An inventory Model for seasonal Goods with time Dependent Seasonal Revenue and Profit Loss in Fuzzy Sense

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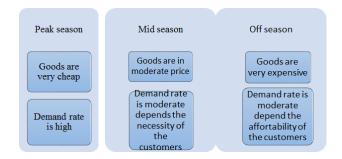
Abstract

Background/Objective: To develop fuzzy set theory and the fuzziness in the inventory problem, the purpose is to find the optimal order quantity corresponds to the total cost and the associated cost also fuzzy. **Methods/Statistical Analysis:** This approach is to find the quantity which has the minimum cost with maximum profit. When the profit gained from selling one unit of the item without deterioration and other shortage cost must be avoided. **Findings:** Mathematical model has been developed in two steps. (1) The Fuzzy economic order quantity and with the fuzzy total cost, (2) The Fuzzy revenue and the related with fuzzy net profit, (3)Fuzzy economic order quantity and the total cost values with seasonal/ time dependent revenue corresponding to the profits. **Applications/Improvements:** Our aim is to find total cost and profit with fuzzy sense. Numerical examples are given and sensitivity analysis is carried out to conclude the result.

Keywords: Economic Order Quantity, Fuzzy Profit, Mid-Season and Off Seasons, On Season, Revenue, Total Cost, Trapezoidal Fuzzy Numbers

1. Introduction

Seasons are given by god as well as weather conditions heavy rain fall no rainfall will affects the seasons some times. Seasonal goods are vegetables, root vegetables, fruits, spices, grains and all eatables etc. In the season time all vendors will come into the business with attractive prices, quality of the product may vary, but availability of the product in high level with less price. In this paper we divide the seasons into three on season, midseason and off season. In the offseason goods are in very expensive price some vendors are will away from the trade by the time demand will be moderate depend the affordability of the customers, in midseason goods are in different price depends the necessity of the customers.



1.2. Representation of Seasons.

The fuzzy optimization was writtenvery first by¹. The process through certain situations over time related to deterioration and customers demand²,observed that when a product is introduced in the market consumers buying behavior and categorize them according to their affordability to purchase³.Fuzzy models constitute a

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wide range of useful tools in assessing the business promotions⁴. Various researchers made their writings in developing inventory models using fuzzy concept are as follows. The inventory problem with unexpected time⁵. The economic order quantity formula by wilson's corresponds to the fuzziness with the additional costs in the fuzzy set environment⁶. Fuzzy models for the newsboyproblem⁷. The model with trapezoidal fuzzy number⁸.A method to find the fuzzy profit when the demand with seasons and selling cost are represented asfuzzy numbers9. An economic order quantity model with fuzzy numbers depends on time, demand and cost coefficients¹⁰. An Economic order quantity model withreliability and demand-dependent unit cost¹¹. The case of revenue and seasonal variable quantity in consumer products¹². Inventory model for decaying items keeping selling price fuzzy sense and developing the model for total relevant cost also the triangular fuzzy numbers¹³. Optimization of fuzzy production inventory model¹⁴. Model for multiple items with profit finding, to maintain appropriate methods used to handle the shortages¹⁵. The mathematical model for buyer and seller also developed the weibull deterioration used to find the realistic solutions¹⁶. Multilevel inventory techniques used for reduce the relevant cost¹⁷. Two parameter weibulldistribution used to reduce the deterioration¹⁸ Two stage vendor scheduling to minimize relevant cost in different times with triangular fuzzynumbers¹⁹. The standard of manufacturing technique is to obtained optimal fuzzy rule²⁰. ²¹analysed fuzzy trapezoidal rule for average mean value method. Fuzzy optimization plays the main role in data mining and share market tradings^{22,23}.

The objective is to determine the total cost in maximizing the resultant profit by considering the fuzzy demand, ordering, holding, selling Price. The solution procedure is presented using the Left and Right functions of the fuzzy Operations with respect to their corresponding Seasonal Price values.

2. Notation and Assumptions

- H is the Holding cost per unit cycle.
- O is the setup cost per order.
- L_{s_n} is the Length of the plan in on season.
- L_{sm} is the Length of the plan in midseason.
- L_{sf} is the Length of the plan in off season.

- L is the length of the plan.
- r is the order quantity per cycle.
- D is the total demand.
- D_{s_n} is the Demand in on season.
- D_{s_m} is the Demand in midseason.
- D_{cf} is the Demand in off season.
- SP_s is the selling price in on season.
- Sp_{sm} is the selling price in midseason.
- SP_{se} is the selling price in off season.
- r* is the Economic order quantity.
- $\tilde{T}c$ is the Fuzzy total cost.
- \tilde{H} is the Fuzzy Holding cost.
- \tilde{O} is the Fuzzy Ordering cost.
- . Re is the Fuzzy total Revenue.
- $\tilde{D}q$ is the Fuzzy demand quantity.
- \widetilde{Sp} is the selling price.
- $\tilde{N}p$ is the Fuzzy Net Profit.

2.1 Fuzzy Operators,

- is the Fuzzy addition.
- is the Fuzzy Subtraction.
- is the Fuzzy Multiplication.
- the fuzzy division.

2.2 Assumptions

- Seasons are fixed time Horizon.
- Selling cost is fixed.
- Shortages and Deteriorations avoided.
- Ordering and holding cost are constant.
- Sn is the on season is fixed for 3 months.
- Sf is the off season is fixed for 3 months.
- Sm_(1,2) is the midseason is fixed for

3. Formulation of the Inventory Model

3.1Economic Order Quantity By the Wilson EOQ formula,

$$EOQ = r^* = \sqrt{\frac{2OD}{HL}}$$
(1)

3.2. For Seasonal Goods

Now we are applying fuzzy variables for Economic order quantity formula:

(10)

(12)

$$[EOQ]_{Sn} = (r^*)_{Sn} = \sqrt{\frac{2OD}{H(Sp)_{Sn}}}$$
(2)

$$[EOQ]_{Sm} = (r^*)_{Sm} = \sqrt{\frac{2OD}{H(Sp)_{Sm}}} \qquad Here, Sm = (Sm_1, Sm_2)$$
(3)

$$[EOQ]_{Sf} = (r^*)_{Sf} = \sqrt{\frac{2OD}{H(Sp)_{Sf}}}$$

$$\tag{4}$$

3.2.1 Total Cost

Total cost is addition of ordering and holding cost respectively,

$$\tilde{T}c = \frac{HL_r}{2} + \frac{OD}{r} \quad and \quad r = r^*$$
(5)

3.3 Fuzzy Inventory Model

L et $\tilde{H} = (\tilde{H}_1, \tilde{H}_2, \tilde{H}_3, \tilde{H}_4)$ and $\tilde{O} = (\tilde{O}_1, \tilde{O}_2, \tilde{O}_3, \tilde{O}_4)$ are trapezoidal fuzzy numbers and operator on total cost, then we get,

By using fuzzy operators,

(ie)
$$Lr^{*} = \sqrt{\frac{2D(\tilde{O}_{1} + \tilde{O}_{2} + \tilde{O}_{3} + \tilde{O}_{4})}{(\tilde{H}_{1} + \tilde{H}_{2} + \tilde{H}_{3} + \tilde{H}_{4})L}}$$
(6)

From(5),
$$\tilde{T}c = \left(\tilde{H} \otimes \frac{L_r}{2}\right) \oplus \left(\tilde{O} \otimes \frac{D}{r}\right)$$
 (7)
 $\tilde{T}c = \left(\left(\tilde{H}_1, \tilde{H}_2, \tilde{H}_3, \tilde{H}_4\right) \otimes \left(\frac{L_r}{2}\right)\right) \oplus \left(\left(\tilde{O}_1, \tilde{O}_2, \tilde{O}_3, \tilde{O}_4\right) \otimes \left(\frac{D}{r}\right)\right)$
 $\tilde{T}c = \left(\tilde{H}_1 \frac{L_r}{2}, \tilde{H}_2 \frac{L_r}{2}, \tilde{H}_3 \frac{L_r}{2}, \tilde{H}_4 \frac{L_r}{2}\right) \oplus \left(\tilde{O}_1 \frac{D}{r}, \tilde{O}_2 \frac{D}{r}, \tilde{O}_3 \frac{D}{r}, \tilde{O}_4 \frac{D}{r}\right)$
 $\tilde{T}c = \left(\tilde{H}_1 \frac{L_r}{2} + \tilde{O}_1 \frac{D}{r}, \tilde{H}_2 \frac{L_r}{2} + \tilde{O}_2 \frac{D}{r}, \tilde{H}_3 \frac{L_r}{2} + \tilde{O}_3 \frac{D}{r}, \tilde{H}_4 \frac{L_r}{2} + \tilde{O}_4 \frac{D}{r}\right)$
 $\tilde{T}c = \left(\Omega_1, \Omega_2, \Omega_3, \Omega_4\right)(say)$ (8)

In the Left and Right form,

$$A_{L}(\alpha) = \Omega_{1} + (\Omega_{2} - \Omega_{1})\alpha$$
$$A_{R}(\alpha) = \Omega_{4} - (\Omega_{4} - \Omega_{3})\alpha$$

By using signed distance method,

$$\tilde{T}c = \frac{1}{2} \int_{0}^{1} \left[A_{L}(\alpha) + A_{R}(\alpha) \right] d\alpha$$

$$\begin{split} \tilde{\tau}c &= \left[(\tilde{H}_{1} + \tilde{H}_{4}) \frac{L_{r}}{2} + (\tilde{O}_{1} + \tilde{O}_{4}) \frac{D}{r} \right] + \frac{1}{4} \left[(\tilde{H}_{2} + \tilde{H}_{3} - \tilde{H}_{1} - \tilde{H}_{4}) \frac{L_{r}}{2} + (\tilde{O}_{2} + \tilde{O}_{3} - \tilde{O}_{1} - \tilde{O}_{4}) \frac{D}{r} \right] \\ \tilde{\tau}c &= \left[\left(2(\tilde{H}_{1} + \tilde{H}_{4}) + (\tilde{H}_{2} + \tilde{H}_{3} - \tilde{H}_{1} - \tilde{H}_{4}) \frac{L_{r}}{8} \right) + \left(2(\tilde{O}_{1} + \tilde{O}_{4}) + (\tilde{O}_{2} + \tilde{O}_{3} - \tilde{O}_{1} - \tilde{O}_{4}) \frac{D}{4r} \right) \right] \\ \tilde{\tau}c &= \left(\tilde{H}_{1} + \tilde{H}_{2} + \tilde{H}_{3} + \tilde{H}_{4} \right) \frac{L_{r}}{8} + \left(\tilde{O}_{1} + \tilde{O}_{2} + \tilde{O}_{3} + \tilde{O}_{4} \right) \frac{D}{4r} \\ \tilde{\tau}c &= Z(r) \quad (say) \end{split}$$
(9)

Now we are applying fuzzy variables for the total cost formula with LR form we get,

$$Z(r^{*}) = \left(\tilde{H}_{1} + \tilde{H}_{2} + \tilde{H}_{3} + \tilde{H}_{4}\right) \frac{\left(L_{r}\right)}{8} + \left(\tilde{O}_{1} + \tilde{O}_{2} + \tilde{O}_{3} + \tilde{O}_{4}\right) \frac{D}{4L_{r}}$$

Total cost which includes the seasonal cost on season, mid-season and off season respectively,

$$\left[Z(r^{*})\right]_{Sn} = \left(\tilde{H}_{1} + \tilde{H}_{2} + \tilde{H}_{3} + \tilde{H}_{4}\right) \frac{(L_{r})_{Sn}}{8} + \left(\tilde{O}_{1} + \tilde{O}_{2} + \tilde{O}_{3} + \tilde{O}_{4}\right) \frac{D}{4(L_{r})_{Sn}}$$

$$\begin{split} & (10) \\ & [Z(r^{*})]_{Sm} = \left(\tilde{H}_{1} + \tilde{H}_{2} + \tilde{H}_{3} + \tilde{H}_{4}\right) \frac{(L_{r})_{Sm}}{8} + \left(\tilde{O}_{1} + \tilde{O}_{2} + \tilde{O}_{3} + \tilde{O}_{4}\right) \frac{D}{4(L_{r})_{Sm}} \\ & (11) \\ & [Z(r^{*})]_{Sf} = \left(\tilde{H}_{1} + \tilde{H}_{2} + \tilde{H}_{3} + \tilde{H}_{4}\right) \frac{(L_{r})_{Sf}}{8} + \left(\tilde{O}_{1} + \tilde{O}_{2} + \tilde{O}_{3} + \tilde{O}_{4}\right) \frac{D}{4(L_{r})_{Sf}} \end{split}$$

Here, Lr_r^*

3.4 To find Revenue with Profit with Fuzzy Sense

Let

$$\begin{split} \tilde{D} &= \left(\tilde{D}_{Sn}, \tilde{D}_{Sm1}, \tilde{D}_{Sm2}, \tilde{D}_{Sf} \right), \quad \tilde{Sp} = \left(\left(\tilde{Sp} \right)_{Sn}, \left(\tilde{Sp} \right)_{Sm1}, \left(\tilde{Sp} \right)_{Sm2}, \left(\tilde{Sp} \right)_{Sf} \right) \\ and \quad z(r^{*}) &= \left(\tilde{Z} \left(r^{*} \right)_{Sn}, \tilde{Z} \left(r^{*} \right)_{Sm1}, \tilde{Z} \left(r^{*} \right)_{Sm2}, \tilde{Z} \left(r^{*} \right)_{Sf} \right) \end{split}$$

are the trapezoidal fuzzy numbers, then,

With the use of Operations on trapezoidal multiplication we can get,

Revenue = (Selling Price) \bigotimes (Demand quantity)

$$\begin{split} \tilde{\mathsf{R}}\mathsf{e} &= \left(\left(S\tilde{p} \right)_{Sn}, \left(S\tilde{p} \right)_{Sm1}, \left(S\tilde{p} \right)_{Sm2}, \left(S\tilde{p} \right)_{Sf} \right) \otimes \left(\tilde{D}_{Sn}, \tilde{D}_{Sm1}, \tilde{D}_{Sm2}, \tilde{D}_{Sf} \right) \\ &= \left(\left(S\tilde{p} \right)_{Sn} \cdot \tilde{D}_{Sf}, \left(S\tilde{p} \right)_{Sm1} \cdot \tilde{D}_{Sm1}, \left(S\tilde{p} \right)_{Sm2} \cdot \tilde{D}_{Sm2}, \left(S\tilde{p} \right)_{Sf} \tilde{D}_{Sn} \right) \end{split}$$

$$Profit (or) \ Loss = 1 \oslash Lr \bigotimes \left[\tilde{R}_{e} \ominus \left(\tilde{Z}(r^{*}) \right] \right]$$
(13)

 $\begin{aligned} &Profit/loss = 1 \oslash L_{sn} \bigotimes \left[\tilde{R}_{e} \ominus \tilde{Z}(r*)_{sn} \right] \oplus 1 \oslash L_{sm} \\ &\bigotimes \left[\tilde{R}_{e} \ominus \tilde{Z}(r*)_{sn} \right] 1 \oslash L_{sf} \bigotimes \left[\tilde{R}_{e} \ominus \tilde{Z}(r*)_{sn} \right] \\ &= 1 \oslash L_{sn} \bigotimes \left[(\tilde{S}\tilde{p})_{sn} \widetilde{D}_{sf} \ominus Z(r*)_{sn} \right] \oplus 1 \oslash L_{sm} \bigotimes \left[(\tilde{S}\tilde{p})_{sm} \widetilde{D}_{sf} \ominus Z(r*)_{sn} \right] \\ &\Leftrightarrow \tilde{P}_{sm} \widetilde{D}_{sf} \ominus Z(r*)_{sm} \right] 1 \oslash L_{sf} \bigotimes \left[(\tilde{S}\tilde{p})_{sf} \widetilde{D}_{sn} \ominus Z(r*)_{sf} \right], \\ &here, S_{m} = S_{ml}, S_{m2} \end{aligned}$ (14)

We can use the above mathematical model for finding the profit through the total cost along with economic order quantity. The following examples are derived from the Equations (6), (13) and (14), Simple Mathematical calculations used for finding the values of Total cost, Revenue, Profit as well as corresponding graphical images has been drawn.

4. Problem Calculations

4.1 Example: 1

Let us consider an optimization problem with the following data:

 $\tilde{O} = (40, 25, 25, 10)$

 $\tilde{H} = (36,22.5,22.5,9)$ and $\widehat{D}_s = (400,250,3250,100)$ and $\widetilde{Sp} = (5,12.5,12.5,20)$ And Find the Profit.

4.1.1 Sensitivity Analysis Table (1)

4.1,2 Graphical Representation (1)



Graph 1. Graphical Representation.

Table 1.Sensitivity analysis

4.2 Example2

Let us consider an inventory system with the following data:

 $\tilde{O} = (150, 120, 90, 60)$

 $\tilde{H} = (160, 120, 80, 40)$ and $\widehat{D}_{i} = (500, 400, 300, 200)$ and $\widetilde{Sp} = (10, 20, 30, 40)$ Find the Profit/Loss.

4.2.1 Sensitivity Analysis Table (2)

4.2.2 Graphical Representation (2)



Graph 2. Graphical Representation.

5 Conclusion and Future Research

From the Tables (1) and (2), it can be observed that,

- Demand quantity is increases when in the on season.
- Demand quantity is moderate when in the midseason.
- Demand quantity is decreases when in offseason.
- Selling Price is increases when offseason.
- Selling Price is decreases when in on season.
- Profit is increases when in the on season.
- Loss is increase when in the offseason.
- Revenue is increases when demand increase.
- Ordering and holding costs are same in both a tables but Profit percentage is varies due to demand, affordability of the customers and seasons.

	D	\tilde{O}	\widetilde{H}	r*	Z(r *)	Re	Ñр
On season	400	40	36	13.333327	900.00029	8000	2366.66657
Midseason1	250	25	22.5	6.666666	1012.50008	3125	704.16664
Midseason2	250	25	22.5	6.666666	1012.50008	3125	704.16664
Off season	100	10	9	3.333333	787.500071	500	-95.833357

 Table 2.
 Sensitivity analysis

	D	õ	Ĥ	r*	Z(r *)	Re	Ñp
On season	500	150	160	10.246951	5623.47526	20000	4792.17491
Midseason1	400	120	120	6.480741	6804.777447	12000	1731.740851
Midseason2	300	90	80	4.582576	7102.991885	6000	-367.66396
Off season	200	60	40	3.240370	6480.741397	2000	-1493.58046

A fuzzy inventory model for demand with constant selling price, ordering cost and holding cost has been developed with fuzzy sense. Trapezoidal fuzzy models are found for profit/loss. Without shortages and deterioration has been found. A numerical example is also given in support the theory. A future research is to extend the model under deterioration with shortages.

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