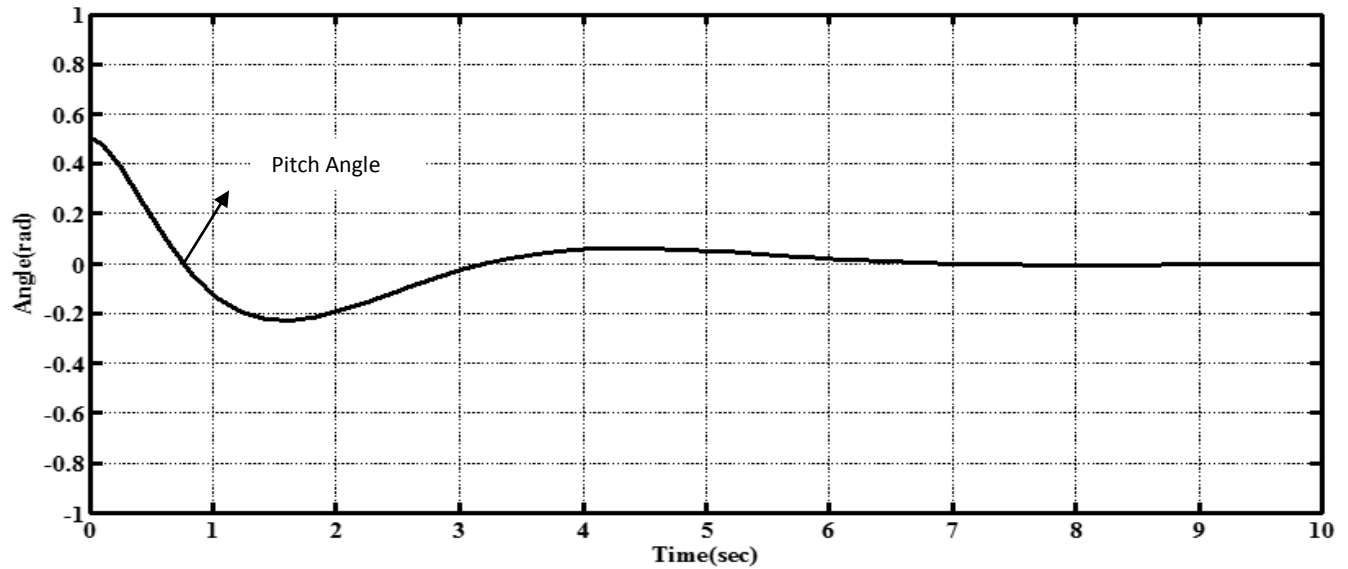


Figure 12. Euler Angles Vs Time Plot (LQR Tuning Test 2).

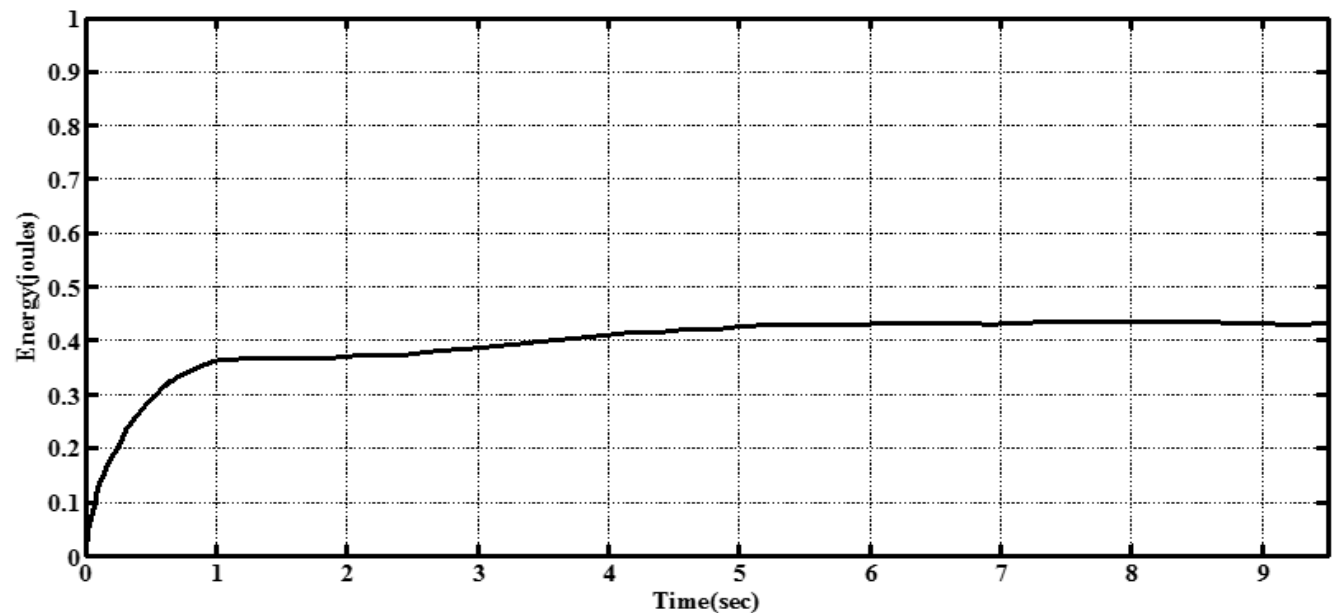


Figure 13. Energy Vs. Time Plot (LQR Tuning Test 2).

As we increase the weight of the inputs and decrease the weights of the states, the state response of the quad copter deteriorates, but in exchange lesser energy is consumed and the endurance is increased. Table 1 shows the two mode of operations and the differences in the energy consumption and pitch overshoot demonstrated by the two different endurance mode.

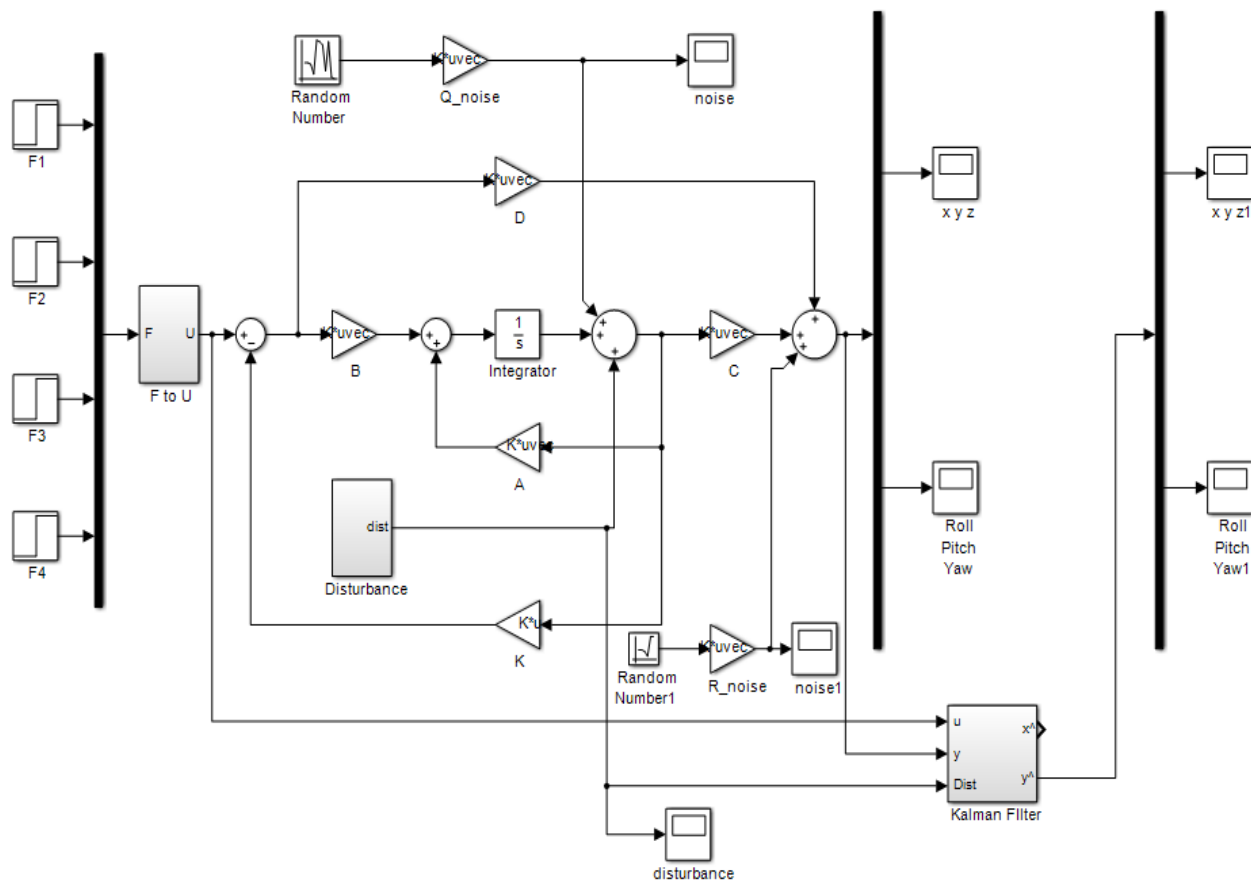
## 2.4 LQG Modeling & Simulation

LQG differs from LQR in the fact that in LQG, a Kalman filter is incorporated to reject the system and output noise. Noise was measured on the actual quad copter and based on the reading, noise was incorporated in the Simulink model as shown in (Figure 14)



**Table 1.** Comparison of Aggressive vs. Endurance Modes of Quad copter

	Aggressive/Less Endurance Mode (Test 1)	Non- Aggressive/High Endurance Mode (Test 2)
Pitch Overshoot	0.11 rad	0.21 rad rradrad
Energy Consumed	3.25 joules	0.44 units

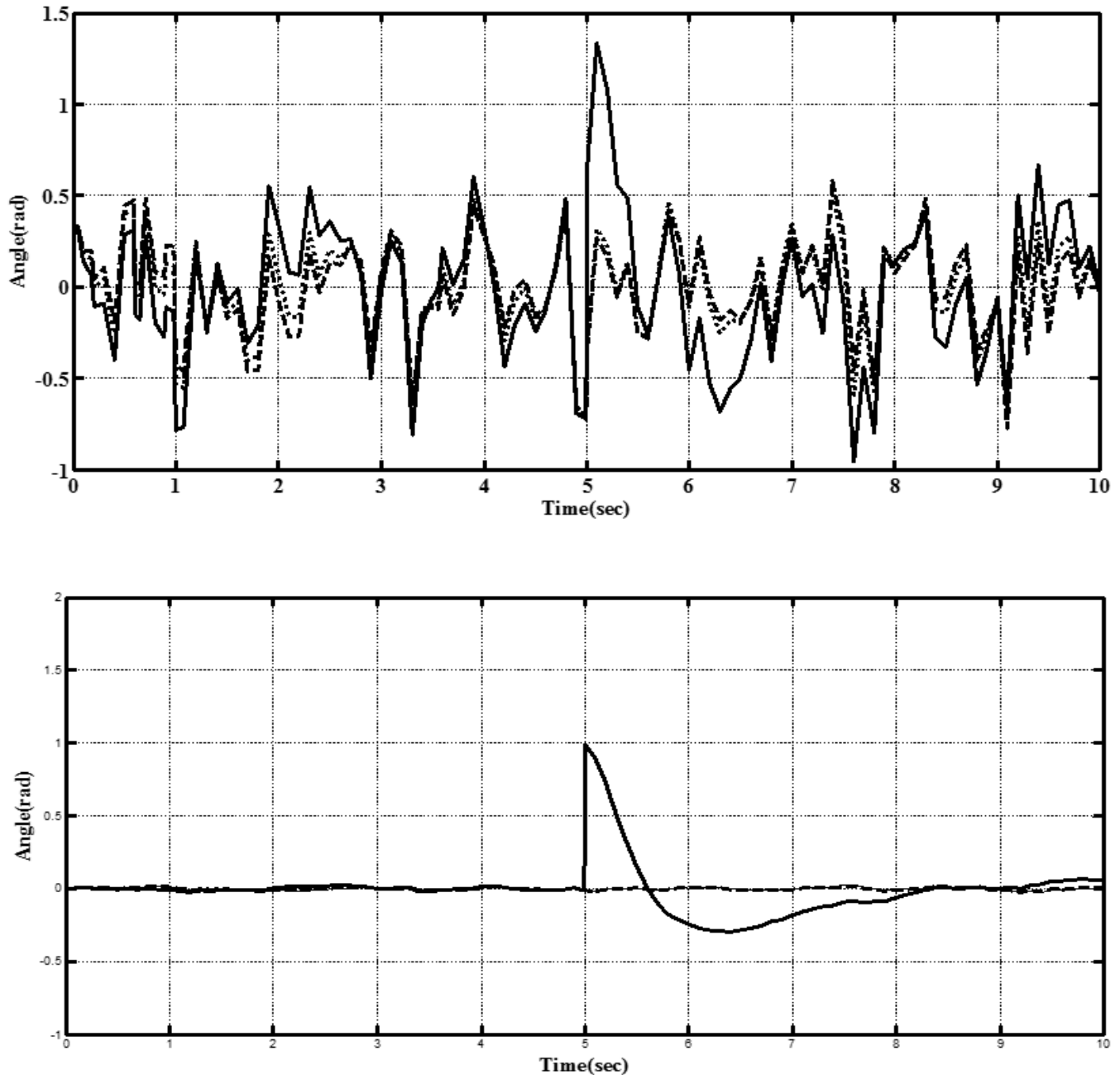


**Figure 14.** LQG Modeled in MATLAB Simulink.

A disturbance of 2 radians was given in the pitch angle and the angular response of three system settings was observed in the presence of noise. It was expected that the system with no filter incorporated would show the worse response as the states were noisy. Kalman filter was

expected to have the best response.

The result is as expected with the Kalman filter performing exceptionally well, as can be witnessed within figure 15. LQR in the presence of noise with no filter incorporated is completely unusable.



**Figure 15.** No Filter vs. Kalman Filter Result.

### 3. Conclusion

Optimal control techniques (LQR & LQG) were studied and implemented in MATLAB Simulink using the 6DOF quad copter state space models. The simulations produced satisfactory results. By changing the weights of Q and R weighing matrices in our LQR controller we optimized our energy consumption. Implementation of Kalman fil-

ter in our LQG controller showed us how important it is when noise is added to the overall system as it is in the real physical world. The implementation of optimal control on the actual 6DOF setup was understood, but not undertaken in this phase. The quad copter was assembled using open-source hardware/software and autonomous flight tests were conducted with encouraging results.

## 4. References

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