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Design and Analysis of an Optimal Inventory Model for Perishable Goods with Fixed Life Time

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Abstract

Background/Objectives: The main purpose of this paper is to plan and implement an optimum inventory model for the deteriorating things that minimizes the entire expected cost over a finite horizon. **Methods/Statistical Analysis**: The model presents a projected price operates for the inventory system with fixed life time. This paper accords with the hassle of ciphering optimum ordering policies for one item for successive intervals. Hyper geometric probability distribution is taken into consideration to examine the expected outdated cost. **Findings:** The optimum inventory model is solved analytically to review the impact of modification within the worth of the parameters. The solution methodology provided within the model helps the choice maker to enforce the first in first out policy by exposing customers to merchandise of a similar age. The results have been explained with numerical examples and graphical illustration. **Applications/Improvements:** First in First out issuing policy is designed to reveal the fact that it minimizes the expected outdates.

Keywords: Fixed Lifetime, Inventory, Lead Time, Optimal, Perishable

1. Introduction

In past and modern years, several researches have been done on distinct deteriorating items with various types of deterioration to model the inventory. Many items undergo quick deterioration, so it is necessary to know their shelf life which will help us to act accordingly. Hence it is very important to study the effect of deterioration in any system. Among the system, one of the major sectors is the food industry where the control of perishable inventory is important as many food particles deteriorate after a fixed shelf life period. Therefore, we can conclude that the life of the items is directly proportional to the size of the space. Because life time inventory is based on age wise profile of item. Hence it is essential to utilize the stock which has a fixed shelf life period before it reaches the valid date. Otherwise, those goods will enter into the category of outdated. Extant reviews of deteriorating inventory have been provided by [1]. The inventoried goods are grouped on the basis of demand variations as well as various states and limiting factors. They briefly discussed the motivations, scope applications and generalization of assorted models to draw the relevant facts about model developments in the last decade. In inventory modeling, most of the researchers assumed that the deterioration of the merchandise begins within the initial instance of their arrival in stocks. During the scheduled section, most of the items would have a span of possessing the essential property or original conditions, specifically when there is no deteriorations[2] observed it as "non-instantaneous deterioration". They are sufficiently successful to suggest an inventory model with allowable delay in payments for non-instantaneous deteriorating things.

[3] designed an inventory model to disclose the approximate expression for the optimal production lot-size, total inventory cycle time and also the mean total value. The speculation is accompanied with the numerical results. He made an impressive effort to incorporate an unfilled

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order backlogging with exponential decaying goods. Based on the shelf-life characteristics, the deteriorating items are classified as definite life time and random life time by [4]. To acknowledge the characteristic distinction between definite life time and random life time[5], used the classification as "age dependent" and "age independent" perishability. Deteriorating items are those that lose their value over time. Some products lose their standard due to some external influence such as technological innovations, launching of new products by opponent in a commercial market, they are normally referred as obsolescence. These products will arrive in the outdated category before it reaches the state of expiry date which in turn will be an additional cost to the outdating perishable goods.

Products such as human blood, photographic films, vegetables, fruits etc., which are remain unused up to its shelf life, will be considered as outdated and it should be disposed from the stock. Therefore, in inventory, the management scientist generally follows the First in First Out or Last in First Out issuing policy for the fixed life time products as it has deterministic shelf life. The literature on deteriorating inventory in determining optimal ordering policies are considered with different scenarios which are associated to demand plan, issuing policies, opinion on inventory etc., Problems on calculating optimum policy for the fastened life time products are studied by [6] and [7]. Nandakumar and [8] designed an inventory model for perishable products with definite life time. To estimate the operation of the resulting heuristics in the model they used myopic bound which was derived on the ordering quantities[9] analysed a (s, S) continuous inventory model with definite life time. They obtained the closed form solution using Markov renewal approach. The complexity in the model was illustrated with the statistical results.

[10] analysed an expanded EOQ –type inventory design for a perishable items in which demand rate is considered as a role of on hand inventory whereas holding cost behaved a non-linear function. For both the cases approximate optimal solution are obtained. The change of nonlinearity in the holding cost are studied with the computational results[11] designed an inventory model possessing time proportional demand with no shortages. Related research in modeling the inventory system with deteriorating products are carried by Balkhi and [12] and [13]. Inventory model with stock dependent depletion rate have been studied by various scientist like [14],[15] etc. [16] analysed an EOQ model with shortages. He explained the optimum inventory replenishment policy under inflationary conditions for non-instantaneous perishable items over a finite horizon using discount cash flow technique. The deterioration impact and inflation on the optimum inventory policy are learned with the assistance of sensitivity analysis and numerical examples.

One for one period policy for deteriorating items was developed by[17]. In designing (1, T) ordering policy for non-deteriorating goods, the time interval between two successive orders and the value of the order are kept unchanging. It is also witnessed that the entire cost rate is not dependent from the lead time. Optimal solution is obtained using search algorithm technique whereas the convexity of the model is analysed by defining the domain of the optimal solution with proposition. To assess the responsiveness of T towards the system constants, a statistical analysis has been carried out. A comparative study has been done with (s-1, S) policy to illustrate the performance of (1,T) model[18] proposed an optimum ordering protocol for a single item with rigid life period[19] designed an Inventory Model for perishable goods with Elapsing Date and Shortages for multi item. Problem considered in this paper is to resolve an optimum Inventory model for perishable items with a positive lead time over a definite horizon with FIFO policy. First in First out issuing policy is designed to disclose the fact that it minimizes the expected outdates. A statistical analysis is accomplished to review the behaviour of optimal policies with by varying various constants.

2. Assumptions and Notations

The successive assumptions and notations are used to design the expected optimal inventory model for deteriorating items.

- Discrete periods of time.
- Length of the period is rigid.
- Lead time is fixed.
- Demands in consecutive periods are not dependent
- Issuing policy is considered to be FIFO.
- Periodic study and order-up-to order policy with constant 'T' is used.
- In the inventory system, units lapse after the age t_n periods.

C - Setup cost

C_h - Holding cost

C_R - Replenishment cost
C_s - Shortage cost
D(t) - Demand in period t
d_l - Probability mass function of the hyper geometric distribution
E {TRC [T]} - Expected Total relevant cost function for the cycle with T setup cost.
E(.) - The expectation function
G - Expected outdate quantity
l - Amount demanded
O - Expected ordered quantity
P_i - Inventory position
t - Consecutive periods
T - Inventory level at the beginning of each cycle
t_n - Fixed life time
θ - Mean demand

3. Mathematical Formulation

The system is predicated on the chance that a product within the inventory model that is sold in a cycle. The feasibility that a product will be depleted is given as

$$P = \frac{\theta}{T} \tag{1}$$

$$O = E(O(t)) = \left\{ \frac{\theta}{\left[1 - \left(1 - \frac{\theta}{T} \right)^{t_n} \right]} \right\} t \tag{2}$$

$$G = E(G(t)) = \left[\frac{\theta}{\left(1 - \frac{\theta}{T} \right)^{t_n}} \right] t \tag{3}$$

Because expected quantity order is the total of expected demand and expected obsolete, i.e., O=D+G.

Let us assume that 'O' is the expected quantity which is to be ordered at the bound of the time interval 't' as well as the quantity which is to be received at the beginning of the time interval 't+1'. In the same way, at the end of the time interval t, let us assume that G number of units are outdated. In brief, the probability of a product which is not sold in single cycle is taken as $(1-P)$ and the probability of an item which is outdated will be $(1-P)^{t_n}$. This is truly established on the reality that execution during the time cycle T are not dependent. Succinctly, since $(1-P)<1$ outdate decreases with increasing t_n. This outcome was conjecture by Nahimas. Then employing (2) in (3) we can establish the outcome for expected outdated units.

$$G = \left[\frac{\theta(1-P)^{t_n}}{1-(1-P)^{t_n}} \right] t \tag{4}$$

For the everyday obsolesce amount of cross-match blood, let P be the ratio of cross-match blood that is really charged.

The probability of running out of stock is given by

$$P_r\{D(t) > T\} = 1 - \sum_{l=0}^{T} d_l \tag{5}$$

where, E(D(t))=θ and d_k =probability {D(t)=k}

The expected shortage quantity is given by

$$\sum_{l=T+1}^{\infty} (l-T)d_l = \theta - \sum_{l=1}^{T} l d_l t_l - T(1 - \sum_{l=1}^{T} d_l t_l) \tag{6}$$

$$\text{Ordering cost} = C_R O = C_R \left[\frac{\theta}{\left(1 - (1 - \frac{\theta}{T}) \right)^{t_n}} \right] t \tag{7}$$

$$\text{Expected holding cost} = C_h \sum_{l=1}^{T} (T-l) d_l t_l \tag{8}$$

$$\text{Expected shortage cost} = C_s \sum_{l=T+1}^{\infty} (l-T) d_l t_l \tag{9}$$

Expected outdate cost=

$$G \sum_{l=1}^{t_n} (T-l)[P(X=x)] =$$

$$G \sum_{l=1}^{t_n} (T-l) \left[\frac{{}_mC_k ({}_{N-m}C_{n-k})}{{}_NC_n} \right] \tag{10}$$

Using (7) to (10) we obtain

E [TRC (T)] = C + C_RO+ Expected holding cost +Expected shortage cost +

Expected outdate cost

E[TRC(T)]=

$$C+C_R(T-P_I)t+C_h\sum_{l=1}^{T}(T-l)d_lt_l+C_s\sum_{l=T+1}^{\infty}(l-T)d_lt_l+G\sum_{l=1}^{t_n}(T-l)\left[\frac{{}_mC_k({}_{N-m}C_{n-k})}{{}_NC_n}\right]. \quad (11)$$

E[TRC(T-1)]=

$$C+C_R(T-P_I-1)t+C_h\sum_{l=1}^{T-1}(T-1-k)d_kt_k+C_s\sum_{l=T}^{\infty}(l-T-1)d_kt_k+G\sum_{k=1}^{t_n}(T-1-l)\left[\frac{{}_mC_k({}_{N-m}C_{n-k})}{{}_NC_n}\right] \quad (12)$$

From (11) and (12)

E[TRC(T)]=

$$C+C_R(T-P_I-1)t+C_h\sum_{l=1}^{T-1}(T-1-l)d_lt_l+C_S\sum_{l=T}^{\infty}(l-T-1)d_lt_l+G\sum_{l=1}^{t_n}(T-1-l)\left[\frac{{}_mC_k({}_{N-m}C_{n-k})}{{}_NC_n}\right]-C-$$
$$C_R(T-P_I)t-C_h\sum_{l=1}^{T}(T-l)d_lt_l-C_s\sum_{l=T+1}^{\infty}(l-T)d_lt_l-G\sum_{l=1}^{t_n}(T-l)\left[\frac{{}_mC_k({}_{N-m}C_{n-k})}{{}_NC_n}\right]$$

E[TRC(T)]=

Table 1. Summary of physical parameters

ETRC(T)	ETRC(T-1)
137.33	137.33
146.35	146.35
145.82	145.82
153.33	153.33
153.75	153.75
138.19	138.19
146.65	146.65
132.3	132.3
140.85	140.85
144.1	144.1
138.95	138.95
138.95	138.95
129.65	129.65
142.11	142.11
137.66	137.66

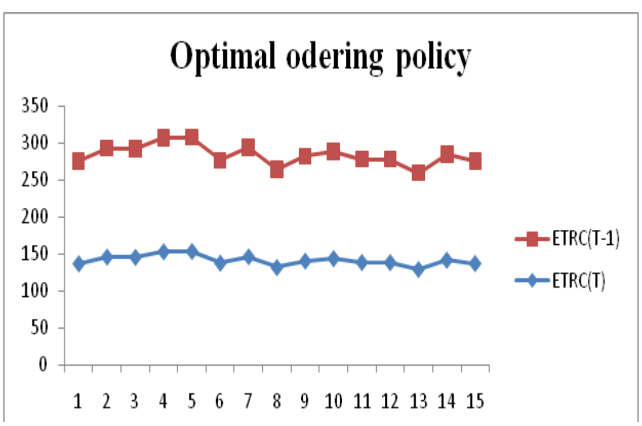

Figure 1. Expected total relevant cost function for the period with the setup cost.

$$-C_Rt-C_h(T-l)d_lt_l+C_s(l-T+1)d_lt_l-G$$

[where l=T-1]

The minimum value of E [TRC (T)] existing at t* gratifying the above equation.

4. Numerical Calculations

A comprehensive statistical investigation is carried out to analyse the behavior of an optimal policy by altering the values of the assorted constants like setup cost, holding cost, ordering cost etc., From the table it is evident that FIFO policy minimizes the expected outdates.

By using various values for the parameters in the proposed inventory system, the optimum ordering quantity that minimizes the expected cost for a deteriorating item over a definite horizon is retrieved. Since C is the setup cost simulated to be independent of the amount ordered or produced, the minimum value of E{TCS(T)} secured at t*. From Figure 1 it is evident that FIFO policy minimizes the expected outdates.

5. Conclusion

In this paper, an optimal inventory model that minimizes the entire expected cost for the deteriorating items with a positive lead time over a finite horizon is designed. The model is analysed for consecutive periods. Hyper geometric distribution is projected to evaluate the expected outdated cost. Based on the numerical calculations, it is evident that the expected obsoletes are minimized in applying First In First Out (FIFO) policy. Graphical representation reveals the fact of minimizing the total expected cost for the deteriorating items with a positive lead time over a finite horizon. The solution technique provided in the model assists the choice maker to carry out the FIFO policy by revealing purchasers to items of the similar phase. Hence it is important to consider the advantages of FIFO policy in formulating inventory model for perishable goods. The designed inventory model will be useful for the perishable goods industry which applies the FIFO issuing policy.

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