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EOQ Models with Optimal Replenishment Policy for Perishable Items taking Account of Time Value of Money

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Abstract

The main purpose of this research is to establish replenishment models and develop optimal replenishment policies for perishable items taking account of time value of money. This paper follows the Discounted Cash Flow (DCF) approach to investigate inventory replenishment problem over a fixed planning horizon. We develop model, to establish optimal solutions with and without backlogging and Show that the total variable cost is minimize and convex by the help of software. Numerical examples are to demonstrate the applicability of the proposed models and sensitivity analysis with respect to the parameters of the system is carried out.

Keywords: Deteriorating, Inventory, Inflation, Time Discounting

1. Introduction

Deterioration is a key factor in the study of inventory, which describes the deteriorating nature of the items; however, academia has not reached a general opinion on the definition of the deteriorating items. According to the definition, deteriorating items can be classified into two categories. Items that become decayed, damaged, evaporative, or expired through time, like meat, vegetables, fruit, medicine, flowers, film and so on are in the first category. The second category refers to the items that lose part or total value through time because of new technology or the introduction of alternatives, like computer chips, mobile phones, fashion and seasonal goods, and so on. Both of the two categories have the characteristic of short life cycle. For the first category, the items have a short natural life cycle. After a specific period (such as durability), the natural attributes of the items will change and then lose useable value and economic value, for the second category, the items have a short market life cycle. After a period of popularity in the market, the items lose the original economic value due to the changes in consumer preference, product upgrading and other reasons. There are several kinds of deteriorating rate in the present study, such as constant deteriorating rate, deteriorating rate is a linear increasing function of time, deteriorating rate is two-parameter Weibull distributed, and deteriorating rate is three-parameter Weibull distributed. Among them, the constant deteriorating rate is the easiest one and the three-parameter Weibull distribution deteriorating rate is more complex. Some studies which belong to the first category have made extensive study in this factor.

Models mentioned above, the inflation and the time value of money disregarded. This has happened mostly because of the belief that the inflation and the time value of money would not influence the inventory policy variables to any significant degree. However, owing to Asia, Russia and Brazil financial crisis, most of the countries have suffered from large-scale inflation and sharp decline in the purchasing power of money for last several years. Therefore, it is important to investigate how inflation and time value influence in various inventory policies

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2. Literature Review

All previous reports [1-7] developed inventory models in which Demand treated as Constant. In [8-10] Developed For Time-Dependent Demand. In [11-14] used inventory level-dependent demand. Chauhan et al.[15] discuss an inventory model with life time items for power demand rate with partial backlogging and variable holding cost. Wee and Law[66] studied price-dependent demand. Among them, ramp type demand is a special type of time-dependent demand. Hill[34] was the first to introduce the ramp type demand to the inventory study. Then Mandal and Pal[44] introduced the ramp type demand to the inventory study of the deteriorating items. After that, many researchers have extensively studied this type of demand, Deng et al.[22], Skouri et al.[58], Chauhan et al.[12], Singh[56] established optimal ordering policy with time dependent ramp type demand, life time and variable rate of deterioration, Kumar et al.[41] proposed fuzzy EOQ models with ramp type demand rate, partial backlogging and time dependent deterioration rate. Although the constant demand assumption helps to simplify the problem, it is far from the actual situation where demand is always in change. In order to make research more practical, many researchers have studied other forms of demand. Like time-dependent demand has attracted considerable attention. Wang and Wang[64] developed a model to determine optimal ordering policy for deteriorating items under inflation, partial backlogging, and time-dependant demand. In addition, Papachristos and Skouri[51], Chu and Chen[16], Khanra and Chaudhuri[39], Dye et al.[25] all conducted research on deteriorating items inventory under the premise that the demand is time-dependent. In fact, the real situation is complex and the demand is always affected by several factors such as time, inventory level, price, and so on. Balkhi and Benkherouf[2], Pal et al.[49], Hsu et al.[36] combined several of the factors together and considered the impact of the combination of the demand, in this premise the optimal inventory strategy was discussed, Wen et al.[67] developed an inventory model in which the demand is affected by time and inflation, Panda et al.[50] consider retailers optimal pricing and economic order quantity in stock and price sensitive demand environment. Yadav et al.[69] describe application of minimax distribution free procedure and Chebyshev inequality for backorder discount inventory model with effective investment to reduce lead-time and defuzzification by signed distance method, Mishra and Singh[45] proposed Computational approach to an inventory model with ramp-type demand and linear deterioration.

According to the study of Wee HM[65]. The first category of deteriorating items refer to the items that become decayed, damaged, evaporative, expired, invalid, devaluation and so on through time. For the second category, constant deteriorating rate is the most common one in model establishment. Such as Ghare and Schrader[26], Shah and Jaiswal[55], Padmanabhana and Vrat[48], Bhunia and Maiti[4], Chang et al.[9], Shah et al.[54] established an EOQ model for deteriorating items. Lodree & Uzochukwu[43], developed constant deterioration rate replenishment policy. Chauhan et al.[13] discuss volume flexibile inventory model for unsystematic rate of deterioration with lost sales. Chaudhary et al.[10] discuss an inventory model with time dependent demand and deterioration under partial backlogging. In recent research, more and more studies have begun to consider the relationship between time and deteriorating rate. In this situation there are several scenarios; including deteriorating rate is a linear increasing function of time Raafat[52]Goyal and Giri[30] made comprehensive literature reviews on deteriorating inventory items in 1991 and 2001 respectively. Huang and Huang[38], Liao and Huang[42] studied the economic order quantity problem with deteriorating items taking the time - value of money into consideration. Chauhan et al.[14], Kumar et al.[40], Singh and Singh[57] considered Imperfect production process with exponential demand rate, Weibull deterioration under inflation, Wu et al.[68] considered a problem to determine the optimal replenishment stock dependent demand of the non instantaneous deteriorating item, Soni[59] has discuss Optimal replenishment policies for deteriorating items with stock sensitive demand under two-level trade credit for limited capacity, Duan and Liao[24]proposed Optimization of replenishment policies for decentralized and centralized capacitated supply chains under various demands, In addition, Balkhi and Benkherouf[1] presented the optimal replenishment schedule for the production lot size model with deteriorating items, where the demand and production are allowed to vary with time in an arbitrary way with and without shortages, Trippi and Lewin[62] adopted a discounted cash flow (DCF) approach to obtain the present value of average inventory costs over an infinite horizon. Dohi et.al[23] discussed inventory systems with and without backlogging allowed for an infinite time span, taking into account time value from a viewpoint different from that of Moon and Yun[46] employed the DCF approach to fully recognize the time value of money to develop a finite planning horizon EOQ model where the planning horizon is a random variable. Hariga[33] studied the effects of the inflation and the time value of money on the replenishment policies of items with time continuous non-stationary demand over a finite planning horizon. Although all the models mentioned in this paragraph consider the inflation and the time value of money, they do not consider deterioration. Combining both factors of deterioration and the time value of money, Bose et al.[7] first explored an inventory model under inflation and time value of money for deteriorating items. They extended the constant demand to a linear time-dependent. Chung et al.[17] discussed the inventory replenishment policy over a finite planning horizon for a deteriorating item taking account of time value and presented a line search technique to decide the optimal interval which has positive inventories. Hou and Lin[35] discussed a cash flow oriented EOQ model with deteriorating items under permissible delay in payments, Ouyang et.al.[47] studied retailer's ordering policy for non-instantaneous deteriorating items with quantity discount for stock-dependent demand and stochastic backorder rate, Chauhan and Singh[11] consider ameliorating inventory model with weibull deteriorating under the effect of inflation, Borgonovo and Peccati[5] discussed global sensitivity analysis in inventory management. Borgonovo and Peccati[6] consider Finite Change Comparative Statics for Risk Coherent Inventories. Chung and Gwo[20] discussed dynamic pricing, promotion and replenishment policies for a deteriorating item under permissible delay in payments.

Teng et al.[61] established various inventory replenishment policies to solve the problem of determining the timing and the number of replenishments, analytically comparing various models, and identified the best alternative among them based on minimizing total relevant costs. They presented four possible systems for inventory replenishment control.

The systems of Teng et al.[61] was described as follows:

System 1: The traditional model starts with an instant replenishment and ends with no shortages, it's used by Goswami and Chaudhuri[29].

System 2: It starts with replenishment and ends with shortages, which can be seen in Hariga[32].

System 3: It starts with shortages and ends without shortages, which was suggested by Goyal et al.[31].

System 4: It starts and ends with shortages, which was first introduced by Teng et al.[61].

Furthermore, they proved that a policy for the possibility to end the planning horizon with shortages is less expensive to operate than a policy without shortages at the ending stage. In the described model, the inventory replenishment model generated by Chung et al.[17] to the situation of system 2. (System 2 can be depicted graphically in Figure 1, which extended by Chung and Lin[19] by applying the DCF approach to determine the optimal number of replenishments for the items.

All the models discussed above have the characteristics of a constant deteriorating rate. While the constant rate simplifies the problems, it cannot reflect the real situation of the deterioration.

We propose and develop a model for time varying decaying items and establish optimal replenishments policy corresponding to cycle length during the time horizon H, consisting of positive and negative inventories periods for both with and without shortage to get a more generalize results. In addition, we prove that the total variable cost functions are convex. Numerical examples are to demonstrate the applicability of the proposed models and sensitivity analysis with respect to the parameters of the system carried out.

The develop article organized as follows. Section 1 is introductory part and section 2 shows the background of past study, the notation and assumptions used throughout this study is in section 3, the limitations of the models are implied through these assumptions. In section 4.1, the mathematical models described without shortage in order to minimize the total cost in the planning horizon. In section 4.2 established an optimal solution procedure to find out the optimal number of replenishments and optimal total cost. Section 4.3 and section 4.4 appear for numerical example to illustrate the model and sensitivity analysis is to discuss the effect of various parameters and section 4.5 show the variability of our model with the classical EOQ model. Similarly, in section 5 and its subsection discussed with shortage and the conclusion drawn in section 6.

3. Assumptions and Notations

a) The replenishment rate is infinite and the lead time is zero.
b) A single item considered over a prescribed period of *H* units of time.
c) The product demand rate, α units per year, is known constant in planning period.
d) The production rate is higher than the sum of consumption and deterioration rates.
e) The time varying decaying rate $\theta(t) = \theta + bt$ where $0 \le b < 1$, $\theta \ll 1$ is only apply to on-hand inventory.
f) m denotes the number of replenishment periods during the time horizon *H*.
g) When an inventory system allows shortages, $m+1$ replenishments made during the entire time horizon *H*. The last replenishment made at a time $t = H$ just to replenish any shortages generated in the last cycle.
h) Two models analyzed; Model-I in which shortage not allows and Model-II in which shortage allow and complete backlogging is permitted with a finite shortage cost C_2 per unit per unit time.
i) C the unit cost of items, C_1 the inventory holding cost per unit per unit time and *A* the ordering cost per order.
j) C_R, total replenishment costs; C_P, total purchasing costs; C_H, total holding costs; C_s, total shortage costs.
k) BQ, backorder quantity during shortage and Q, an optimal order quant.

Two models are analysed; Model-I in which Shortage is not permitted and Model-II in which shortage is permitted and complete backlogged.

4. Model I: Without Shortages

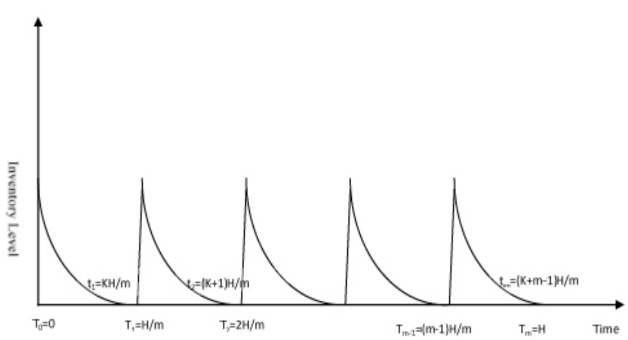

Figure 1. Representation of the inventory cycles without Shortage.

4.1 Model Formulation

Procurement of the inventory level in system is shown in figure-1. For the evolution of the model, we assume that, **m** cycles during the real time horizon **H** each of length T, that **T=H/m**. Hence, the reorder times over the planning horizon H are $T_j = jT$ (j= 0,1, 2,…..,m). Initially, consider the inventory level I (t) during the first replenishment cycle. This inventory level is depleted by the effects of demand and deterioration rate. So, the variation of I (t) with respect to t is demonstrate by the following differential equation:

$$\frac{dI(t)}{dt} = -\alpha - \left(\theta + bt\right)I(t) \qquad 0 \le t \le T \tag{1}$$

With the boundary condition I (T) = 0. The solution of (1) can be represented by

$$I(t) = \alpha\left[(T-t) + \frac{\theta}{2}\left(T^2 - t^2\right) + \frac{\left(b+\theta^2\right)}{6}\left(T^3 - t^3\right)\right] \times \left[1 - \left\{\theta t + \left(b-\theta^2\right)\frac{t^2}{2}\right\}\right] \quad 0 \le t \le T \tag{2}$$

Since there are m replenishments in the entire time horizon *H*,

The present values of the total replenishment costs are given by

$$C_R = A\sum_{j=0}^{m-1} e^{-RT_j} = A\frac{\left(1-e^{-RH}\right)}{\left(1-e^{\frac{-RH}{m}}\right)} \tag{3}$$

The present values of total purchasing costs are

$$C_P = C\sum_{j=1}^{m} I(0)e^{-RT_{j-1}} = \alpha C\sum_{j=1}^{m}\left\{T + \frac{\theta T^2}{2} + \left(b+\theta^2\right)\frac{T^3}{6}\right\}\{1-0\}e^{-T_{j-1}}$$

$$C_P = \alpha C\left[T + \frac{\theta T^2}{2} + \left(b+\theta^2\right)\frac{T^3}{6}\right]\frac{\left(1-e^{-RH}\right)}{\left(1-e^{-RH/m}\right)} \tag{4}$$

The present values of the holding costs during the first replenishment cycle are

$$H_1 = C_1\int_0^T I(t)e^{-Rt}dt$$

$$H_1 = \alpha C_1\left[\left\{T + \frac{\theta T^2}{2} + \frac{\left(b+\theta^2\right)}{6}T^3\right\}\left(\frac{e^{-Rt}}{-R} + \frac{1}{R}\right) + \left\{1 + \theta T + \frac{\theta^2 T^2}{2} + \frac{\left(b+\theta^2\right)\theta T^3}{6}\right\}\left\{\frac{Te^{-RT}}{R} + \frac{e^{-RT}}{R^2} - \frac{1}{R^2}\right\}\right] \tag{5}$$

Hence, the present values of the total holding costs during the entire time horizon *H* are given as

$$C_H = \sum_{j=1}^{m} H_1 e^{-RT_{j-1}}$$

$$= \alpha C_1 \left[\left\{ \frac{H}{m} + \frac{\theta H^2}{2m^2} + \frac{(b+\theta^2)H^3}{6m^3} \right\} \left(\frac{e^{-RH/m}}{-R} + \frac{1}{R} \right) \right.$$

$$\left. + \left\{ 1 + \frac{\theta H}{m} + \frac{\theta^2 H^2}{2m^2} + \frac{(b+\theta^2)\theta H^3}{6m^3} \right\} \left\{ \frac{He^{-RT}}{Rm} + \frac{e^{-RH/m}}{R^2} - \frac{1}{R^2} \right\} \right] \left(\frac{1-e^{-RH}}{1-e^{-RH/m}} \right) \tag{6}$$

The net present total variable cost TC of the system during the entire time Horizon H is the sum of the replenishment cost C_R, the purchasing cost C_p and holding cost C_H

$$TC(m) = C_R + C_P + C_H$$

So,

$$TC(m) = \left[A + \alpha C \left[T + \frac{\theta T^2}{2} + (b+\theta^2)\frac{T^3}{6} \right] + \alpha C_1 \left\{ \frac{H}{m} + \frac{\theta H^2}{2m^2} + (b+\theta^2)\frac{H^3}{6m^3} \right\} \left\{ \frac{e^{-RH/m}}{-R} + \frac{1}{R} \right\} \right.$$

$$+ \alpha C_1 \left\{ 1 + \frac{\theta H}{2m} + \frac{\theta H^2}{2m^2} + (b+\theta^2)\frac{\theta H^3}{6m^3} \right\} \tag{7}$$

A mathematical model is derived to obtain the optimal replenishment when TC (m) is minimized. Minimize TC (m); such that m > 0.

4.2 Optimal Solution Procedure

Our problem is to determine the optimal value of m that minimize the total system cost $TC(m)$. The optimization technique is used to minimize m as follow

Step 1: Since the number of cycle in the planning horizon m, is an integer value, start by choosing an integer value of m ≥1.

Step 2: For m=1, take the derivative of TC (m) with respect to m and equate the results to zero, the necessary conditions for optimality are $dTC(m)/dm = 0$ this equation solve for m (which is shown in appendix theorem.1)

Step 3: Using m found in step 2, substitute into equation (7) and derive TC (m).

Step 4: Repeat step 2 and 3 for all possible m values until the minimum TC (m^*) is found. The TC (m*) constitute the optimal solution that satisfied the following conditions:

$$d^2TC(m)/dm^2 > 0 \text{ For } m^* \text{ and } TC(m+1) \geq TC(m)$$

Step 5: For optimal value m^*, we can find the optimal-order quantity is $Q = \alpha \left[\left(\frac{H}{m} \right) + \frac{\theta}{2}\left(\frac{H^2}{m^2} \right) + \frac{(b+\theta^2)}{6}\left(\frac{H^3}{m^3} \right) \right.$ by using Eq. (2).

4.3 Numerical Example

The numerical example is derived here to illustrate the effect of the general model-I developed in this paper with the following data:

The demand parametric value are α=600unit/year, the deterioration rate of the on hand inventory per unit time is θ =.20, A= $ 120.00, C_1=$1.75 per unit per year, C=$2 per unit, R=0.50, b=0.05, the time horizon H=10 yr.

By using the solution procedure described in the previous section, the computational result are Shown in Table-1, from this table, we see that that the optimal replenishment number m^*=11, the total variable cost TC become minimum. During the first replenishment cycle order quantity Q^*=601.803 and minimum total variable cost $TC^*(m)$ = 16284.7, then we have the time interval between replenishment is T=(H/m) = 10/11=0.901 year

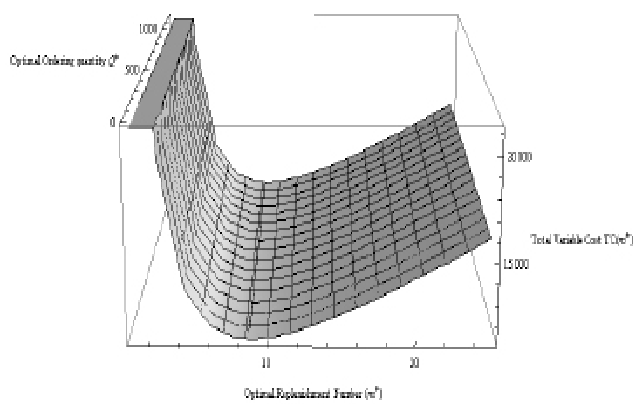

Figure 2. Representation of convexity of function TC (m).

Table 1

m	1	2	3	5	7	10	11*	13	16	19	22	25
TC(m)	149172	45889.1	28992.7	19850.8	17274.3	16299.5	16284.7*	16487	17136.2	18014	19021.3	20109
Q	21000	5625	3000	1512	1005.83	669	601.803*	501.138	400.635	333.722	285.969	250.176

4.4 Sensitivity Analysis

In order to study how the parameters affect the optimal solution, we conduct the sensitivity analysis. The change in the values of parameters can take place due to uncertainties in any decision making situation. In order to examine the implications of these changes, the sensitivity analysis will be of great help in decision making. Using the numerical example given in the preceding section, the sensitivity analysis of various parameters has been done.

Sensitivity analysis with respect to various parameters on ordering quantity and total system cost for model without shortages.

Table 2

Parameter	**θ =.20**										
% Change	-50	-40	-30	-20	-10		10	20	30	40	50
TC	15396.2	15571.4	15747.9	15925.6	16104.5	16284.7*	16466.3	16649.1	16833.4	17019.1	17206.2
Q	574.756	580.045	585.394	590.804	596.273	601.803*	607.393	613.043	618.753	624.523	630.353
Parameter	b=.050										
% Change	-50	-40	-30	-20	-10		10	20	30	40	50
TC	16249.6	16256.6	16263.7	16270.7	16277.7	16284.7	16291.7	16298.8	16305.8	16312.8	16319.8
Q	599.925	600.301	600.676	601.052	601.427	601.803	602.179	602.554	602.93	603.306	603.681
Parameter	C_1=1.75										
% Change	-50	-40	-30	-20	-10		10	20	30	40	50
TC	12396.7	13174.3	13951.9	14729.5	15507.1	16284.7	17062.3	17839.9	18617.5	19395.1	20172.7
Q	601.803	601.803	601.803	601.803	601.803	601.803	601.803	601.803	601.803	601.803	601.803
Parameter	C=5										
% Change	-50	-40	-30	-20	-10		10	20	30	40	50
TC	12193.5	13011.7	13830.1	14648.2	15466.5	16284.7	17103.1	17921.2	18739.4	19557.7	20375.9
Q	601.803	601.803	601.803	601.803	601.803	601.803	601.803	601.803	601.803	601.803	601.803
Parameter	A=120										
% Change	-50	-40	-30	-20	-10		10	20	30	40	50
TC	16121.6	16154.2	16186.8	16219.5	16252.1	16284.7	16317.3	16350.2	16382.6	16415.2	16447.9
Q	601.803	601.803	601.803	601.803	601.803	601.803	601.803	601.803	601.803	601.803	601.803
Parameter	R=.5										
% Change	-50	-40	-30	-20	-10		10	20	30	40	50
TC	82905.1	52320.2	35891.8	26275.1	20257.5	16284.7	13543.7	11581.2	10131.4	9030.9	8176
Q	601.803	601.803	601.803	601.803	601.803	601.803	601.803	601.803	601.803	601.803	601.803
Parameter	α=600										
% Change	-50	-40	-30	-20	-10		10	20	30	40	50
TC	8305.5	9901.3	11497.2	13093	14688.9	16284.7	17880.6	19476.4	21072.2	22668.1	24263.9
Q	300.902	361.082	421.262	481.443	541.623	601.803	661.983	722.164	782.344	842.524	902.705
Parameter	H=10										
% Change	-50	-40	-30	-20	-10		10	20	30	40	50
TC	17578.4	17033.6	16659.3	16425.2	16306.9	16284.7	16343	16469	16652.9	16886.5	17163.4
Q	285.969	346.585	408.435	471.561	536.004	601.803	669	737.635	807.748	879.381	952.573

Results of the sensitivity analysis are exhibited in Table 2.

- Effect of changing deteriorating parameter (θ and b) is moderately sensitive for the total system cost (TC) and the order quantity (Q).
- Effect of changing the unit cost (C), holding cost (C_1) and ordering cost (A) is also moderately sensitive for to the total system cost (TC) but no effect of the order quantity (Q).
- Effect of changing discount rate parameter (R) is highly sensitive for to the total system cost (TC) but no effect on the order quantity (Q).
- Effect of changing consumption rate parameter (α) is highly sensitive for to the total system cost (TC) and the order quantity (Q).

The main conclusions one can draw from the sensitivity analysis are as follows:

- The order quantity (Q) is more sensitive to consumption rate parameter (α) as compared to deterioration rate (θ), while no effect of holding cost (C_1) and discount rate parameter (R).
- The total system cost (TC) is more sensitive to consumption rate parameter (α), the unit cost (C), the discount rate parameter (R) and deterioration rate (θ) as compared to ordering cost (A) and holding cost (C_1).

4.5 Special Case (Undiscounted)

In this section, we explain the important particular case *R*=0, and compare the differences with classical EOQ model. When *R*=0, then the total (undiscounted) variable cost, TC (T) is

$$C_h = \alpha C_1\left(T^2 + \frac{T^3\theta}{2} + \frac{T^4(b+\theta^2)}{6}\right)\frac{H}{T}\left(\left(\frac{H}{T}+1\right)\Big/2\right);$$

$$C_R = A\left(\frac{H}{T}\right)\left(\frac{H}{T}-1\right)\Big/2;$$

$$C_p = \left(\alpha\ C\ \left(H/T\right)\left(H/T+1\right)\Big/2\right)\left[T + \frac{\theta\ T^2}{2} + (b+\theta^2)\frac{T^3}{6}\right].$$

$$\mathrm{TC(T)} = C_R + C_p + C_H$$

The necessary condition for minimum total variable cost is $\frac{dTC(T)}{dT} = 0$, giving

$$-\frac{AH^2}{2T^3} - \frac{AH(\frac{H}{T}-1)}{2T^2} - \frac{H^2\{\frac{1}{6}CT^3\alpha(b+\theta^2)) + \frac{1}{2}CT^2\alpha\theta + CT\alpha\}}{2T^3}$$

$$-\frac{C_1H^2\alpha\{\frac{1}{6}T^3(b+\theta^2) + \frac{T^2\theta}{2} + T\}}{2T^2} + \frac{H\left(\frac{H}{T}+1\right)\left(\frac{1}{2}CT^2\alpha(b+\theta^2) + CT\alpha\theta + C\alpha\right)}{2T}$$

$$+\frac{1}{2}C_1H\alpha\left(\frac{H}{T}+1\right)\left(\frac{1}{2}T^2(b+\theta^2) + T\theta + 1\right)$$

$$-\frac{H\left(\frac{H}{T}+1\right)\left(\frac{1}{6}CT^3\alpha(b+\theta^2) + \frac{1}{2}CT^2\alpha\theta + CT\alpha\right)}{2T^2} = 0 \quad (8)$$

The optimum value of T can be obtained from expression (8) by using the optimization method. Taking the optimum value of T, we can also obtain the optimum-order quantity by Eq. (2) is

$$Q = \left[T + \frac{\theta\ T^2}{2} + (b+\theta^2)\frac{T^3}{6}\right]\alpha \quad (9)$$

When *R*=0 and *H*=1.0, with the same values of different parameters as taken in the numerical example of Section 3.3, Eq. (7) to (9) we get the optimal replenishment cycle length, T, order quantity, *Q*, replenishment cost C_R, purchasing cost C_P, holding cost C_H and total system cost, TC as follow:

$$T^* = 0.919015\ ;\ TC^* = 2502.48\ ;\ Q^* = 609.07\ ;$$

$$C_R^* = 5.75325\ ;\ C_P^* = 1383.89 \quad C_H^* = 1112.84$$

Now, we find that the optimal-order quantity and all the relative system costs under classical EOQ model as using the same parameter value are as follow:

$$TC^*(EOQ) = 1701.99\ ;\ Q^*(EOQ) = 286.85\ ;$$

$$C_R^*(EOQ) = 250.99\ ;\ C_P^*(EOQ) = 1200\ ;$$

$$C_H^*(EOQ) = 250.99$$

When we compare the values among Q^*, $C_R{}^*$, $C_P{}^*$, $C_H{}^*$, TC^* and $Q^*(EOQ)$, $C_R{}^*(EOQ)$, $C_P{}^*(EOQ)$, $C_H{}^*(EOQ)$, $TC^*(EOQ)$, we notice that $Q^* > Q^*(EOQ)$, $C_R{}^* < C_R{}^*(EOQ)$, $C_P{}^* > C_P{}^*(EOQ)$, $C_H{}^* > C_H{}^*(EOQ)$, $TC^* > TC^*(EOQ)$

This phenomenon is due to deterioration rate. So that when an inventory system included deterioration, the system will increase order quantity to avoid shortage but accompany higher purchasing and holding cost, so the total system cost is greater than that of the classical EOQ model.

5. Model-II: With Shortages Under Complete Backlogging

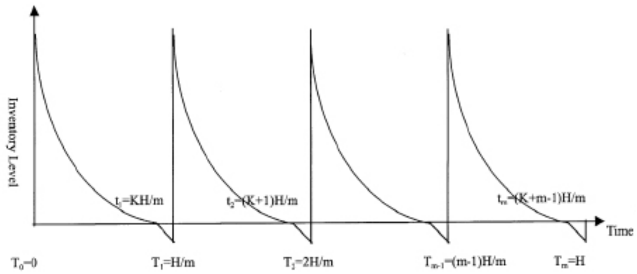

Figure 3. Pictorial representation of the inventory cycles with Shortage.

5.1 Model Formulation

Similarly as previously developed model in section 3, suppose that the planning horizon H is divided into m equal parts of length $T = H/m$ hence, the reorder times over the planning horizon H are $T_j = j\ T$ (j=0 to m) . We further assume that the period for which there is no-shortage in each interval [j T, (j+1) T] is a fraction of the scheduling period T and is equal to $KT(0 < K < 1)$. Shortages occur at time $t_j = (K + j - 1)\ T \quad (j = 1\ to\ m)$, Model is illustrated in Figure 3.

First, let us consider the level of inventory $I(t)$ at time t during the first replenishment cycle, that is $0 \le t \le T$. This inventory level is depleted by the effects of demand and deterioration rate. Therefore, the variation of $I(t)$ with respect to time is governed by the following differential equation:

$$\frac{dI(t)}{dt} = -\alpha - (\theta + bt) I(t), \qquad 0 \le t \le t_1 \tag{10}$$

As to the level of shortage, $S(t)$ during the first replenishment cycle may be represented by the following differential equation, since demand (backlogging) rate is constant.

$$\frac{dS(t)}{dt} = \alpha, \qquad t_1 \le t \le T \tag{11}$$

with the boundary condition $I(t_1) = 0$, the solution of (10) and (11) can be represented by

$$I(t) = \alpha\left[(t_1 - t) + \frac{\theta}{2}\left(t_1^2 - t^2\right) + \frac{\left(b+\theta^2\right)}{6}\left(t_1^3 - t^3\right)\right]$$

$$\times\left[1 - \left\{\theta t + (b - \theta)\frac{T^2}{2}\right\}\right] 0 \le t \le t_1 \tag{12}$$

$$S(t) = \alpha\,(t - t_1), \qquad t_1 \le t \le T \tag{13}$$

Since there are m+1 replenishments in the entire time horizon H, the present values of the total replenishment costs are given by

$$C_R = A\sum_{j=0}^{m} e^{-RT_{j-1}} = A\frac{\left(e^{RH/m} - e^{-RH}\right)}{\left(e^{RH/m} - 1\right)} \tag{14}$$

Let I_1 be the initial inventory level and let S_1 be the maximum shortage quantity during the first replenishment cycle. Using equations (12) and (13), we get

$$I_1 = \alpha\left[t_1 + \frac{\theta t_1^2}{2} + \frac{1}{6}\left(b+\theta^2\right)t_1^3\right] = \alpha\left[1 + \frac{\theta}{2}\frac{KH}{m} + \frac{1\left(b+\theta^2\right)}{6}\frac{K^2H^2}{m^2}\right]\frac{KH}{m} \tag{15}$$

$$S_1 = \alpha\,(T - t_1) = \alpha\left(\frac{H}{m} - \frac{KH}{m}\right) = \alpha\,(1 - K)\frac{H}{m} \tag{16}$$

Because shortages during the first replenishment cycle should be backordered during the next replenishment cycle and shortages during the last cycle is replenished at time $T_m = H$ Therefore, the present values of total purchasing costs during the entire time horizon H are

$$C_P = C\left\{\sum_{j=1}^{m} I_1 e^{-RT_{j-1}} + \sum_{j=1}^{m} S_1 e^{-RT_j}\right\}$$

$$= C\left[\alpha\sum_{j=1}^{m}\left\{1 + \frac{\theta}{2}\frac{KH}{m} + \frac{1}{6}\left(b+\theta^2\right)\frac{K^2H^2}{m}\right\}\frac{KH}{m}\frac{\left(1 - e^{-RH}\right)}{\left(1 - e^{-RH/m}\right)}\right.$$

$$\left. + \alpha\,(1 - K)\frac{H}{m}\frac{\left(1 - e^{-RH}\right)}{\left(e^{RH/m} - 1\right)}\right] \tag{17}$$

The present values of the holding costs during the first replenishment cycle are

$$HC_1 = C_1\int_0^{t_1} I(t) e^{-Rt}\,dt$$

$$= C_1\alpha\int_0^{t_1}\left[(t_1 - t) + \frac{\theta}{2}\left(t_1^2 - t^2\right) + \frac{\left(b+\theta^2\right)}{6}\left(t_1^3 - t^3\right)\right]$$

$$\times\left[1 - \left\{\theta t + \left(b - \theta^2\right)\frac{t^2}{2}\right\}\right] e^{-Rt}\,dt$$

$$= \alpha C_1\left[\left\{t_1 + \frac{\theta t_1^2}{2} + \frac{\left(b+\theta^2\right)}{6}t_1^3\right\}\left\{\frac{e^{-Rt_1}}{-R} + \frac{1}{R}\right\}\right.$$

$$\left. + \left\{1 + \theta t_1 + \frac{\theta^2 t_1^2}{2} + \frac{\left(b+\theta^2\right)\theta t_1^3}{6}\right\}\left\{\frac{t_1 e^{-Rt_1}}{R} + \frac{e^{t_1}}{R^2} - \frac{1}{R^2}\right\}\right]$$

Hence the present values of the total holding costs during the entire time horizon H are given as

Now, $C_H = \sum_{j=1}^{m} HC_1 e^{-RT_{j-1}} = \alpha C_1 \left[\left\{ \frac{H}{m} + \frac{\theta H^2}{2m^2} + \frac{(b+\theta^2)H^3}{6m^3} \right\} \left(\frac{e^{-RH/m}}{-R} + \frac{1}{R} \right) \right.$

$$\left. + \left\{ 1 + \frac{\theta H}{m} + \frac{\theta^2 H^2}{2m^2} + \frac{(b+\theta^2)\theta H^3}{6m^3} \right\} \left\{ \frac{He^{-RT}}{Rm} + \frac{e^{-RH/m}}{R^2} - \frac{1}{R^2} \right\} \right] \left(\frac{1-e^{-RH}}{1-e^{-RH/m}} \right) \quad (18)$$

Present values of the total shortage costs during the first replenishment cycle are

$$SC_1 = C_2 \int_{t_1}^{T_1} S(t) e^{-Rt} dt = \alpha C_2 \int_{t_1}^{T_1} (t - t_1) e^{-Rt} dt$$

$$= \frac{\alpha C_2}{R^2} \left[R(K-1) \frac{He^{-RH/m}}{m} \left\{ e^{-R(K-1)H/m} - 1 \right\} e^{-RH/m} \right]$$

Hence, the present values of the total shortage costs during the entire time horizon H are

$$C_S = \sum_{j=1}^{m} SC_1 e^{-RT_{j-1}} = \frac{\alpha C_2}{R^2} \left[(K-1)\frac{HR}{m} + \left\{ e^{R(1-K)H/m} - 1 \right\} \right] e^{-RH/m} \frac{(1-e^{-RH})}{(e^{RH/m}-1)} \quad (19)$$

Consequently, the present value of the total variable cost of the system during the entire time horizon H is

$$TC(m,K) = C_R + C_P + C_H + C_S$$

So, the present value of total variable cost during the entire time horizon H is

$$TC(m,K) = AD + C\alpha \left[1 + \frac{\theta}{2}\frac{KH}{m} + \frac{1}{6}(b+\theta^2)\frac{K^2H^2}{m^2} \right] \frac{KH}{m} E + C\alpha (1-K)\frac{H}{m} F$$

$$+ C_1 \alpha \left[\left\{ 1 + \frac{\theta HK}{2m} + \frac{1}{6}(b+\theta^2)\frac{H^2K^2}{m^2} \right\} \frac{KH}{mR} \left(1 - e^{-RKH/m} \right) \right.$$

$$+ \left\{ 1 + \frac{\theta HK}{m} + \frac{\theta^2 H^2 K^2}{2m^2} + (b+\theta^2)\frac{\theta H^3 K^3}{6m^3} \right\} \left\{ \frac{H}{mR} e^{-RKH/m} \right.$$

$$\left. \left. + \frac{e^{-RH/m}}{R^2} - \frac{1}{R^2} \right\} \right] E$$

$$+ \frac{C_2 \alpha}{R^2} \left[(K-1)\frac{HR}{m} + \left\{ e^{R(1-K)H/m} - 1 \right\} \right] F \quad (20)$$

where $D = \frac{e^{RH/m} - e^{-RH}}{e^{RH/m} - 1}$; $E = \frac{1-e^{-RH}}{1-e^{-RH/m}}$; $F = \frac{1-e^{-RH}}{e^{RH/m}-1}$;

5.2 Optimal Solution Procedure

The present value of total variable cost function TC(m ,K) is a function of two variables K and m where K is a continuous variable and m is a discrete variable. For a given value of m, the necessary condition for $TC(m,K)$ to be minimized is $dTC(m,K)/dK = 0$ which gives

$$dTC(m,K)/dK = -\frac{c(1-e^{-HR})H\alpha}{\left(-1+e^{\frac{HR}{m}}\right)m} + \frac{C_2(1-e^{-HR})H\alpha \left\{ -\frac{HRe^{\frac{HR(1-K)}{m}}}{m} + \frac{H(K-1)}{m} + \frac{HK}{m} \right\}}{\left(-1+e^{\frac{HR}{m}}\right)mR^2}$$

$$+ \frac{c(1-e^{-HR})HK\alpha \left\{ \frac{H\theta}{2m} + \frac{H^2K(b+\theta^2)}{18m^2} \right\}}{\left(-1+e^{\frac{HR}{m}}\right)m} + \frac{c(1-e^{-HR})H\alpha \left\{ 1 + \frac{HK\theta}{2m} + \frac{H^2K^2(b+\theta^2)}{36m^2} \right\}}{\left(-1+e^{\frac{HR}{m}}\right)m}$$

$$+ \left(\frac{\alpha C_1 (1-e^{-HR})}{\left(1-e^{-\frac{HR}{m}}\right)} \right) \left(\frac{KH\left(1-e^{-\frac{HKR}{m}}\right)\left(\frac{H\theta}{2m} + \frac{H^2K(b+\theta^2)}{3m^2}\right)}{mR} + \frac{H\left(1-e^{-\frac{HKR}{m}}\right)\left(1 + \frac{HK\theta}{2m} + \frac{H^2K^2(b+\theta^2)}{6m^2}\right)}{mR} \right.$$

$$+ \frac{H^2 e^{-\frac{HKR}{m}} K \left(1 + \frac{HK\theta}{2m} + \frac{H^2K^2(b+\theta^2)}{6m^2} \right)}{m^2} + \left(\frac{-1}{R^2} + \frac{e^{-\frac{HR}{m}}}{R^2} + \frac{He^{-\frac{HKR}{m}}}{mR} \right)$$

$$\left. \left(\frac{H\theta}{2m} + \frac{H^2K\theta^2}{m^2} + \frac{H^3K^2\theta(b+\theta^2)}{2m^3} \right) - \frac{H^2 e^{-\frac{HKR}{m}} \left(1 + \frac{HK\theta}{2m} + \frac{H^2K^2\theta^2}{m^2} + \frac{H^3K^3\theta(b+\theta^2)}{6m^3} \right)}{m^2} \right) = 0 \quad (21)$$

Furthermore, Theorem 2 in the appendix shows that $d^2TC(m,K)/dK^2$ is positive. So, for a given positive integer m, the optimum value of K can be obtained from Eq. (21) by using the Newton-Raphson method. Now we let (m^*,K^*) denote the optimal solution to $TC(m,K)$ and let $(m, K(m))$ denote the optimal solution to $TC(m,K)$ when m is given. If $\tilde{m}$ is the smallest integer such that $TC(\tilde{m}, K(\tilde{m}))$ less than each value of $TC(\tilde{\tilde{m}}, K(\tilde{\tilde{m}}))$ in the interval $\tilde{m}+1 \le \tilde{\tilde{m}} \le \tilde{m}+10$. Then we take $(\tilde{m}, K(\tilde{m}))$ as the optimal solution to $TC(m,K)$. Hence $(\tilde{m}, K(\tilde{m})) = (m^*,K^*)$. using the optimal solution

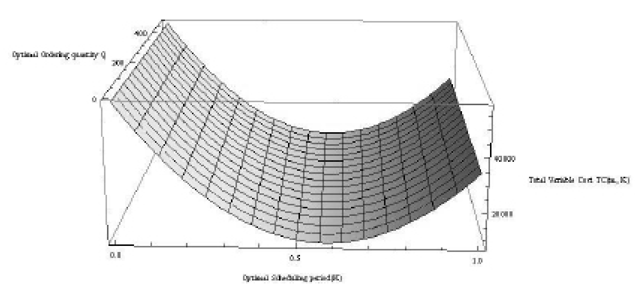

Figure 4. A graphical representation showing the convexity of function TC(m,K).

procedure described above, we can find the optimal order quantity and backorder to be

$$Q=\alpha\left[1+\frac{\theta}{2}\frac{KH}{m}+\frac{\left(b+\theta^{2}\right)}{6}\frac{K^{2}H^{2}}{m^{2}}\right]\frac{KH}{m}$$

$$BQ=\alpha\left(1-K\right)\frac{H}{m}$$

Respectively, by Equations. (15) and (16).

Table 3

M	TC	K	Q	BQ
1	34061.5	0.340757	3097.34	3955.46
3	13377.8	0.540745	1329.13	918.51
4	10783.7	0.566533	995.73	650.2
6	8625.95	0.586718	652.507	413.282
8	7781.37	0.594208	482.445	304.344
13	7083.35	0.6005	290.843	184.385
15	6989.14	0.601511	250.833	159.396
18	6920.66	0.602474	207.884	132.509
20	6903.81	0.602904	186.57	119.129
21*	**6901.13***	**0.603078***	**177.47***	**113.406***
22	6901.46	0.603231	169.214	108.21
23	6904.33	0.603366	161.691	103.47
26	6924.77	0.603689	142.66	91.4563
31	6985.12	0.604051	119.258	76.6352
33	7015.26	0.604157	111.913	71.9715
37	7082.3	0.604324	99.6383	64.1637
38	7100.18	0.604359	96.9789	62.4697
39	7118.42	0.604391	94.4577	60.8629
40	7136.99	0.604422	92.0642	59.3368

5.3 Numerical Examples and Sensitivity Analysis

The numerical example is devised here to illustrate the effect of the general model I developed in this paper with the following data:

The demand parametric value are α=600unit/year, the deterioration rate of the on hand inventory per unit time is θ=.20, *A*= $ 120.00, C_1=$1.75 per unit per year, *C*=$2 per unit, *R*=0.50, b=0.05, the time horizon *H*=10 yr.

By using the solution procedure described in the previous section, the computational result are Shown in Table3 From this table ,we see that that the optimal replenishment number m^*=21 ,the total variable cost TC become minimum. During the first replenishment cycle order quantity Q^* = 177.47 and minimum total variable cost $TC^*(m)$ = 6901.13 the optimal Values of backorder BQ = 113.406, we then have the time interval between replenishment is T=(H/m) = 10/21=0.476 year.

5.4 Sensitivity Analysis

Using the numerical example given in the preceding section, the sensitivity analysis of various parameters has been done. The sensitivity analysis with respect to various parameters on order quantity (Q), backorder (BQ) and total system cost (TC) for model with shortages.

Table 4

Parameter	θ=.20										
% Change	-50	-40	-30	-20	-10		10	20	30	40	50
TC	6837.66	6850.55	6863.35	6876.04	6888.64	6901.13	6913.53	6925.83	6938.03	6950.14	6962.16
K	0.616381	0.613672	0.610988	0.608327	0.605691	0.603078	0.600489	0.597922	0.595377	0.592855	0.590355
Q	178.845	178.57	178.295	178.02	177.744	177.47	177.195	176.92	176.645	176.371	176.097
BQ	109.605	110.379	111.146	111.906	112.66	113.406	114.146	114.879	115.606	116.327	117.041
Parameter	**b=.050**										
% Change	-50	-40	-30	-20	-10		10	20	30	40	50
TC	6900.77	6900.84	6900.92	6900.99	6901.06	6901.13	6901.2	6901.27	6901.35	6901.42	6901.49
K	0.603204	0.603179	0.603154	0.603129	0.603103	0.603078	0.603053	0.603028	0.603003	0.602978	0.602952
Q	177.449	177.453	177.457	177.461	177.465	177.47	177.474	177.478	177.482	177.486	177.491
BQ	113.37	113.377	113.385	113.392	113.399	113.406	113.413	113.421	113.428	113.435	113.442

(*Continued*)

Parameter	C_1=1.75										
% Change	-50	-40	-30	-20	-10		10	20	30	40	50
TC	6762.84	6790.6	6818.31	6845.96	6873.57	6901.13	6928.65	6956.12	6983.55	7010.95	7038.3
K	0.612136	0.610221	0.60836	0.60655	0.60479	0.603078	0.601412	0.599789	0.598209	0.596669	0.595169
Q	180.217	179.636	179.071	178.522	177.989	177.47	176.964	176.473	175.994	175.527	175.073
BQ	110.818	111.365	111.897	112.414	112.917	113.406	113.882	114.346	114.797	115.237	115.666
Parameter	**C=2**										
% Change	-50	-40	-30	-20	-10	0	10	20	30	40	50
TC	3749	4383.78	5016.35	5646.75	6275	6901.13	7525.17	8147.14	8767.07	9384.99	10000.9
K	0.662266	0.650217	0.638274	0.626438	0.614707	0.603078	0.591552	0.580125	0.568798	0.557567	0.546433
Q	195.469	191.795	188.16	184.56	180.997	177.47	173.977	170.519	167.094	163.703	160.345
BQ	96.4953	99.9381	103.35	106.732	110.084	113.406	116.7	119.964	123.201	126.409	129.59
Parameter	**A=120**										
% Change	-50	-40	-30	-20	-10		10	20	30	40	50
TC	6619.45	6675.78	6732.12	6788.46	6844.79	6901.13	6957.47	7013.8	7070.14	7126.48	7182.82
K	0.603078	0.603078	0.603078	0.603078	0.603078	0.603078	0.603078	0.603078	0.603078	0.603078	0.603078
Q	177.47	177.47	177.47	177.47	177.47	177.47	177.47	177.47	177.47	177.47	177.47
BQ	113.406	113.406	113.406	113.406	113.406	113.406	113.406	113.406	113.406	113.406	113.406
Parameter	**R=.5**										
% Change	-50	-40	-30	-20	-10	0	10	20	30	40	50
TC	6326.61	7847.83	8061.52	7796.93	7368.4	6901.13	6444.97	6018.3	5626.11	5267.82	4940.75
K	0.605178	0.615722	0.621061	0.620819	0.614822	0.603078	0.585768	0.563217	0.53587	0.504267	0.469016
Q	178.106	181.305	182.927	182.853	181.032	177.47	172.226	165.409	157.162	147.661	137.098
BQ	112.806	109.794	108.268	108.337	110.051	113.406	118.352	124.795	132.609	141.638	151.71
Parameter	**α=600**										
% Change	-50	-40	-30	-20	-10		10	20	30	40	50
TC	3732.25	4366.03	4999.8	5633.58	6267.35	6901.13	7534.91	8168.68	8802.46	9436.23	10070
K	0.603078	0.603078	0.603078	0.603078	0.603078	0.603078	0.603078	0.603078	0.603078	0.603078	0.603078
Q	88.7348	106.482	124.229	141.976	159.723	177.47	195.217	212.964	230.71	248.457	266.204
BQ	56.7031	68.0437	79.3843	90.725	102.066	113.406	124.747	136.087	147.428	158.769	170.109
Parameter	**H=10**										
% Change	-50	-40	-30	-20	-10		10	20	30	40	50
TC	6639.84	6747.54	6809.32	6847.78	6875.92	6901.13	6927.46	6957.04	6990.9	7029.43	7072.71
K	0.604477	0.604247	0.603992	0.603712	0.603408	0.603078	0.602724	0.602345	0.60194	0.601511	0.601056
Q	87.6235	105.42	123.304	141.274	159.33	177.47	195.691	213.993	232.375	250.833	269.367
BQ	56.5033	67.8435	79.2017	90.5801	101.981	113.406	124.858	136.339	147.851	159.396	170.976
Parameter	**C_2=3**										
% Change	-50	-40	-30	-20	-10		10	20	30	40	50
TC	6937.07	6943.76	6941.24	6932.24	6918.47	6901.13	6881.02	6858.73	6834.69	6809.22	6782.58
K	0.490692	0.524116	0.550291	0.571338	0.588626	0.603078	0.615338	0.625869	0.635012	0.643024	0.650103
Q	143.588	153.625	161.508	167.862	173.091	177.47	181.189	184.387	187.167	189.605	191.761
BQ	145.517	135.967	128.488	122.475	117.535	113.406	109.903	106.895	104.282	101.993	99.9706

Results are summarized in Table 4

- Effect of changing deteriorating parameter (θ) is slightly sensitive for order quantity (*Q*), backorder (BQ) and total system cost (TC).
- Effect of changing the unit cost (C), holding cost (C_1) and ordering cost (A) is moderately sensitive for order quantity (*Q*), backorder (BQ) and total system cost (TC).
- Effect of changing consumption rate parameter (α) is highly sensitive for order quantity (*Q*), backorder (BQ) and total system cost (TC).
- Effect of changing consumption rate parameter (R) is highly sensitive for order quantity (*Q*), backorder (BQ) and total system cost (TC).

The main conclusions one can draw from the sensitivity analysis are as follows:

- The order quantity (*Q*) and backorder (BQ) are more sensitive to consumption rate parameter (α), ordering cost (*A*), the unit cost (*C*), discount rate parameter (*R*) and shortage cost (C_2) as compared to deterioration rate (h), and carrying cost (C_1).
- The total system cost (TC) is more sensitive to consumption rate parameter (a), the unit cost (*C*) and discount rate parameter (*R*) as compared to deterioration rate (h), ordering cost (*A*), carrying cost (C_1) and shortage cost (C_2).
- Order quantity and backorder are more sensitive than the total system cost to changes in the parameters of ordering cost (*A*), carrying cost (C_1) and shortage cost (C_2); this indicates that the cost penalty is less for errors in the estimation of these parameters. Manager should estimate these parameters reasonably instead of estimating these parameters with precision.

5.5 Special Case

In this section, we study the important particular case *R*=0, and compare the differences with classical EOQ model. When *R*=0, then the total (undiscounted) variable cost, TC (T,K) is

$$\mathrm{TC(T,K)}=\left\{A+\frac{c_1}{12}\left(-\frac{3}{5}K^5T^5\theta\left(-b+\theta^2\right)\right.\right.$$

$$+\frac{1}{6}K^6T^6\left(b^2-\theta^2\right)-\frac{1}{3}K^3T^3\left(6\,KT\left(b-\theta^2\right)\right.$$

$$+K^2T^2\left(b^2+3b\theta-\theta^3(3+\theta)\right)-6\theta(-1+\theta\,KT)\Big)+6K^2T^2(-1+\theta\,KT)$$

$$-2K^2T^2\left(6+KT\left(b+3\theta+\theta^2\right)\right)(-1+\theta\,KT)+\frac{\theta}{2}K^2T^2+\frac{b+\theta^2}{6}K^3T^3$$

$$\left.+(1\text{-}K)T\right)+\frac{c_2\alpha T^2(1-K)^2}{2}\Bigg\}\left(\frac{\frac{H}{T}\left(\frac{H}{T}+1\right)}{2}\right)$$

The necessary condition for minimum total variable cost is $\frac{dTC(T,K)}{dT}=0,\ \frac{dTC(T,K)}{dK}=0$, giving

$$\frac{dTC(T,K)}{dT}=$$

$$\left\{\left(\frac{1}{2T}\right)H\left(1+\frac{H}{T}\right)\left(c\alpha\left(1+T\,K^2\theta+\frac{1}{2}T^2K^3\left(b+\theta^2\right)\right)\right)\right.$$

$$+\left(\frac{1}{12}\right)\left(6\,T^2K^3\theta+12\,T\,K^2(-1+T\theta)-3\,T^4K^5\theta\left(-b+\theta^2\right)\right.$$

$$+\left(\frac{1}{2}\right)T^4K^4\left(b\,K\,\theta+K\,\theta^3\right)+T^5K^6\left(b^2-\theta^4\right)$$

$$-2\,T^2K^3(-1+T\,K\,\theta)(b+\theta(3+\theta)\Big)$$

$$+2\,T^3K^4\left(\theta^2(-4+T\,K\,\theta)+b\,(2+T\,K\,\theta)\right)$$

$$-2\,T^2K^3\theta\left(6+T\,K\left(b+3\theta+\theta^2\right)\right)\Big)$$

$$-4\,T\,K^2(-1+T\,K\,\theta)\left(6+T\,K\left(b+3\theta+\theta^2\right)\right)\Big)$$

$$-\frac{1}{3}T^3K^3\left(-6\,K\,\theta^2+6\,K\left(b-\theta^2\right)\right.$$

$$+2\,T\,K^2\left(b^2+3\,b\theta-\theta^3(3+\theta)\right)\Big)\Big)$$

$$-T^2K^3(-6\,\theta(-1+T\,X\theta)+6\,T\,K(b-\theta^2)$$

$$+T^2K^2\left(b^2+3\,b\,\theta-\theta^3(3+\theta)\right)\Big)\Big)\Big)\,C_1$$

$$+T(1-K)^2\alpha\,C_2)-\left(\frac{H^2}{2\,T^3}\right)(A+C\,\alpha\,(T(1-K)$$

$$+T\,X+\frac{T^2K^2}{2}\theta+\frac{T^3K^3}{6}(b+\theta^2))+\left(\frac{1}{12}\right)$$

$$\left(6\,T^2K^2(-1+T\,K\,\theta)-\frac{3T^5K^5\theta\,(-b+\theta^2)}{5}\right.$$

$$+\frac{T^6K^6\,(b^2-\theta^4)}{6}+\frac{T^4K^4}{2}\,(\theta^2\,(-4+TK\theta)$$

$$+b\,(2+TK\theta))-2T^2K^2\,(-1+TK\theta)$$

$$(6+TK\,(b+\theta\,(3+\theta)))-\frac{T^3K^3}{3}$$

$$(-6\theta\,(-1+TK\theta)+6TK\,(b-\theta^2)$$

$$+T^2K^2\,(b^2+3b\theta-\theta^3\,(3+\theta))))\,C_1+\frac{T^2\,(1-K)^2\,\alpha\,C_2}{2})$$

$$-\frac{H}{2T^2}\,(1+\frac{H}{T})\,(A+C\,\alpha\,(T\,(1-K)$$

$$+TK+\frac{T^2K^2\theta}{2}+\frac{T^3K^3}{6}\,(b+\theta^2))+\frac{1}{12}\,(6T^2K^2$$

$$(-1+TK\theta)-\frac{3T^5K^5}{5}\,\theta\,(-b+\theta^2)$$

$$+\frac{T^6K^6}{6}\,(b^2-\theta^4)+\frac{T^4K^4}{2}\,(\theta^2\,(-4+TK\theta)+b\,(2+TK\theta))$$

$$-2T^2K^2\,(-1+TK\theta)\,(6+TK\,(b+\theta(3+\theta)))$$

$$-\frac{T^3K^3}{3}(-6\theta\,(-1+TK\theta)$$

$$+6TK\,(b-\theta^2)+T^2K^2\,(b^2+3b\theta-\theta^3\,(3+\theta))))\,C_1$$

$$+\frac{T^2\,(1-K)^2\alpha\,C_2}{2}\}=0 \tag{22}$$

$$\frac{dTC(T,K)}{dK}=\{\frac{1}{2T}H(\frac{H}{T}+1)(\frac{1}{12}C_1\{K^5T^6(b^2-\theta^4)$$

$$-\frac{1}{3}K^3T^3\left(2KT^2(b^2+3b\theta-\theta^4-3\theta^3)\right)+6T(b-\theta^2)$$

$$-6T\theta^2)-K^2T^3(K^2T^2(b^2+3b\theta-\theta^4-3\theta^3)+6KT(b-\theta^2)$$

$$-6\theta(KT\theta-1))-3K^4T^5\theta(\theta^2-b)$$

$$+\frac{1}{2}K^4T^4(bT+T\theta^3)+2K^3T^4(b(KT\theta+2)+\theta^2(KT\theta-4))$$

$$-2K^2T^3(b+\theta(\theta+3))(KT\theta-1)$$

$$-2K^2T^3\theta(KT(b+\theta(\theta+3))+6)-4KT^2(KT\theta-1)$$

$$(KT(b+\theta(\theta+\theta))+6)+6K^2T^3\theta$$

$$+12KT^2(KT\theta-1))+C\,\alpha(\frac{1}{2}K^2T^3(b+\theta^2)+KT^2\theta)$$

$$+C_2(-(1-K))T^2\alpha)\}=0 \tag{23}$$

The optimum value of T can be obtained from expression by using the numerical method. Taking the optimum value of T and K, we can find that the optimum-order quantity and Back order by Eq. (15) and (16) as

$$Q=\alpha\left[1+\frac{\theta KT}{2}+\frac{(b+\theta^2)K^2T^2}{6}\right]KT \tag{24}$$

$$BQ=\alpha\,(1-K)T \tag{25}$$

respectively.

When R=0 and H=1.0, with the same values of different parameters as taken in the numerical example of Section 3.3, Eqs. (22) to (25) yield that the optimal order quantity, Q, Backorder, BQ, replenishment cost, CR, purchasing cost, CP, holding cost, CH ,Shortage Cost, CS and total system cost, TC are

$Q^*=2546.65$; $BQ^*=168.349$ $C_R^*=18.9322$

$C_P^*=856.683$ $C_H^*=1.73174$ $C_S{}^*=11.1785$

$TC^*=888.525$

Using the same parameter values, we find that the optimal-order quantity and all the relative system costs under classical EOQ model are follow:

$Q^*(EOQ)=360.95$; $BQ^*(EOQ)=132.982$;

$C_R^*(EOQ)=199.47$; $C_P^*(EOQ)=3000$

$C_H^*(EOQ)=125.98$; $C_S^*(EOQ)=73.50$;

$TC^*(EOQ)=3398.94$

When we compare the values among Q^*, $C_R{}^*$, $C_P{}^*$, $C_H{}^*$, TC^* and $Q^*(EOQ)$, $C_R{}^*(EOQ)$, $C_P{}^*(EOQ)$, $C_H{}^*(EOQ)$, $TC^*(EOQ)$, we observed that $Q^*> Q^*(EOQ)$, $C_R{}^* < C_R{}^*(EOQ)$, $C_P{}^* < C_P{}^*(EOQ)$, $C_H{}^* < C_H{}^*(EOQ)$, $BQ^*> BQ^*(EOQ)$, $TC^*< TC^*(EOQ)$

This phenomenon is due to deterioration rate. So that when an inventory system incorporates deterioration, the system will increase order quantity to for that accompany lower holding cost. So that the total system cost is lesser than that of the classical EOQ model.

6. Conclusions

This model incorporates some realistic features that are likely to be associated with the inventory of some kinds of goods. First, items deterioration over time is a natural feature for goods. Second, occurrence of shortages and backorder the quantity in inventory is a natural phenomenon in real situations. Third, the DCF approach permits a proper recognition of the financial implication of the opportunity cost in inventory analysis. In keeping with this reality, these factors are incorporated into the present models.

We have given an analytic formulation of the problem on the framework described above and have presented an optimal solution procedure to find optimal inventory replenishment policies. From this research results, we have also recognize that an inventory policy for perishable items which permit backorders result in smaller discounted total cost than a policy which does not permit backorders.

Finally, the sensitivity of the solution to changes in the values of different parameters has been discussed. It is seen that changes in the consumption rate parameter (α), the unit cost (*C*) and unit shortage cost (C_2) have significant effects on the order quantity. The total system cost is sensitive to changes in the consumption rate parameter (α), the unit cost (*C*) and discount rate parameter (*R*). This behavior is different from that of the classical EOQ model. This phenomenon is due to the factor of deterioration.

Appendix

Theorem 1. TC(m) is convex with respect to m.

$$\mathrm{TC}(m)=\Bigg[A+\alpha C\left[T+\frac{\theta T^2}{2}+\left(b+\theta^2\right)\frac{T^3}{6}\right]$$

$$+\alpha C_1\left\{\frac{H}{m}+\frac{\theta H^2}{2m^2}+\left(b+\theta^2\right)\frac{H^3}{6m^3}\right\}\left\{\frac{e^{-RH/m}}{-R}+\frac{1}{R}\right\}$$

$$+\alpha C_1\left\{1+\frac{\theta H}{2m}+\frac{\theta H^2}{2m^2}+\left(b+\theta^2\right)\frac{\theta H^3}{6m^3}\right\}$$

$$\left\{\frac{He^{-RH/m}}{Rm}+\frac{e^{-RH/m}}{R^2}-\frac{1}{R^2}\right\}\Bigg]\cdot\frac{\left(1-e^{-RH}\right)}{\left(1-e^{-RH/m}\right)}$$

Therefore, we have

$$\frac{d\{\mathrm{TC}(m)\}}{dm}=\left(\frac{HR(1-e^{-HR})e^{-\frac{HR}{m}}}{m^2(1-e^{-\frac{HR}{m}})^2}\right)\left\{A+\frac{CH^3\alpha(b+\theta^2)}{6m^3}\right.$$

$$+C_1\alpha\left(\frac{e^{-\frac{HR}{m}}}{R^2}+\frac{He^{-\frac{HR}{m}}}{mR}-\frac{1}{R}\right)\left(\frac{H^3\theta(b+\theta^2)}{6m^3}+\frac{H^2\theta}{2m^2}+\frac{H\theta}{2m}+1\right)$$

$$\left.+C_1\alpha\left(\frac{1}{R}-\frac{e^{-\frac{HR}{m}}}{R}\right)\left(\frac{H^3(b+\theta^2)}{6m^3}+\frac{H^2\theta}{2m^2}+\frac{H}{2m}\right)+\frac{CH^2\alpha\theta}{2m^2}+\frac{CH\alpha}{m}\right\}$$

$$+\frac{(1-e^{-HR})}{(1-e^{-\frac{HR}{m}})}\left\{-\frac{CH^3\alpha(b+\theta^2)}{2m^4}\right.$$

$$+\frac{C_1H^2\alpha e^{-\frac{HR}{m}}\left(\frac{H^3\theta(b+\theta^2)}{6m^3}+\frac{H^2\theta}{2m^2}+\frac{H\theta}{2m}+1\right)}{m^3}$$

$$-\frac{C_1H\alpha e^{-\frac{HR}{m}}\left(\frac{H^3(b+\theta^2)}{6m^3}+\frac{H^2\theta}{2m^2}+\frac{H}{2m}\right)}{m^2}$$

$$+C_1\alpha\left(\frac{e^{-\frac{HR}{m}}}{R^2}+\frac{He^{-\frac{HR}{m}}}{mR}-\frac{1}{R}\right)\left(-\frac{H^3\theta(b+\theta^2)}{2m^4}-\frac{H^2\theta}{m^3}-\frac{H\theta}{2m^2}\right)+$$

$$\left.C_1\alpha\left(\frac{1}{R}-\frac{e^{-\frac{HR}{m}}}{R}\right)\left(-\frac{H^3(b+\theta^2)}{2m^4}-\frac{H^2\theta}{m^3}-\frac{H}{m^2}\right)-\frac{CH^2\alpha\theta}{m^3}-\frac{CH\alpha}{m^2}\right\}=0$$

$$\frac{d^2\mathrm{TC}(m)}{dm^2}=\left(\frac{e^{-HR/m}(1-e^{-HR})H^2R^2}{(1-e^{-HR/m})^2m^4}\right)\left\{\frac{C\alpha(\theta^2+b)H^3}{6m^3}+\frac{C\alpha\theta H^2}{2m^2}+\frac{C\alpha H}{m}+A\right.$$

$$+\alpha\ C_1\left(\frac{1}{R}-\frac{e^{-HR/m}}{R}\right)\left(\frac{(\theta^2+b)H^3}{6m^3}+\frac{\theta H^2}{2m^2}+\frac{H}{m}\right)$$

$$\left.+\alpha\ C_1\left(\frac{He^{-HR/m}}{mR}-\frac{1}{R}+\frac{e^{-HR/m}}{R^2}\right)\left(\frac{\theta(\theta^2+b)H^3}{6m^3}+\frac{\theta H^2}{2m^2}+\frac{\theta H}{2m}+1\right)\right\}$$

$$+\left(\frac{2e^{-2HR/m}(1-e^{-HR})H^2R^2}{(1-e^{-HR/m})^3m^4}\right)\left\{\frac{C\alpha(\theta^2+b)H^3}{6m^3}+\frac{C\alpha\theta H^2}{2m^2}+\frac{C\alpha H}{m}+A\right.$$

$$+\alpha\ C_1\left(\frac{1}{R}-\frac{e^{-HR/m}}{R}\right)\left(\frac{(\theta^2+b)H^3}{6m^3}+\frac{\theta H^2}{2m^2}+\frac{H}{m}\right)\cdot$$

$$\left.+\alpha\ C_1\left(\frac{He^{-HR/m}}{mR}-\frac{1}{R}+\frac{e^{-HR/m}}{R^2}\right)\left(\frac{\theta(\theta^2+b)H^3}{6m^3}+\frac{\theta H^2}{2m^2}+\frac{\theta H}{2m}+1\right)\right\}$$

$$+\left(\frac{2e^{-2HR/m}(1\text{-}e^{-HR})HR}{(1\text{-}e^{-HR/m})^2 m^2}\right)\left\{-\frac{C\alpha(\theta^2+b)H^3}{2m^4}+\frac{C\alpha\theta H^2}{m^3}\right.$$

$$+\frac{\alpha C_1H^2e^{-HR/m}\left(\frac{\theta(\theta^2+b)H^3}{6m^3}+\frac{\theta H^2}{2m^2}+\frac{\theta H}{2m}+1\right)}{m^3}$$

$$-\frac{C\alpha H}{m^2}-\frac{\alpha C_1He^{-HR/m}\left(\frac{\theta(\theta^2+b)H^3}{6m^3}+\frac{\theta H^2}{2m^2}+\frac{\theta H}{2m}\right)}{m^2}$$

$$+\alpha C_1\left(\frac{1}{R}-\frac{e^{-HR/m}}{R}\right)\left(-\frac{(\theta^2+b)H^3}{2m^4}-\frac{\theta H^2}{m^3}-\frac{H}{m^2}\right)$$

$$\left.+\alpha C_1\left(\frac{He^{-HR/m}}{mR}-\frac{1}{R}+\frac{e^{-HR/m}}{R^2}\right)\left(-\frac{\theta(\theta^2+b)H^3}{2m^4}-\frac{\theta H^2}{m^3}-\frac{\theta H}{2m^2}\right)\right\}$$

$$-\left(\frac{2e^{-2HR/m}(1\text{-}e^{-HR})HR}{(1\text{-}e^{-HR/m})^2 m^3}\right)\left\{\frac{C\alpha(\theta^2+b)H^3}{6m^3}+\frac{C\alpha\theta H^2}{2m^2}+\frac{C\alpha H}{m}\right.$$

$$+A+\alpha C_1\left(\frac{1}{R}-\frac{e^{-HR/m}}{R}\right)\left(\frac{(\theta^2+b)H^3}{6m^3}+\frac{\theta H^2}{2m^2}+\frac{H}{m}\right)$$

$$\left.+\alpha C_1\left(\frac{He^{-HR/m}}{mR}-\frac{1}{R}+\frac{e^{-HR/m}}{R^2}\right)\left(\frac{\theta(\theta^2+b)H^3}{6m^3}+\frac{\theta H^2}{2m^2}+\frac{\theta H}{2m}+1\right)\right\}$$

$$+\left(\frac{1\text{-}e^{-HR}}{1\text{-}e^{-HR/m}}\right)\left\{\frac{2C\alpha(\theta^2+b)H^3}{m^5}\right.$$

$$+\frac{R\alpha C_1H^3e^{-HR/m}\left(\frac{\theta(\theta^2+b)H^3}{6m^3}+\frac{\theta H^2}{2m^2}+\frac{\theta H}{2m}+1\right)}{m^5}+\frac{3C\alpha\theta H^2}{m^4}$$

$$-\frac{R\alpha C_1H^2e^{-HR/m}\left(\frac{\theta(\theta^2+b)H^3}{6m^3}+\frac{\theta H^2}{2m^2}+\frac{H}{2m}\right)}{m^4}$$

$$+\frac{2\alpha C_1H^2e^{-HR/m}\left(-\frac{\theta(\theta^2+b)H^3}{2m^4}-\frac{\theta H^2}{m^3}-\frac{\theta H}{2m^2}\right)}{m^3}$$

$$-\frac{3\alpha C_1H^2e^{-HR/m}\left(\frac{\theta(\theta^2+b)H^3}{6m^3}+\frac{\theta H^2}{2m^2}+\frac{\theta H}{2m}+1\right)}{m^4}.$$

$$+\frac{2C\alpha H}{m^3}-\frac{2\alpha C_1He^{-HR/m}\left(-\frac{\theta(\theta^2+b)H^3}{2m^4}-\frac{\theta H^2}{m^3}-\frac{H}{2m^2}\right)}{m^2}$$

$$+\frac{2\alpha C_1He^{-HR/m}\left(\frac{(\theta^2+b)H^3}{6m^3}+\frac{\theta H^2}{2m^2}+\frac{H}{m}\right)}{m^3}$$

$$+\alpha C_1\left(\frac{1}{R}-\frac{e^{-HR/m}}{R}\right)\left(\frac{2(\theta^2+b)H^3}{m^5}+\frac{3\theta H^2}{m^4}+\frac{2H}{m^3}\right)$$

$$\left.+\alpha C_1\left(\frac{He^{-HR/m}}{mR}-\frac{1}{R}+\frac{e^{-HR/m}}{R^2}\right)\left(\frac{2\theta(\theta^2+b)H^3}{m^5}+\frac{3\theta H^2}{m^4}+\frac{\theta H}{m^3}\right)\right\}>0$$

For: $e^{-\frac{HR}{m}}$, $\left(\frac{1}{R^2}+\frac{H}{mR}\right)$, $\left(-\frac{1}{R^2}+\left(\frac{1}{R^2}+\frac{H}{mR}\right)e^{-\frac{HR}{m}}\right)$, $\left(1-e^{-HR}\right)$ and $\left(1-e^{-\frac{HR}{m}}\right)$ all are greater than zero. There will exist a unique solution that satisfies $\frac{\text{dTC(m)}}{\text{dm}}=0$. Note: If $R>\frac{m}{H}$ Thus, we have $\frac{d^2\text{TC(m)}}{\text{dm}^2}>0$. This is the complete proof.

Theorem 2. When m is given TC(m,K) is convex with respect to K.

$$\text{TC(m,K)}=A\frac{(e^{-HR/m}\text{-}e^{-RH})}{(e^{-HR/m}\text{-}1)}+\frac{\pm c}{m(1\text{-}e^{HR/m})}\left\{1+\frac{\theta KH}{2m}+\frac{(b+\theta^2)K^2H^2}{36m^2}\right\}KH(1\text{-}e^{-RH})$$

$$+\frac{\alpha cH(1-K)(1-e^{-RH})}{m(1-e^{-RH/m})}+\frac{B\alpha}{mR}\left(KH(1+\frac{\theta KH}{2m}+\frac{(b+\theta^2)K^2H^2}{6m^3})(1-e^{-\frac{RKH}{m}})\right.$$

$$\left.+(1+\frac{\theta HK}{2m}+\frac{\theta^2K^2H^2}{2m^2}+\frac{(b+\theta^2)\theta K^3H^2}{6m^3})(\frac{He^{-RKH/m}}{Rm}+\frac{e^{-RH/m}}{R^2}-\frac{1}{R^2})\right)\frac{(1-e^{-RH})}{(1-e^{-HR/m})}$$

$$+\frac{\alpha Hd}{mR^2}\left(\frac{(K-1)HK}{m}+(e^{-\frac{R(k-1)H}{m}}-1)\right)\frac{(1-e^{-RH})}{(1-e^{-\frac{RH}{m}})}$$

Therefore, we get

$$\frac{d\{TC(m,K)\}}{dK}$$

$$=\frac{B\alpha(1-e^{-HR})}{1-e^{-HR/m}}\left\{\frac{H^2Ke^{-HR/m}}{m^2}\left(\frac{H^2K^2(b+\theta^2)}{6m^2}+\frac{HK\theta}{2m}+1\right)\right.$$

$$+\frac{H(1-e^{-HR/m})}{mR}\left(\frac{H^2K^2(b+\theta^2)}{6m^2}+\frac{HK\theta}{2m}+1\right)$$

$$+\frac{HK(1-e^{-HRK/m})}{mR}\left(\frac{H^2K(b+\theta^2)}{3m^2}+\frac{H\theta}{2m}\right)+\left(\frac{He^{-HRK/m}}{mR}+\frac{e^{-HR/m}}{R^2}-\frac{1}{R^2}\right)$$

$$\left(\frac{H^3K^2\theta(b+\theta^2)}{2m^3}+\frac{H^2K\theta^2}{m^2}+\frac{H\theta}{2m}\right)$$

$$-\frac{H^2e^{-HKR/m}}{m^2}\left(\frac{H^3K^3\theta(b+\theta^2)}{6m^3}+\frac{H^2K^2\theta^2}{2m^2}+\frac{HK\theta}{2m}+1\right)$$

$$+\frac{cH\alpha(1-e^{-HR})}{m(1-e^{-HR/m})}\left(\frac{H^2K^2(b+\theta^2)}{36m^2}+\frac{HK\theta}{2m}+1\right)$$

$$+\frac{cH\alpha(1-e^{-HR})}{m(1-e^{-HR/m})}\left(\frac{H^2K(b+\theta^2)}{18m^2}+\frac{H\theta}{2m}\right)-\frac{cH\alpha(1-e^{-HR})}{m(e^{-HR/m}-1)}$$

$$+\frac{dH\alpha(1-e^{-HR})}{mR^2(e^{-HR/m}-1)}\left(-\frac{H\,\mathrm{R}e^{H(1-K)R/m}(b+\theta^2)}{m}+\frac{H(K-1)}{m}+\frac{HK}{m}\right)\Bigg\}=0$$

$$\frac{d^2\{TC(m,K)\}}{dK^2}=\frac{B\alpha(1-e^{-HR})}{1-e^{-HR/m}}\left\{\left[\left(\frac{H^3K(1-e^{-HRK/m})(b+\theta^2)}{3m^3R}\right)\right.\right.$$

$$+\frac{2H^2e^{-HKR/m}}{m^2}\left(\frac{H^2K^2(b+\theta^2)}{6m^2}+\frac{HK\theta}{2m}+1\right)$$

$$+\frac{2H^2Ke^{-HKR/m}}{m^2}\left(\frac{H^2K(b+\theta^2)}{3m^2}+\frac{H\theta}{2m}\right)+\frac{2H^2(1-e^{-HKR/m})}{mR}$$

$$\left(\frac{H^2K(b+\theta^2)}{3m^2}+\frac{H\theta}{2m}\right)$$

$$-\frac{H^3KRe^{-HKR/m}}{m^3}\left(\frac{H^2K^2(b+\theta^2)}{6m^2}+\frac{HK\theta}{2m}+1\right)$$

$$-\frac{2H^2e^{-HKR/m}}{m^2}\left(\frac{H^3K^2\theta(b+\theta^2)}{2m^3}+\frac{H^2K\theta^2}{m^2}+\frac{H\theta}{2m}\right)$$

$$+\frac{H^3Re^{-HKR/m}}{m^3}\left(\frac{H^3K^3\theta(b+\theta^2)}{6m^3}+\frac{H^2K^2\theta^2}{2m^2}+\frac{HK\theta}{2m}+1\right)$$

$$+\left(\frac{He^{-HKR/m}}{mR}+\frac{e^{-HR/m}}{R^2}-\frac{1}{R^2}\right)\left(\frac{H^3K\theta(b+\theta^2)}{m^3}+\frac{H^2\theta^2}{m^2}\right)\Bigg]\Bigg\}$$

$$+\frac{cH^3K\alpha(1-e^{-HR})}{18m^3(1-e^{-HR/m})}+\frac{2cH\alpha(1-e^{-HR})}{m(1-e^{-HR/m})}\left(\frac{H^2K(b+\theta^2)}{18m^2}+\frac{H\theta}{2m}\right)$$

$$+\frac{dH\alpha(1-e^{-HR})}{mR^2(e^{HR/m}-1)}\left(\frac{H^2\,\mathrm{R}^2e^{H(1-K)R/m}(b+\theta^2)}{m^2}+\frac{2H}{m}\right)>0$$

For $e^{\frac{HR}{m}}, e^{-\frac{H(-1+2K+m)R}{m}}, \left(1-e^{-HR}\right), e^{\frac{HKR}{m}}, e^{-\frac{H(-1+2K)R}{m}}$ *and* $\left(-\frac{1}{R^2}+\frac{H}{mR}\right)$ all are greater than zero. There will exist a unique solution K^* that satisfies $\frac{dTC(m,K^*)}{dK}=0$ When $0<K^*<1$. Note: If $R>\theta$, $R>\frac{m}{H}$ Thus, we have $\frac{d^2TC(m,K^*)}{dK^2}>0$. This is the complete proof.

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