Indian Journal of Science and Technology, Vol 9(16), DOI: 10.17485/ijst/2016/v9i16/88766, April 2016 ISSN (Print) : 0974-6846
ISSN (Online) : 0974-5645

Criteria on Existence of Solutions for Fractional Order Impulsive Differential System with Infinite Delay

R. P. Pathak and Piyusha S. Somvanshi*

Department of Mathematics, National Institute of Technology Raipur, Opp Ayurvedic College, GE Rd, Raipur – 492001, Chhattisgarh, India; rppathak.maths@nitrr.ac.in, piyusha.susheel@gmail.com*

Abstract

The paper is mainly concerned with the existence and uniqueness of solution for a class of Cauchy initial value problem of fractional order with impulses and infinite delay. The criteria on existences and uniqueness are obtained via successive approximation and solution operator. Finally an example is given to support our main result.

Keywords: Caputo Fractional Derivative, Existence and Uniqueness, Fractional Impulsive Differential Equation, Infinite Delay

Subject Classification (MSC 2010):34A08, 34A37.

1. Introduction

During the last decades, fractional calculus has been blossomed and grown in pure mathematics as well as scientific applications. But to classify fractional calculus as a young science would be utterly wrong. In fact, the origin of fractional calculus is a far back as classical itself. On the other hand today's mathematical topics which fall under the class of fractional calculus are far from being the "Calculus of fractions" as one might suspect by the notation itself. Instead, integration and differentiation of an arbitrary order would be a better notation for the field of fractional calculus as it is understood today. See[1-5] and reference therein.

In recent years many researchers and scientists have been attracted with the topics related to the existence results for fractional differential systems with delay and impulses. (See[6-8] and references therein).

Motivated by some recent works, we consider the following fractional impulsive differential equations with infinite delay.

$$\begin{cases} D_C^q x(t) = f(t, x(t)), & t \in J = [0,T], t \neq t_k; \\ \Delta x(t_k) = I_k(x(t_k)), & k = 1,2,.......m; \\ x(t) = \varphi(t), & t \in [-\infty, 0], \end{cases} \tag{1}$$

Where D_C^q is the Caputo fractional derivative of order, $0<q<1$, $0 = t_0<t_1<t_2<\ldots.. <t_m< t_{m+1} = T$, $f \in$ ([0,T] × R, R)and $I_k \in \mathbb{C}$ (R, R) are given functions satisfying some assumptions that will be specified later. $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, $x(t_k^+)$ and $x(t_k^-)$ represents the right and left limits of x (t) at $t = t_k$ respectively, and they satisfy that $x(t_k^-) = x(t_k)$. Now define

$$x_t(\theta) = x(t+\theta), \ \theta \in (\infty, 0].$$

Here x_t, represents the history of the state up to present time t.

Further, the construction of paper is in following way. In section 2, we give some preliminaries. In Section 3, we study the existence and uniqueness of mild solutions for the problem (1). At last an example is given to demonstrate the applicability of results in Section 4.

2. Preliminaries

In this section we recall some basic facts, definitions and propositions of fractional calculus which will be needed in the paper. (See[9,10]).

*Author for correspondence

Definition 2.1. (See[11,12]). "*The fractional integral of order q with the lower limit zero for a function f is defined as*

$$I^q f(t)=\frac{1}{\Gamma(q)}\int_0^t \frac{f(s)}{(t-s)^{1-q}}ds, \quad t>0, \quad q>0.$$

Where Γ is the gamma function."

Definition 2.2. (See[11,12]). "*The Riemann-Liouville derivative of order q with the lower limit zero for a function f is defined as:*

$${}^{RL}D^q f(t)=\frac{1}{\Gamma(n-q)}\frac{d^n}{dt^n}\int_0^t (t-s)^{n-q-1}f(s)ds, \qquad n-1<q<n, t>0."$$

Definition 2.3. (See[11,12]). "*The Caputo derivative of order q with the lower limit zero for a functions f is defined as*

$$D_C^q=\frac{1}{\Gamma(n-q)}\int_0^t \frac{f^n(s)}{(t-s)^{q+1-n}}ds, \qquad t>0, \qquad 0<n-1<q<n".$$

Let *J = [0,T]* and *J' = J* \ $\{t_1, t_2, \ldots t_m\}$. We denote ℙℂ (J) = {u : *[0,T]* → R | u ∈ ℂ (J,ℝ), and $u(t_k^-)$ exists and $u(t_k^-)=u(t_k), k=1,2,\ldots,m\}$. Obviously ℙℂ (J) is a Banach space with the norm $||u||=\sup_{t\in J}|u(t)|$."

Lemma 2.4. According to X. Zhang (See[13]), we give following Lemma:

"*Assume that y* ∈ ℂ *([0,T], ℝ) then a function x* ∈ *PC (J) is a solution of Cauchy problem*

$$\begin{cases} D_C^q x(t)=y(t,x(t)), & t\in J=[0,T], t\neq t_k; \\ \Delta x(t_k)=I_k(x(t_k)), & k=1,2,\ldots\ldots.m; \\ x(t)=\varphi(t), & t\in[-\infty,0], \end{cases} \tag{2}$$

Iff x satisfies following equation

$$x(t)=\begin{cases} \varphi(t), \quad t\in[-\infty,0]; \\ \frac{1}{\Gamma(q)}\int_{t_k}^t (t-s)^{q-1}y(s)ds+\sum_{j=1}^k I_j x(t_j) \\ +\sum_{i=0}^{k-1}\frac{1}{\Gamma(q)}\int_{t_i}^{t_i+1}(t_{i+1}-s)^{q-1}y(s)ds, \quad t\in(t_k,t_{k+1}], k=0,1,2,\ldots..m. \end{cases} \tag{3}$$

Proof. Assume that x satisfies the integral equation (3). We have ϕ (0) = 0 and

$$x(t)=\frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}y(s)ds, \qquad t\in[t_0,t_1].$$

since $x(t_1^+)-x(t_1^-)=I_1(x(t_1))$, hence we get

$$x(t_k^+)=I_1(x(t_1))+\frac{1}{\Gamma(q)}\int_0^{t_1}(t_1-s)^{q-1}y(s)ds.$$

It follows that for t∈ $(t_1,t_2]$,

$$x(t)=x(t_1^+)+\frac{1}{\Gamma(q)}\int_{t_1}^t (t-s)^{q-1}y(s)ds$$

$$=\frac{1}{\Gamma(q)}\int_{t_1}^t (t-s)^{q-1}y(s)ds+\frac{1}{\Gamma q}\int_0^{t_1}(t-s)^{q-1}y(s)ds+I_1(x(t_1))$$

In consequence, we can see, by means of, $x(t_2^+)=x(t_2^-)+I_2(x(t_2))$, that

$$x(t_2^+)=\sum_{i=0}^{1}\frac{1}{\Gamma(q)}\int_{t_1}^{t_{i+1}}(t_{i+1}-s)^{q-1}y(s)ds+\sum_{j=1}^{2}I_j(x(t_j)),$$

Which shows that for t ∈ $(t_2,t_3]$.

$$x(t)=\frac{1}{\Gamma(q)}\int_{t_2}^t (t-s)^{q-1}y(s)ds+\sum_{i=0}^{k-1}\frac{1}{\Gamma(q)}\int_{t_i}^{t_{i+1}}(t_{i+1}-s)^{q-1}y(s)ds+\sum_{j=1}^{k}I_j(x(t_j)),$$

By iteration, the solution x(t) for t ∈ $(t_k, t_{k+1}]$ can be written as

$$x(t)=\frac{1}{\Gamma(q)}\int_{t_k}^t (t-s)^{q-1}y(s)ds+\sum_{i=0}^{k-1}\frac{1}{\Gamma(q)}\int_{t_i}^{t_{i+1}}(t_{i+1}-s)^{q-1}y(s)ds+\sum_{j=1}^{k}I_j(x(t_j)),$$

Conversely, if x is a solution of problem (1), then it can be easily seen by direct computation, that $D^q x(t)=y(t)$, $t\neq t_k$, t ∈ [0,T] and $\Delta x(t)=x(t_k^+)-x(t_k^+)=I_k(x(t_k))$where

$$x(t_k^+)=\sum_{i=0}^{k-1}\frac{1}{\Gamma(q)}\int_{t_i}^{t_{i+1}}(t_{i+1}-s)^{q-1}y(s)ds+\sum_{j=1}^{k}I_j(x(t_j)),$$

and

$$x(t_k^-)=\frac{1}{\Gamma(q)}\int_{t_{k-1}}^{t_k}(t-s)^{q-1}y(s)ds+\sum_{i=0}^{k-2}\frac{1}{\Gamma(q)}\int_{t_i}^{t_{i+1}}(t_{i+1}-s)^{q-1}y(s)ds+\sum_{j=1}^{k-1}I_j(x(t_j)).$$

In this way the proof of the Lemma is completed".

3. Main Results

Firstly, set $\mathbb{C}_0$ = {z | z ∈ ℂ ([0, T], ℝ), z(0) = 0}. For each z ∈ $\mathbb{C}_0$, we denote by z the function defined by

$$\bar{z}(t)=z(t),\ 0\le t\le T, and\ \bar{z}(t)=0, \quad -\infty\le t\le 0. \tag{4}$$

If *x* is solution of (1), then *x* (.) can be decomposed as $x(t)=\bar{z}(t)+\varphi(t)$ for $-\infty\le t\le T$, which implies that $x_t=\bar{z}_t+\varphi_t$ for $0\le t\le T$, where

$$\varphi(t)=0,\ 0\le t\le T, and\ \varphi(t)=\varphi(t), \quad -\infty\le t\le 0. \tag{5}$$

Therefore, the problem (1) can be transformed into the following fixed point problem of the operator F : $\mathbb{C}_0 \to \mathbb{R}$,

$$Fz(t) = \frac{1}{\Gamma(q)} \int_{t_k}^{t} (t-s)^{q-1} f(s, \bar{z}_s + \phi_s) ds$$
$$+ \sum_{i=0}^{k-1} \frac{1}{\Gamma(q)} \int_{t_i}^{t_{i+1}} (t_{i+1} - s)^{q-1} f(s, \bar{z}_s + \phi_s) ds$$
$$+ \sum_{j=1}^{k} I_j(\bar{z}(t_j)), \; t \in (t_k, t_k + 1], \; k = 0,1,......m. \quad (6)$$

Now, let us present our main result.

Theorem 3.1. *For the functions $f \in \mathbb{C}([0,T] \times \mathbb{R}, \mathbb{R})$ and $I_k : \mathbb{R} \to \mathbb{R}$, assume the following conditions hold.*

(H1) There exists a continuous function a : [0,T] $\to \mathbb{R}^+$ satisfying

$$|f(t,u_t) - f(t,v_t)| \le a(t) \sup_{s \in [0,t]} |u(s) - v(s)|, \; u,v \in R, \; t \in [0.T];$$

(H2) There exists a constant $L_k > 0$ such that $|I_k(u) - I_k(v)| \le L_k |u - v|$, $k = 1, 2, \ldots, m$;

(H3) $\sum_{i=1}^{m+1} \frac{a_i T^q}{\Gamma(q+1)} + \sum_{j=1}^{m} L_j < 1, where\; a_k = \sup_{t \in (i_k, t_{k+1})} a(t);$

(H4) There exists a constant $M > 0$ such that $|f(t,\varphi t)| \le M$, where φ is defined in (5).

Proof. We complete the proof, via method of successive approximations. Define a sequence of functions z_n : [0,T] $\to \mathbb{R}$, n = 0,1,2,…. as follows:

$$z_0(t) = 0, \quad z_n(t) = Fz_{n-1}(t). \quad (7)$$

Since $z_o(t) = 0$, it is easy to see from (4) that $(\bar{z}_o)_s = 0$ for s ∈ [0,T]. Thus we have

$$|z_1(t) - z_0(t)| \le \frac{1}{\Gamma q} \int_{t_k}^{t} (t-s)^{q-1} |f(s,\phi_s)| ds + \sum_{j=1}^{k} I_j(0)$$
$$+ \sum_{i=0}^{k-1} \frac{1}{\Gamma q} \int_{t_i}^{t_{i+1}} (t_{i+1} - s)^{q-1} |f(s,\phi_s)| ds$$
$$\le \frac{M(t-t_k)^q}{\Gamma(q+1)} + \sum_{i=1}^{k} \frac{M(t_i - t_{i-1})^q}{\Gamma(q+1)} + \sum_{j=1}^{k} I_j(0)|$$
$$\le \sum_{i=1}^{m+1} \frac{M(t_t - t_{i-1})^q}{\Gamma(q+1)} + \sum_{j=1}^{m} |I_j(0)| := N_0, \; k = 1,2,...,m,$$

it follows that $\|z_1 - z_0\| \le N_0$. Furthermore,

$$|z_n(t) - z_{n-1}(t)|$$
$$\le \frac{1}{\Gamma(q)} \int_{t_k}^{t} (t-s)^{q-1} |f(s,(\bar{z}_{n-1})_s + \phi_s) - f(s,(\bar{z}_{n-2})_s + \phi_s)| ds$$
$$+ \sum_{i=0}^{k-1} \frac{1}{\Gamma(q)} \int_{t_i}^{t_{i+1}} (t_{i+1} - s)^{q-1} |f(s,(\bar{z}_{n-1})_s + \phi_s) - f(s,(\bar{z}_{n-2})_s + \phi_s)| ds$$
$$+ \sum_{j=1}^{k} |I_j(\bar{z}_{n-1}(t_j)) - I_j(\bar{z}_{n-2}(t_j))|$$
$$\le \frac{1}{\Gamma(q)} \int_{t_k}^{t} (t-s)^{q-1} a(s) \underset{r \in [0,s]}{SUP} |\bar{z}_{n-1}(r) - \bar{z}_{n-2}(r)| ds$$
$$+ \sum_{i=0}^{k-1} \frac{1}{\Gamma(q)} \int_{t_i}^{t_{i+1}} (t_{i+1} - s)^{q-1} a(s) \underset{r \in [0,s]}{SUP} |\bar{z}_{n-1}(r) - \bar{z}_{n-2}(r)| ds$$
$$+ \sum_{j=1}^{k} |L_j(\bar{z}_{n-1}(t_j) - \bar{z}_{n-2}(t_j)|$$
$$\le \left[a_k \frac{(t-t_k)^q}{\Gamma(q+1)} + \sum_{i=1}^{k} a_i \frac{(t_i - t_{i-1})^q}{\Gamma(q+1)} + \sum_{J=1}^{K} L_j \right] \|\bar{z}_{n-1} - z_{n-2}\|$$
$$\le \left(\sum_{i=1}^{m+1} a_i \frac{T^q}{\Gamma(q+1)} + \sum_{J=1}^{m} L_j \right) \|\bar{z}_{n-1} - z_{n-2}\|$$
$$:= N \|z_{n-1} - z_{n-2}\|, \quad (8)$$

which implies that $\|z_n - z_{n-1}\| \le N \|z_{n-1} - z_{n-2}\|$ with $N < 1$. Note that for any $m > n > 0$, we have

$$\|z_n = z_n\| \le \|z_{n+1} - z_n\| + \|z_{n+2} - z_{n+1}\| + ... + \|z_m = z_{m-1}\|$$
$$\le (N^n + N^{n+1} + ... + N^{m-1}) \|z_1 = z_0\|$$
$$\le \frac{N^n}{1-N} \|z_1 = z_0\|. \quad (9)$$

If m,n are sufficiently large numbers then it follows form the above inequalities with $N < 1$ that $\|z_m - z_n\| \to 0$. Thus $\{z_n(t)\}$ is a Cauchy sequence in $\mathbb{PC}(J)$. Since $\mathbb{PC}(J)$ is a complete Banach space, then $\|z_n - z\| \to 0$ as $n \to \infty$ for some $z \in \mathbb{PC}(J)$, which means the $z_n(t)$ is uniformly convergent to $z(t)$ with respect to t.

Thus, we will reach to the conclusion that z(t) is a solution of the equation (1). Observe that

$$\left| \frac{1}{\Gamma(q)} \int_{t_k}^{t} (t-s)^{q-1} f(s,(\bar{z}_n)_s + \phi_s) ds - \frac{1}{\Gamma(q)} \int_{t_k}^{t} (t-s)^{q-1} f(s,\bar{z}_s + \phi_s) ds \right|$$
$$\le \frac{1}{\Gamma(q)} \int_{t_k}^{t} (t-s)^{q-1} |f(s,(\bar{z}_n)_s + \phi_s) - f(s,\bar{z}_s + \phi_s)| ds$$

$$\leq \frac{1}{\Gamma(q)}\int_{t_k}^{t} a(t)(t-s)^{q-1}\sup_{r\in[0,s]}|\bar{z}_n(r)-\bar{z}(r)|\,ds$$

$$= \frac{1}{\Gamma(q)}\int_{t_k}^{t} a(t)(t-s)^{q-1}\sup_{r\in[0,s]}|\bar{z}_n(r)-z(r)|\,ds$$

Since $z_n(t) \to z(t)$ as $n \to +\infty$, for any $\varepsilon > 0$, there exists a sufficiently large *number* $n_0 > 0$ such that for all $n > n_0$, we have

$$|z_n(r)-z(r)| < \min\left\{\frac{\Gamma(q+1)}{\sum_{i=0}^{m} a_i T^q}\varepsilon, \frac{1}{\sum_{j=1}^{m} L_j}\varepsilon\right\}.$$

Therefore,

$$\left|\frac{1}{\Gamma(q)}\int_{t_k}^{t}(t-s)^{q-1} f(s,(\bar{z}_n)_s,+\phi_s)ds - \frac{1}{\Gamma(q)}\int_{t_k}^{t}(t-s)^{q-1} f(s,\bar{z}_s+\phi_s)ds\right| < \varepsilon, \tag{10}$$

and

$$\left|\sum_{i=0}^{k-1}\frac{1}{\Gamma(q)}\int_{ti}^{t_{i+1}}(t_{i+1}-s)^{q-1} f(s,(\bar{z}_n)_s,+\phi_s)ds - \sum_{i=0}^{k-1}\frac{1}{\Gamma(q)}\int_{ti}^{t_{i+1}}(t_{i+1}-s)^{q-1} f(s,(\bar{z}_s)_s,+\phi_s)ds\right|$$

$$\leq \sum_{i=0}^{k-1}\frac{1}{\Gamma(q)}\int_{ti}^{t_{i+1}}(t_{i+1}-s)^{q-1}|f(s,(\bar{z}_n)_s,+\phi_s)-f(s,(\bar{z}_s+\phi_s)|ds$$

$$\leq \sum_{i=0}^{k-1} a(t_i)\frac{(t_i-t_{i-1})^q}{\Gamma(q+1)}\sup_{r\in[0,s]}|z_n(r)-z(r)|ds < \varepsilon, \tag{11}$$

and

$$\left|\sum_{j=1}^{k} I_j(\bar{z}_n(t_j)) - \sum_{j=1}^{k}|I_j(\bar{z}(t_j))\right|$$
$$\leq \sum_{j=1}^{k} L_j(\bar{z}_n(t_j)-\bar{z}(t_j)|$$
$$= \sum_{j=1}^{k} L_j|z_n(t_j)-z(t_j)|| < \varepsilon. \tag{12}$$

In lieu of the same, for a sufficiently large number $n > n_0$.

$$|z(t)-Fz(t)|$$

$$\leq |z(t)-z_{n+1}(t)|+|z_{n+1}(t)-Fz_n(t)|+|Fz_n(t)-Fz(t)|$$

$$\leq |z(t)-z_{n+1}(t)|+\left|z_{n+1}(t)-\left[\frac{1}{\Gamma(q)}\int_{t_k}^{t}(t-s)^{q-1}f(s,(\bar{z}_n)_s+\phi_s)ds\right.\right.$$

$$\left.\left.+\sum_{i=0}^{k-1}\frac{1}{\Gamma(q)}\int_{t_i}^{t_{i+1}}(t_{i+1}-s)^{q-1} f(s,(\bar{z}_n)_s+\phi_s)ds+\sum_{j=1}^{k} I_j(\bar{z}_n(t_j))\right]\right|$$

$$+\left|\frac{1}{\Gamma(q)}\int_{t_k}^{t}(t-s)^{q-1}f(s,(\bar{z}_n+\phi_s)ds-\frac{1}{\Gamma(q)}\int_{t_k}^{t}(t-s)^{q-1}f(s,(\bar{z}_n)_s+\phi_s)ds\right|$$

$$+\left|\sum_{i=0}^{k-1}\frac{1}{\Gamma(q)}\int_{t_i}^{t_{i+1}}(t_{i+1}-s)^{q-1} f(s,(\bar{z}_s+\phi_s)ds\right.$$
$$\left.-\sum_{i=0}^{k-1}\frac{1}{\Gamma(q)}\int_{t_i}^{t_{i+1}}(t_{i+1}-s)^{q-1} f(s,(\bar{z}_s)_s+\phi_s)ds\right|$$

$$+\left|\sum_{j=1}^{k} I_j(\bar{z}_n(t_j))-\sum_{j=1}^{k} I_j(\bar{z}_n(t_j))\right|$$

Thus, as per convergence of the two previous and equations (10)-(12), one obtains that $|z(t) - Fz(t)| \to 0$, which implies that z is a solution of (1).

Finally, we prove the uniqueness of the solution. Assume that $z_1, z_2 : [0,T] \to \mathbb{R}$ are two solution of (1) , Note that

$$|z_1(t)-z_2(t)|$$

$$\leq \frac{1}{\Gamma(q)}\int_{t_k}^{t}(t-s)^{q-1} a(s)\sup_{r\in[0,s]}|\bar{z}_1(r)-\bar{z}_2(r)|ds$$

$$+\sum_{i=0}^{k-1}\frac{1}{\Gamma(q)}\int_{t_i}^{t_{i+1}}(t_{i+1}-s)^{q-1} a(s)\sup_{r\in[0,s]}|\bar{z}_1(r)-\bar{z}_2(r)|ds+\sum_{j=1}^{k} L_j|\bar{z}_1(t_j)-\bar{z}_2(t_j)|$$

$$\leq\left(\sum_{i=i}^{m+1}\frac{a_i T^q}{\Gamma(q+1)}+\sum_{j=1}^{m} L_j\right).\ \|z_1-z_2\|.$$

According to the condition (H3), the uniqueness of the problem (1) follows immediately, which completes the proof.

4. An Example

In this section we give an example to illustrate the above results. Consider the following impulsive partial hyperbolic fractional differential equations with infinite delay.

$$\begin{cases} D^q u(t) = \dfrac{e^t}{(9+e^t)}\dfrac{|u_t|}{(1+|u_t|)} & t\in[0,1],\ t\notin\frac{1}{2}, 0<q<1; \\ \Delta u\left(\frac{1}{2}\right) = \dfrac{|u(\frac{1}{2}^-)|}{1/4+|u(\frac{1}{2}^-)|} & \\ u(t) = \varphi(t) = \dfrac{e^{-t}-1}{2}, & -\infty\leq t\leq 0, \end{cases} \tag{13}$$

where $0 < q < 1$, $\Gamma(q+1) < \dfrac{3}{10}$

Set,

$f(t,u) = \dfrac{e^{-t}u}{(9+e^t)(1+u)}$, $I(u)=\dfrac{u}{1/4+u}$, for $(t,u) \in [0,1] \times [0,+\infty]$

It is clear that the functions f and I are continuous. Now we have.

$$\begin{aligned}|f(t,u_t)-f(t,v_t)| &= \frac{e^{-t}}{(9+e^t)}\frac{(|u_t|-|v_t|)}{(1+|v_t|)(1+|v_t|)} \\ &\le \frac{e^{-t}}{(9+e^t)}|u_t - v_t| \\ &\le a(t) \sup_{s\in[0,1]} |u(s)-v(s)|,\end{aligned}$$

Where a(t) = $\frac{1}{10}$ So the condition (H1) of Theorem 3.1 is satisfied. Also we have

$$\begin{aligned}|I(u)-I(v)| &= \left|\frac{u}{\frac{1}{4}+u} - \frac{v}{\frac{1}{4}+v}\right| \\ &= \frac{1}{4}\left|\frac{|u|-|v|}{(\frac{1}{4}+u)(\frac{1}{4}+v)}\right| \\ &= \frac{1}{4}|u-v|,\end{aligned}$$

Where L = $\frac{1}{4}$. So the condition (H2) is also satisfied. Now it is easy to conclude

$$\sum_{i=1}^{m+1}\frac{a_1 T^q}{\Gamma(q+1)} + \sum_{j=1}^{m} L_j = \frac{1}{10}\frac{1}{\Gamma(q+1)}\frac{1}{4} < 1$$

and

$$|f(t,u_t)| = \frac{e^{-t}}{(9+e^t)}\frac{|u_t|}{(1+|u_t|)} \le \frac{e^{-t}}{(9+e^t)} \le \frac{1}{10}, t\in[0,1].$$

Thus, the equation (4) satisfies all the conditions given in Theorem 3.1, which implies that the (13) has a unique solution.

5. Conclusion

In this paper, existence and uniqueness for a class of Cauchy initial value problem with impulses and infinite delay is discusses. A better and simple method is set to get criteria of existence and uniqueness of the solution to such problems by using successive approximation. Such type of equations arise in real world phenomena like oscillation with discontinuities, etc. One can deduce slightly different results by taking different time variables.

6. Competing Interest

Both the authors do not have any competing interests.

7. Author's Contribution

Both the researchers have given equal input in preparing the paper.

8. References

1. Benchohra M, Hamani S. The method of upper and lower solutions and impulsive fractional differential inclusions. Nonlinear Analysis: Hybrid System. 2009; 3:433–40.
2. Benchohra M, Berhoun F. Impulsive fractional differential equations with variable times. Computers and Mathematic with Applications. 2010; 59:1245–52.
3. Laxmikantham V, Vatsala AS. Basic theory of fractional differential equations, Nonlinear analysis; Theory Methods and Applications. 2008; 69(8):2677–82.
4. Belaebi A, Benchohra M, Ouahab A. Uniqueness results for fractional functional differential equations with infinite delay in Frechet space. Appl. Anal. 2006; 85:1459-70.
5. Lin Wei. Global existence and Chaos control of fractional differential equations. Journal of Mathematical Analysis and Applications. 2007; 332:709–26.
6. Ahemad B, Sivasundaram S. Existences results for nonlinear impulsive hybrid boundary value problems involving fractional differential equation. Nonlinear Analysis: Hybrid Systems. 2009; 3(3):251–58.
7. Agarwal RP, Benchohra M, Slimani BA. Existence results for differential equations with fractional order and impulses. Memoirs on Differential Equations and Mathematical Physics. 2008; 44:1–21.
8. Benchohra M, Slimani BA. Existences and uniqueness of solutions to Impulsive fractional differential equations. Electronoc Journal of Differential Equations. 2009; 10:1–11.
9. Kilbas A, Srivastva HM, Trujillo JJ. Amsterdam: Elsevier Science B. V.: Theory and Applications of Fractional Differential Equations. North-Holland Mathematics Studies. 2006; 204.
10. Miller KS, Ross B. New York: Wiley: An Introduction to the Fractional Calculus and Differential Equations. 1993.
11. Jinrong Wang. Fractional finite time delay evolution systems and optimal controls in infinite dimensional spaces. Journal of Dynamical Systems and Control. 10/21/2011.
12. Haiyong Qin, Zuo Xin, Liu J Wei, Lischen Liu. Approximate controllability and optimal controls of fractional dynamical systems of order 1<q<2 in Banach spaces. Advances in Difference Equations. 2015.
13. Zhang X. The existence and uniqueness of mild solutions for impulsive fractional equations with non local conditions and infinite delay. Nonlinear Analysis Hybrid System. 2010; 11.