

Fair Allocation of Reactive Power Contribution by Generators in Ancillary Services Market

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Abstract

Objectives: In re-structured power systems, knowledge of reactive power supplied by the ancillary services and generators is required separately for accurate and fair cost allocation. The main objective is to price the reactive power based on each generator contributions and static capacitor contributions to individual loads. **Methods/Analysis:** A Y-bus matrix method is used to obtain the contribution of individual buses to loads. But this bus contribution doesn't give the fair cost allocation. So, the individual generator contributions should be found. **Findings:** To find the individual generator contributions a new methodology is proposed in this paper. This new methodology takes the bus inflows and outflows into consideration for finding the bus injections at the generator buses. Using this data, the contribution of generators to the loads can be obtained which gives rise to fair and transparent cost allocation. **Applications:** This method is applied on simple 5 bus system and on IEEE 30 bus system and obtained results are tabulated and compared with other existing methods.

Keywords: Ancillary Services, Contribution of Generators, Cost Allocation, Reactive Power, Re-structured Power Systems

1. Introduction

Deregulation is fairly a new paradigm in the electric power industry. Unlike other industries the goal of deregulation is to enhance competition, bring new choices and economic benefits. In deregulated environment generation, transmission and distribution are made as separate entities. There will be a number of generating companies which will compete to supply power to customers so that customers can make a choice of particular generating unit¹. In deregulation, pricing of electricity is done in bidding process i.e. generators bid for highest price and customers bid for lowest price. Loads consume both real and reactive power but when pricing is done solely based on real power, fair allocation is not possible. For example, one customer may consume more real power and less reactive power, whereas other customers may consume more reactive power and less real power. If the pricing is done only for real power, then the customer who consumes more reactive power and violates the system network cannot be penalized. Hence,

it is necessary to find a proper cost allocation method duly taking into account the reactive power contribution².

Reactive power allocation is one of the most important research areas on which researchers are working. Derivation of optimal spot prices using Optimal Power Flow (OPF) is done keeping in view different customer characteristics, metering and communication costs. It has been shown that spot pricing can improve efficiency in production and also get maximum social benefits but here, the objective function includes maximization of social benefits instead of minimizing the production cost³. A real time pricing of reactive power is carried out using modified OPF model and shown that this real time pricing gives better result when compared to traditional power factor penalties scheme in terms of efficiency⁴. But here, incremental cost of reactive power is done. Some research is also done on LMP (Locational Marginal Pricing). An ACOPF (AC Optimal Power flow) based on direct optimization technique is used for LMP calculation. A comparison of both AC and DC power flows is done for determining the marginal price details

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of each bus⁵. Another reference suggested that recovering of operational cost alone will not give the total price of reactive power, capital cost of that reactive power production should also be added i.e. static capacitor investment. But this method neglected production cost of generators reactive power⁶. Research has been focussed on finding the cost of reactive power consumed by loads as follows. First one was tracing the power flow from individual generators to loads using upstream and downstream looking algorithms. A supplement charge allocation in the open access using topological distribution factors has been discussed^{7,8}. Contributions of generators and loads using graph theory approach which makes the power system into state graphs consisting of commons and links was proposed. It uses the recursive equations to solve the real and reactive power that each generator contributes to each load⁹. The above methods are based on a strong proportionality assumption which can neither be proved nor disproved. A review of cost allocation methodologies is given¹⁰. Allocation of transmission supplementary charge to loads is done using the MW-Mile method. Here, the charge is separated based on used capacity and un-used capacity. The charge of un-used capacity is uniformly given to loads¹¹. Reactive power pricing is done based on incremental cost analysis, by determining the change in line flows for 1MVar increase in generation. Further research is carried out on finding the incremental cost using OPF¹². MW-Mile method used for real power pricing does not take into account the reactive power. To account for the reactive power also, MVA-Mile method has been used. But the MVA-Mile method resulted in lesser revenue reconciliation. To overcome this drawback a new MW+MVar-Mile method is developed which gives more revenue reconciliation when compared to MVA-Mile method, as both real and reactive powers are taken separately. This method also facilitates in finding the appropriate share of real and reactive powers¹³. But in this method the contributions of particular generator are not given. To find the static VAr compensator location optimally, a new index for reactive power spot price was given¹⁴. The contribution of generators to the load reactive power has not been considered in this method. Provision of reactive power is treated as an ancillary service and is derived from the distributed generation using Renewable sources¹⁵. A modified Y-bus matrix method is used for pricing the

reactive power. When a shunt capacitor is added the total contribution of that particular shunt capacitor is shown more than its generation in this method¹⁶. Line charging susceptances are taken as separate reactive power sources using improved and enhanced Y-bus method to match the added shunt capacitor generation and its contribution¹⁷. Using the contributions, reactive power cost allocation is done¹⁸.

It is worth noting that, in the above discussed Y-bus based methods, the obtained contributions are not actual generator contributions. They represent the bus contributions to which the generator is connected. So, a new method is proposed for finding the generator actual contributions. In this proposed method the bus inflows, outflows, generator injections and generator ratios are found out to calculate the generator contributions. Cost allocation based on these generator contributions will thus give fair cost allocation.

Organization of this paper is as follows, Section 2 explains the mathematical modelling of the cost allocation methodology, Section 3 presents proposed algorithms, Section 4 depicts the results obtained from the case studies on a simple 5 bus system and on IEEE 30 bus system and Section 5 summarizes the conclusions derived.

2. Mathematical Modelling

Starting from the converged N-R load flow solution, using the converged voltages (VL_q) all loads are converted into equivalent admittances YL_q as follows.

$$YL_q = \frac{1}{VL_q} \left(\frac{SL_q}{VL_q} \right)^* \quad (1)$$

where, SL_q is the apparent power of load bus q .

Consider network equation using bus admittance matrix for n bus system with g generators and l loads.

$$Y_{bus} \times V_{bus} = I_{bus} \quad (2)$$

Let bus no 1 to g are generator buses and $g+1$ to n are load buses. The bus admittance matrix can be divided into 4 sub matrices as follows.

$$\begin{bmatrix} YGG & YGL \\ YLG & YLL \end{bmatrix} \begin{bmatrix} VG \\ VL \end{bmatrix} = \begin{bmatrix} IG \\ IL \end{bmatrix} \quad (3)$$

where,

$$\begin{aligned}
 YGG &= \begin{bmatrix} Y_{1,1} & \cdots & Y_{1,g} \\ \vdots & \ddots & \vdots \\ Y_{g,1} & \cdots & Y_{g,g} \end{bmatrix} & YGL &= \begin{bmatrix} Y_{1,g+1} & \cdots & Y_{1,n} \\ \vdots & \ddots & \vdots \\ Y_{g,g+1} & \cdots & Y_{g,n} \end{bmatrix} \\
 YLG &= \begin{bmatrix} Y_{g+1,1} & \cdots & Y_g + Y_{1,g} \\ \vdots & \ddots & \vdots \\ Y_{n,1} & \cdots & Y_{n,g} \end{bmatrix} & YLL &= \begin{bmatrix} Y_{g+1,g+1} & \cdots & Y_{g+1,n} \\ \vdots & \ddots & \vdots \\ Y_{n,g+1} & \cdots & Y_{n,n} \end{bmatrix} \\
 VG &= \begin{bmatrix} V_1 \\ \vdots \\ V_g \end{bmatrix} & VL &= \begin{bmatrix} V_{g+1} \\ \vdots \\ V_n \end{bmatrix} & IG &= \begin{bmatrix} I_1 \\ \vdots \\ I_g \end{bmatrix} & IL &= \begin{bmatrix} I_{g+1} \\ \vdots \\ I_n \end{bmatrix}
 \end{aligned}$$

Now, this re-arranged Y-bus matrix is modified by adding load equivalent admittance YL from eq1 to YLL matrix. So, a new YLL is obtained which is represented as YLLn

Hence, equation 3 can be written as,

$$\begin{bmatrix} YGG & YGL \\ YLG & YLLn \end{bmatrix} \begin{bmatrix} VG \\ VL \end{bmatrix} = \begin{bmatrix} IG \\ IL \end{bmatrix} \quad (4)$$

From above,

$$[YGG] [VG] + [YGL] [VL] = [IG] \quad (5)$$

$$[YLG] [VG] + [YLLn] [VL] = [IL] \quad (6)$$

As all loads are converted into equivalent admittances there will be no current injection at those load buses. So, IL is equated to zero. Therefore

$$[YLG] [VG] + [YLLn] [VL] = [0] \quad (7)$$

$$[VL] = -[YLLn]^{-1} + [YLG] [VG] \quad (8)$$

Eq 8 gives the relation between load bus voltages and generator bus voltages. Let

$$[YB] = -[YLLn]^{-1} [YLG] \quad (9)$$

$$[VL] [YB] [VG] \quad (10)$$

Now, the voltage of particular load bus q can be written as a function of all generator bus voltages.

$$VL_q = \sum_{p=1}^g YB_{q,p} \times VG_p \quad (11)$$

Let,

$$\Delta VL_{p,q} = \text{Im} \{ \Delta VL_{p,q} \times IL_q \} \quad (12)$$

The reactive power contribution from bus p to the load q can be written as

$$QL_{p,q} = \text{Im} \{ \Delta VL_{p,q} \times IL_q^* \} \quad (13)$$

Here, IL_q is the load current obtained using the formula

$$IL_q = \left(\frac{SL_q}{VL_q} \right) \quad (14)$$

Here, it is to be noted that, eq 13 gives bus contributions to loads rather than generator contributions. But to find the actual generator contributions the following procedure is proposed in this paper.

Let,

$Q_{out}(p)$ is the reactive power outflow from bus p.

$Q_{in}(p)$ is the reactive power inflow to bus p.

$Q_{inj}(p)$ is the pth bus reactive power injection.

From the reactive power outflows $Q_{out}(p)$ and inflows $Q_{in}(p)$ of each generator bus (say pth generator), the bus injections are calculated using the formula

$$Q_{inj}(p) = Q_{out}(p) - Q_{in}(p) \quad (15)$$

It is worth mentioning here that half line charging susceptances $[(Y)c(p)]$ also supply reactive power $[(Q)c(p)]$ to the bus and that value is calculated as,

$$Q_c(p) = V_p^2 \times Y_c(p) \quad (16)$$

The total bus injection $Q_{inj}(p)$ is the sum of reactive power generated by the synchronous generator $QG(p)$ and the half line charging susceptances $Q_c(p)$

$$Q_{inj}(p) = QG(p) + Q_c(p)$$

To obtain $(QG(p))$ the reactive power generation of the synchronous generator, (Q_c) the reactive power supply from half line charging is subtracted from the bus injections at the generator buses.

$$QG(p) = Q_{inj}(p) - Q_c(p) \quad (17)$$

A simple example to explain the calculation of inflows, outflows and injections using Figure 1.

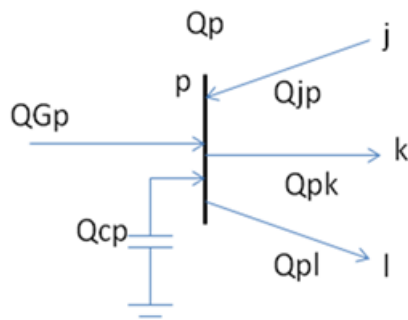


Figure 1. Illustration of bus inflows and outflows.

In Figure 1, p^{th} generator bus is connected to j , k and l^{th} buses. The p^{th} bus has two outflows (Q_{pk} and Q_{pl}) and one inflow (Q_{jp}).

$$Q_{\text{out}}(p) = Q_{pk} + Q_{pl}$$

$$Q_{\text{in}}(p) = Q_{jp}$$

The bus injection is,

$$Q_{\text{inj}}(p) = Q_{pk} + Q_{pl} - Q_{jp}$$

Actual reactive power generation is,

$$Q_G(p) = Q_{\text{inj}}(p) \times Q_c(p)$$

Having obtained the actual reactive power contribution by the individual generators, the percentage of each generator contribution to the reactive power outflow from that bus can be determined using the formula,

$$Q_{G_{\text{ratio}}}(p) = \frac{Q_{G_p}}{Q_{\text{out}}(p)} \quad (18)$$

From eq 13 reactive power contributions of any generator bus has been calculated. But this contribution is not totally from the synchronous generator connected to that bus. The ratio calculated above facilitates in knowing the actual contribution from any generator to load.

Now, to obtain actual generator contributions these generator ratios are multiplied with particular bus contributions.

$$QGL_{p,q} = Q_{G_{\text{ratio}}}(p) \times Q_{L_{p,q}} \quad (19)$$

After finding the actual generator reactive power contributions, the cost allocation of reactive power can be given as,

$$GGL_{p,q} = QGL_{p,q} \times C_p \quad (20)$$

Here, $GGL_{p,q}$ is the reactive power cost paid by load q to generator p . $QGL_{p,q}$ is the contribution of reactive power from generator p to load q . C_p is per unit cost of reactive power for each generator.

$$TC_q = \sum_{p=1}^g GGL_{q,p} \quad (21)$$

Eq 21 gives the total cost paid by each load to all the generators.

3. Proposed Algorithm

Step 1: Get the N-R load flow solution

Step 2: The matrix $[Y][V] = [I]$ is rearranged into the form of $\begin{bmatrix} YGG & YGL \\ YLG & YLL \end{bmatrix} \begin{bmatrix} VG \\ VL \end{bmatrix} = \begin{bmatrix} IG \\ IL \end{bmatrix}$.

Step 3: All loads are converted into equivalent admittances YL using equation 1.

Step 4: In admittance matrix the YLL sub matrix is modified by adding equivalent admittance (YL) to it and the modified matrix is $\begin{bmatrix} YGG & YGL \\ YLG & YLLn \end{bmatrix} \begin{bmatrix} VG \\ VL \end{bmatrix} = \begin{bmatrix} IG \\ IL \end{bmatrix}$.

Step 5: Load voltages are calculated as a function of generator voltages using the equations 5 to 8.

Step 6: using eq 13 the bus reactive power contributions are obtained.

Step 7: From the converged power flow solution, calculate the inflows and outflows of each generator.

Step 8: Bus injections are calculated using outflows and inflows using the eq 15.

Step 9: Now, the reactive power supplied by half line charging susceptances are calculated using the eq 16.

Step 10: Using eq's 17 and 18 actual reactive power generation and ratios are obtained respectively.

Step 11: Actual generator contributions can be obtained using the eq 19.

Step 12: Now, the pricing can be done to loads by these generator contributions using the equations 20 and 21.

So, now a fair cost allocation can be done to loads.

4. Results and Discussions

The proposed methodology is conducted on a simple five bus system. The Figure for the bus system is shown in Figure 2.

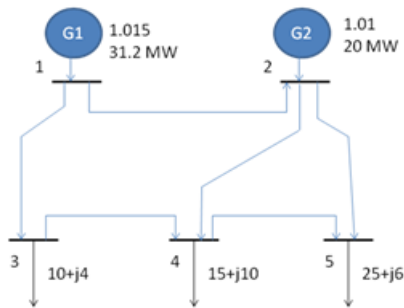


Figure 2. 5 bus system.

The line data of this system is shown in Table 1.

Table 1. Line data

Line no.	From bus	To bus	R(pu)	X(pu)	B/2 (pu)
1	1	2	0.05	0.25	0.01
2	1	3	0.09	0.35	0.01
3	2	4	0.25	0.6	0.01
4	2	5	0.06	0.25	0.01
5	3	4	0.15	0.5	0.01
6	4	5	0.1	0.45	0.01

Using Y-bus matrix method the reactive power bus contributions obtained are tabulated in Table 2.

Table 2. Bus reactive power contributions to each load

Load bus	Reactive power of load (MVAR)	Supplied by bus1 (MVAR)	Supplied by bus2 (MVAR)
3	4	2.93	1.07
4	10	2.79	7.22
5	6	0.47	5.53

Actual generator contributions:

Obtained bus injections by calculating bus inflows and outflows are depicted in Table 3.

Table 3. Bus injections at each generator bus in MVAR

Buses	Bus outflow (Q_{out}) MVAR	Bus inflow (Q_{in}) MVAR	Bus injections (B_{inj})
Bus1	4.974	1.081	3.893
Bus2	13.053	0	13.053

At generator buses by subtracting the reactive power supplied by the half line charging susceptances (Q_c) from the bus injections, actual reactive power generations can be obtained which are shown in Table 4.

Table 4. Reactive power generation at each generator bus in MVAR

Buses	Bus injections MVAR	Q_c MVAR	generator reactive power. (G_{inj})
1	3.893	2.06	1.833
2	13.053	2.04	11.013

Generator ratios give the ratio of individual generator reactive power to the reactive power outflow from that bus. This generator ratio is shown in Table 5.

Table 5. Generator ratios

Buses	Bus outflow (Q_{out}) MVAR	generator reactive power. (G_{inj}) MVAR	Generator ratios (G_{ratios})
1	4.974	1.833	0.3685
2	13.053	11.013	0.8437

Finally, the actual generator contributions and cost allocations are obtained and depicted in Table 6.

Here, the cost factor is taken as 150 Rs/MVAR from reference¹⁸. The total load reached by the generators is 13.9338MVAR. The remaining load is supplied by the half line charging susceptances present at the load buses.

The reactive power supplied by half line charging susceptances present at load buses is 5.69 MVAR.

Using proportional sharing principle, cost allocations and reactive power contributions are shown in Table 7.

Table 6. Actual generator reactive power contributions in MVAR and cost allocation in Rs/hr

Bus no	Reactive power of load (MVAR)	Supplied by G1 (MVAR)	Cost of G1 (Rs/hr)	Supplied by G2 (MVAR)	Cost of G2 (Rs/hr)	Total cost (Rs/hr)
3	4	1.0803	162.05	0.9013	135.19	297.24
4	10	1.0269	154.04	6.0857	912.85	1066.89
5	6	0.1725	25.875	4.6671	700.06	725.94

Table 7. Allocating the reactive power contribution and cost by the Proportional sharing method

Bus no	Reactive power of load (MVAR)	Supplied by G1 (MVAR)	Cost of G1 (Rs/hr)	Supplied by G2 (MVAR)	Cost of G2 (Rs/hr)	Total cost (Rs/hr)
3	4	3.64	462	0.36	231	693
4	10	1.87	882	8.14	577.5	1459.5
5	6	0	528	6	357	885

So, now when the bus contributions are taken the total cost of loads 3, 4 and 5 is 3001.5 Rs/hr.

But when generator contributions are taken the total cost allocation to loads by generators is 2090.07 Rs/hr.

Following the above tables, some observations are discussed below

- Knowledge of reactive power flow along with real power flow is also necessary for calculating the reactive power consumption shared by the loads.
- From the power flow analysis, it is noticed that real power flow in lines is larger than reactive power flow. When losses are considered the reactive power loss in lines is greater than real power loss because of more line reactance than line resistance. So, it is likely inappropriate to neglect the losses or share them in

the same proportion as that of flows which is done in proportional sharing method.

- By comparing the Tables 6 and 7, the cost allocation to loads is reduced when the actual generator contributions are taken. But when bus contributions are taken cost allocation is more which is not fair.

To illustrate the effectiveness of the proposed methodology an IEEE 30 bus system is taken for the allocation of reactive power cost. The input data of the system is taken from¹⁶. There are 6 generators and as an ancillary service a static capacitor is kept at bus 7 with the capacity of 10MVAR.

The contributions of all generators along with static capacitor of IEEE 30 bus system is shown in Table 8.

Table 8. Reactive power allocation to loads with capacitor added in MVAR

load bus (j)	Load	From G1	From G2	From G3	From G4	From G5	From G6	From G7
8	1.2	0.368	0.321	0.362	0.018	0.075	0.025	0.031
9	0	0	0	0	0	0	0	0
10	2	0.088	0.251	0.663	0.216	0.115	0.325	0.342
11	1.6	0.304	0.614	0.598	-0.01	0.146	-0.1	0.046
12	7.5	0.409	0.881	1.552	0.462	0.283	3.149	0.765
13	0	0	0	0	0	0	0	0
14	1.6	0.086	0.206	0.387	0.099	0.073	0.497	0.254
15	2.5	0.127	0.304	0.603	0.158	0.11	0.768	0.43
16	1.8	0.09	0.211	0.454	0.163	0.079	0.589	0.214
17	5.8	0.257	0.675	1.763	0.796	0.295	1.244	0.77
18	0.9	0.044	0.113	0.248	0.062	0.045	0.227	0.161
19	3.4	0.159	0.414	0.981	0.309	0.172	0.798	0.567
20	0.7	0.033	0.087	0.21	0.061	0.037	0.15	0.122
21	11	0.449	1.295	4.073	1.231	0.619	1.411	2.123
22	0	0	0	0	0	0	0	0
23	1.6	0.067	0.16	0.375	0.106	0.062	0.409	0.421
24	6.7	0.201	0.538	1.614	0.468	0.242	0.876	2.761
25	11	0.383	1.289	3.201	0.104	5.617	0.095	0.211
26	2.3	0	0	0	0	0	0	2.3
27	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0
29	0.9	0.005	0.021	0.055	-0.01	0.011	0	0.814
30	1.9	0.01	0.039	0.086	-0.02	0.021	-0.01	1.765

Here, we get the bus contributions.

For finding actual generator contributions, the outflows, inflows, injections, reactive power by line charging susceptances (Q_c) and generator ratios are calculated and tabulated in Table 9.

Table 9. The outflows, inflows, generator injections, Q_c and generator ratios

bus no	Q _{out} (MVar)	Q _{in} (MVar)	Q _{inj} (MVar)	Q _c (MVar)	QG (MVar)	QG _{ratio}
1	6.181	1.73	4.451	5.03	-0.579	-0.09
2	11.833	0.564	11.269	9	2.269	0.192
3	22.709	0	22.709	2.71	19.999	0.881
4	4.169	0	4.169	0	4.169	1
5	7.884	0	7.884	3.29	4.594	0.583
6	1.557	0	1.557	0	1.557	1
7	10.6	0.64	9.96	0	9.96	0.94

In Table 8, contributions of bus to the loads are given which is the sum of generator contribution and that of line charging susceptance contribution. To determine the actual contribution of generators, multiply the values in Table 8 with QG_{ratio} obtained in Table 9. The calculated values of generator contributions are given in Table 10.

Table 10. The contribution of generation to each load in MVar

Load bus	Actual load	Reactive power supplied by generators to load
8	1.2	0.4633
9	0	0
10	2	1.5539
11	1.6	0.637
12	7.5	5.9939
13	0	0
14	1.6	1.2486
15	2.5	1.9723
16	1.8	1.4321
17	5.8	4.5953
18	0.9	0.7027
19	3.4	2.6698
20	0.7	0.5461
21	11.2	8.7946
22	0	0
23	1.6	1.302
24	6.7	5.5875
25	10.9	6.7054
26	2.3	2.162
27	0	0
28	0	0
29	0.9	0.8173
30	1.9	1.7329

From above Table 10, it is noted that the total reactive load is 64.5MVar, the total generators reactive power supplied to load is 48.91MVar. Remaining reactive power is supplied by line charging susceptances present at the load buses i.e 14.53MVar.

The cost allocation to loads by the contributions of generators is done by multiplying these contributions with the cost factor.

5. Conclusion

An attempt has been made for reactive power price in deregulated electricity market. Effect of half-line charging susceptances in the reactive power generation and allocation has been investigated. It has been demonstrated that the usage of Y-bus based methods give the reactive power contributions made by a bus but they do not refer to actual generator contributions. A new method has been proposed to find actual contributions by the individual generators to loads. Concept of ratio of outflow from a bus to generator reactive powers has been introduced and these ratios facilitates in calculation of actual generator contribution to loads. Effectiveness of the proposed method has been illustrated with the case study conducted of IEEE 30 bus test system.

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