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Mathematical Modeling of Wastewater Vibrational Filtering Process

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Abstract

Background/Objectives: The article describes the development of the tools to design wastewater vibrational filtering process based on mathematical modeling of the motion of contaminating particles. **Methods/Statistical Analysis:** A method of reducing contamination of the filtered surface through forced vibration is considered. To build a mathematical model, we consider the mode of motion of particles without contact with each other. In this case, hydrodynamic resistance forces effect on the particle determined by the difference of water flow velocity at the location of the particle and own velocity of the particle. **Findings:** The proposed mathematical model is based on representation of solid contaminations of the liquid being treated as a set of spherical particles of different diameters, which allows taking into account the basic properties of interaction of particles with the liquid and contact interaction between the particles and between a particle and the filter screen. Dimensionless performance evaluation criteria of vibrational filtering are proposed, as well as dimensionless parameters to evaluate the properties of the filtering surface with respect to the particle diameter. **Applications/Improvements:** As a result of mathematical modeling, nonlinear dependence of the average frequency of passage of particles through the filter surface on vibration parameters and on dimensionless parameters of the filter is established. The range of parameters of the filtering element is found ensuring maximum efficiency of the filtering process.

Keywords: Amplitude, Elastic Modulus, Filtration, Friction Coefficient, Mathematical Model, Vibration

1. Introduction

Wastewater treatment of mechanical-engineering enterprises, containing a significant amount of solid inclusions is associated with great technical difficulties and serious material costs as rapid contamination of the filtering elements results in lower performance of treatment facilities. Therefore, great attention is paid to the improvement of the filtering equipment. One way to enhance equipment performance is the use of vibrational filtering elements. This problem is described in detail in[1–6]. At the same time, the issues related to the development of the tools to design such systems based on mathematical modeling of the motion of solid inclusions are not well studied. This work involves the development of a mathematical model of the interaction of the liquid being treated with the filtering surface and studying the effect of vibration on deposition of solid inclusions on the surface of the filters at different modes of vibrational exposure. Studies were conducted to determine the conditions under which throughput of the filter is maximized.

2. Design Scheme and Mathematical Model

Many works, including[7–16] describe the issues of motion simulation of multiphase media. This article considers a model that describes the filtering process taking into account the deposition of solid particles on the surface of the vibrational fine filter, see Figure 1. The filter comprises housing 2 with the filtering element 1 on the elastic base, equipped with de-balancing vibration exciter 5. Contaminated water 3 enters the housing and after passing through the filtering surface reaches the consumer 4.

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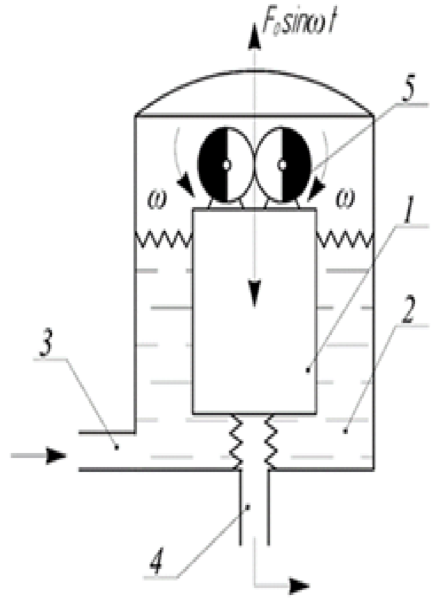

Figure 1. Scheme of the vibrating fine filter for contaminated wastewater.

The presence the exciter 5 with de-balances rotating with frequency ω allows generating vertical oscillations of the filtering surface with frequency ω and amplitude A.

We assume that there are solid particles with different diameters in water. A mathematical model is based on the hypothesis that the particles are not destroyed in interaction with the filter. The forces acting on the particles in contact are associated with elastic interaction between the particles and the surface of the filter, energy dissipation and dry friction. Figure 2 shows a diagram of interaction of the particles with the filtering surface in the process of filtration[17–19].

We assume that contaminating particles have a shape of an elastic sphere and are characterized by two averaged sizes D and d. Large particles with a diameter D does not pass through the filter and fine having a size d smaller than the filter holes may pass through the screen together with water being treated. Thus, fine particles are markers showing the process of treated water transfer through the filtering surface.

To build a mathematical model, we consider the mode of motion of particles without contact with each other. In this case, hydrodynamic resistance forces effect on the particle determined by the difference of water flow velocity at the location of the particle and own velocity of the particle. The scheme of such interaction is shown in Figure 3. Here, the following designations are adopted:

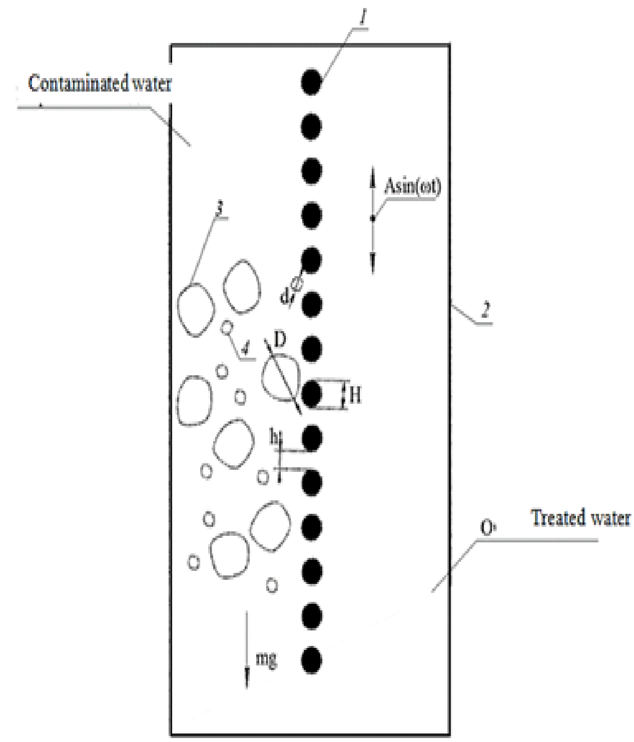

Figure 2. . Scheme of the filtering process: 1 – filtering surface; 2 –filter housing; 3 – particles of solid inclusions; H – fiber diameter of the filtering element; h – minimum distance between the fibers of the filtering element; A – oscillation amplitude; ω – frequency of oscillations; d – diameter of small particles; D – diameter of large particles.

m_i – mass of i-th particle; v_i – particle velocity vector; V_i – known vector of water velocity; r_{Oi} – radius vector determining position of the particle in Oxyz space related to the filter housing.

By applying Newton's second law, we write the equation of motion of a free particle in vector form (1).

$$\begin{aligned} m_i \frac{dv_i}{dt} &= R_i + G_i \\ R_i &= \mu(V_i - v_i) \\ G_i &= (0, 0, m_i g) \\ R_i &= (R_{xi}, R_{yi}, R_{zi}) \end{aligned} \qquad (1)$$

Where R_i, G_i are the forces acting on the particle from water, weight force; μ is the coefficient of interaction between water and particles, g is acceleration due to gravity. Rotational motion of the particle is neglected.

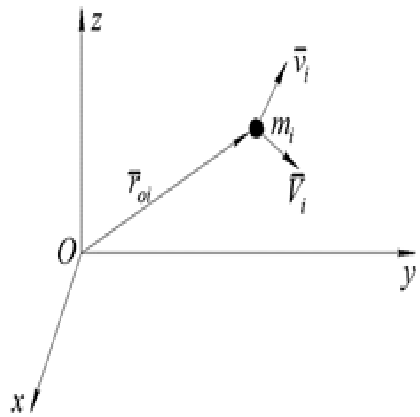

Figure 3. . Scheme of the particle motion in the liquid.

Next, consider the simulation of contact between two particles, which is based on the assumption that the particles are only influenced by the forces of inter particle interaction. We consider the contact of two spherical particles i and j with radiuses R_i and R_j, masses m_i and m_j, moments of inertia I_i and I_j, coordinates of centers of mass (x_i, y_i), (x_j, y_j), linear velocities v_i and v_j and angular velocities ω_i and ω_j, respectively. The distance between the centers of the particles is l_i.

We assume that in collision, the particles contact at one point - point of contact C. P is the contact plane tangential to the particles i and j at point C. Figure 4 shows a scheme of contact interaction of particles with different diameters. The force F_{ij} associated with the contact of two particles is transmitted through the contact patch from one particle to another and includes the normal component F_n, acting along the common normal at point C and the tangential component F_t, acting in the plane of contact P.

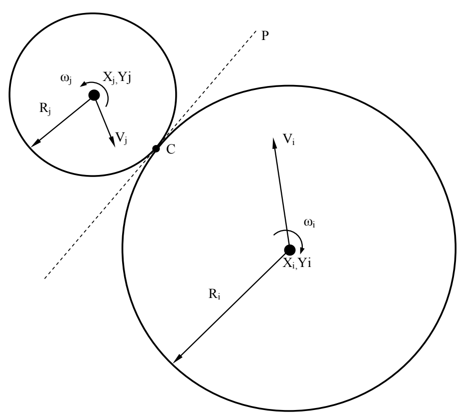

Figure 4. Contact between two spherical particles (beginning of the interaction).

Tangential components do not affect normal components of the motion, if the elastic properties of the materials of two bodies are the same. Let's define all the forces acting on the particle in the inter particle interaction. To do this, we add up all the forces acting on particle *i* at time *t*, which gives a resulting force that determines the center of mass of the particle in contact. Hydrodynamic interaction forces are neglected in this mode.

$$F_t(t) = \sum_{j \in E_i} (F_n + F_t) + m_i g$$

Tangential forces lead to the emergence of rotation moments and summing of all the moments gives the resultant moment.

$$T_t(t) = \sum_{j \in E_i} F_t x n_{ij} R_i$$

The contact between the particles occurs under the condition $l_i = R_i + R_j$ and detachment occurs $F_n = 0$.

In the work, it is assumed that F_n is determined according to the Hooke's law:

$$F_n = \frac{E(l_i - R_i - R_j)S}{l_i}, \quad F_t = f_1 F_n,$$

where E is the modulus of elasticity of the material of particles, f_1 is the coefficient of friction, *S* is the area of contact.

Let's use the Newton's law to record the differential equations of motion of particles in contact, both for progressive and for rotary motion:

$$\begin{aligned} m_i \ddot{x}_i(t) &= F_i(t), \\ I_i \ddot{\varphi}_i &= T_i(t), \end{aligned} \qquad (2)$$

Where $I_i = \frac{2}{3} m_i R_i^2$ in case of a homogeneous sphere.

Particular attention is paid to modeling of the interaction of particles with a large diameter with the filtering surface. Here, as in the contact problem we used a method based on determination of the forces of interaction of the particle with the surface. It is assumed that the surface of the filter is not deformable and the contact of the particle with the mesh occurs under the following condition: $x_i =$ a. The following force is acting on the particle:

$$P_i = \frac{E(x_i - R_i - a)S}{x_i}.$$

Where a is the coordinate determining the location of the filter screen inside the housing. The condition for detachment of the particle from the surface is: $P_i = 0$.

Movement of the particle along the filter screen is under the influence of two forces: Weight force $m_i g$ and friction force P_{ti} determined as follows:

$$P_{ti} = f_2 P_i sign(\dot{\varepsilon}) \text{ at } \dot{\varepsilon} \neq 0, \; P_{ti} = P_{ti}^{o} \text{ at } \dot{\varepsilon} = 0$$

Where f_2 is the coefficient of friction between the particle and the screen surface, ε is the relative velocity of particle movement on the screen, P_t^{i0}=mi ($A\omega^2$-g) is the limit value of the friction force.

Thus, the filtering process in the proposed model is defined by the following parameters: A, ω, m_i, E, f_1,f_2, a, μ, D, d.

To evaluate the effectiveness of the filtering process, we introduce a parameter ν defined as follows:

ν=1/ T

In this equation, the mean time between passage through the screen of small diameter particles T can be determined by the expression:

$$T = \frac{1}{n}\sum_{i=1}^{n} T_i,$$

Where T_i is the time of passage of *i-th* particle, *n* is the number of particles passed through the screen.

Parameter ν essentially determines the time necessary to filter contaminated water. The higher the value, the less time is required to filter a given volume of liquid.

For convenience, later we will use a dimensionless form of parameter $\bar{\nu}$ that is defined as follows:

$$\bar{\nu} = \frac{1}{T\omega}.$$

In addition, let's also introduce two dimensionless parameters: $\bar{\Delta}$ is determined as the ratio of the radius of the small particles to the distance between the characteristic size of the filtering element $\bar{\Delta} = \frac{d}{h}$ and R - determining the properties of the filtering surface:

$$\bar{R} = \frac{h+H}{H}.$$

To integrate the system of differential Equations (1) and (2), a special algorithm was developed that takes into account the change in the right-hand parts, depending on the existence of contact between the particles or lack of it. The accuracy and ease of implementation of the algorithm is of great importance. Under the accuracy we implied local and global accuracy. Local accuracy or approximation error is related to the approximate nature of the numerical integration and is determined by approximation $(\Delta t)^k$, with which the difference scheme approximates the solution of the original differential problem. This requires that when (Δt) tends to zero, the approximate solution converges to the analytical solution. This condition of convergence implies that by reducing the integration step (Δt) we can get a solution with any predetermined accuracy.

3. Results of Modeling

Below are the results of numerical experiments to study the motion on the vibrating surface of the filter for different vibration parameters of the filtering surface. Figure 5 shows the dependence of the efficiency parameter of filtration process ν on the oscillation frequency at different amplitudes. Figure 6 shows graphs with the dependences of the efficiency parameter of filtration process ν on parameter R determining the properties of the filtering surface at different oscillation amplitudes of the separator. Figure 7 shows the dependence of the efficiency parameter of filtration process ν on the size of the filter holes at different oscillation amplitudes.

Analysis of the graphs (Figure 5) shows that both the amplitude and the frequency of surface oscillations significantly affect the duration of the filtration process. It is clearly seen that with the increase in oscillation ampli-

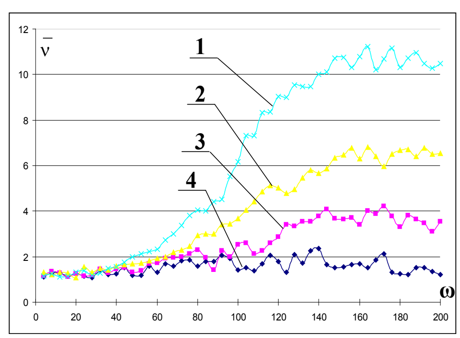

Figure 5. Dependence of the efficiency parameter of filtration process ν on the oscillation frequency of the filter at different amplitudes. 1) A = 0.4 mm; 2) A = 0.3 mm; 3) A = 0.2 mm; 4) A = 0.1 mm;).

tude, the frequency decreases at which the filter efficiency increases. Thus, at the amplitude of A = 0.1 mm, velocity of the particles passing through the filter is almost independent of frequency. If A = 0.4 mm, the behavior of the graphs changes after 20 1/sec. There is a sharp increase in the velocity of passage through the filter with the increase in the frequency of oscillations, but when reaching the frequency of 140, further increase stops and the process stabilizes.

The graphs in Figure 6 show that with the increase in the parameter R and the oscillation amplitude, the efficiency of the filtration process increases. Indeed, a filter

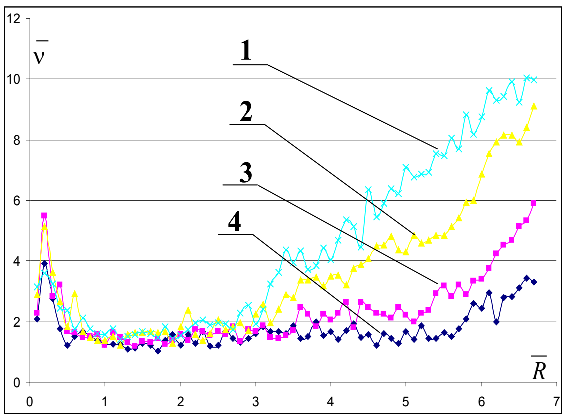

Figure 6. Dependence of the efficiency parameter of filtration process ν on the radius of the separator wire, at different amplitudes of sieve vibration. 1) A = 0.1; 2) A = 0.2; 3) A = 0.3; 4) A = 0.4;).

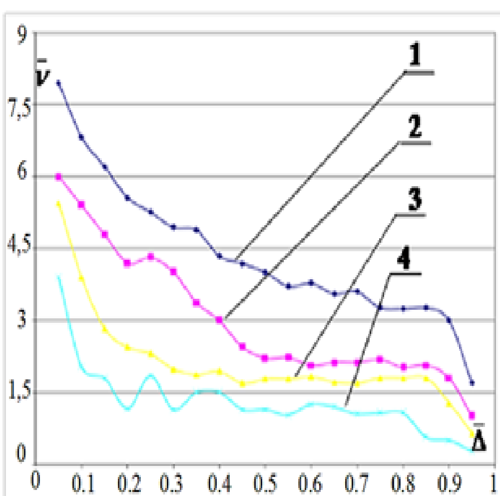

Figure 7. . Dependence of the efficiency parameter of filtration process ν on the specific size of the filtering element at different sieve oscillation amplitudes: 1) A = 0.1; 2) A = 0.2; 3) A = 0.3; 4) A = 0.4.

with a large hole diameter is more efficient. Maximum efficiency of the filtering process is observed at values of R = 6-7. In this case, the value of the efficiency of the filtering process ν exceeds 10, and at the same time, without vibration, this parameter is 1-2.

The graphs in Figure 7 show that with the increase in the parameter $\overline{\Delta}$ and the oscillation amplitude, the efficiency of the filtration process determined by the parameter ν decreases. Indeed, at a given size of the holes of the filtering surface, maximum filter efficiency corresponds to the condition when the particle diameter is much smaller than the size of the hole of the filter screen. With the increase in the particle diameter, filter throughput decreases and within the limit at $\overline{\Delta} = 1$ the filtering process stops.

4. Conclusions

The mathematical model of vibrational filtration is proposed based on representation of solid contaminations of the liquid being treated as a set of spherical particles of different diameters, which allows taking into account the basic properties of interaction of particles with the liquid and contact interaction between the particles and between a particle and the filter screen. The model allows rather realistically reproducing the system behavior under different modes of vibrational movement of the filtering element. Dimensionless performance evaluation criteria of vibrational filtering are proposed, as well as dimensionless parameters to evaluate the properties of the filtering surface with respect to the particle diameter. As a result of mathematical modeling, nonlinear dependence of the average frequency of passage of particles through the filter surface on vibration parameters and on dimensionless parameters of the filter is established. The range of parameters of the filtering element is found ensuring maximum efficiency of the filtering process.

In the future it is planned simulate the filtering process taking into account the properties of a real electric drive of a vibration exciter, the deformability of the filtering surface and to conduct experiments and compare the theoretical data with the experimental ones.

5. References

1. Jatsun SF, Safarov DI, Mishchenko VY, Loktionova OG. Vibratory machines and technology. Baku: Elm; 2004.

2. Safarov DI. The system for wastewater treatment made on the basis of a vibrating filter-coalescer. Ecology Journal of the Central Black Belt Region of the Russian Federation. 2000; 2(1):49–52.
3. Safarov DI. Electromagnetic vibrating filter-coalescer in the system of preparation of coolant-cutting fluid for reuse. Bilgi Journal. Technology series. 2001; 2(1):34–9.
4. Zakharov IS, Safarov DI, Jatsun SF. Designing mechatronic vibration technology machines. Mechatronics, Automation, Control. 2005 Jul; 9(1):39–44.
5. Nigmatulin RI. Fundamentals of mechanics of heterogeneous media. Moscow: Science; 1978.
6. Nigmatulin RI. Dynamics of multiphase media. Moscow: Science; 1987.
7. Savage SB. Instability of unbounded uniform granular shear flow. Journal of Fluid Mechanics. 1992 Aug; 241(1):109–203.
8. Savage SB. Streaming motions in a bed of vibrationally fluidized dry granular material. Journal of Fluid Mechanics. 1988 Sep; 194(1):457–78.
9. Jatsun S, Safarov J. Dynamics of vibrating engine for walking robots. 3rd International Conference GLAWAR; Madrid, Spain. 2000. p. 153–74.
10. Jatsun S, Safarov J. Mathematical model of granular material in a vibrating boiling layers. EUROMECH Colloquium 425; Aberdeen, Scotland. 2001. p. 98–121.
11. Mechanics of granulated media. The theory of rapid motions (coll. of articles). Moscow: Mir; 1985.
12. Jenkins JT. Rapid flows of granular materials. Non-Classical Continuum Mechanics. Cambridge: Cambridge University Press; 1987.
13. Jenkins JT, Savage SB. A theory for the rapid flow of identical, smooth, nearly elastic, spherical particles. Journal of Fluid Mechanics. 1983 May; 130(1):187–202.
14. Luding S, Herrmann HJ, Blumen A. Kinetic theory for granular flow of dense, slightly inelastic Scaling behavior. Physical Review E. 1994; 50(1):3100–8.
15. Lun CKK. Kinetic theory for granular flow of dense, slightly inelastic, slightly rough spheres. Journal of Fluid Mechanics. 1991 Jul; 233(1):539–59.
16. Lun CKK, Bent AA. Numerical simulations of inelastic frictional spheres in simple shear flow. Journal of Fluid Mechanics. 1994 Jan; 258(1):335–53.
17. Lun CKK, Savage SB, Jeffrey PJ, Chepumiv N. Kinetic theories for granular flow: Inelastic particles in Couette flow and slightly inelastic particles in a general flow field. Journal of Fluid Mechanics. 1984 Mar; 140(1):223–56.
18. Patel R, Shah D, Prajapti BG, Patel M. Overview of industrial filtration technology and its applications. Indian Journal of Science and Technology. 2010 Oct; 3(10):1121–27.