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Piecewise Feature Extraction and Artificial Neural Networks: an Approach towards Curve Reconstruction

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Abstract

Background/Objectives: Reconstruction of shapes from unorganized data points is a problem with lot of practical significance in which a piecewise linear approximation to the shape is computed from the sample of the unknown shape. **Methods/Statistical Analysis**: An approach based on reconstruction of curves by using the feature points as control points is presented. At the transmission end the curve is represented using the feature points and at the receiver side the reconstruction of curve is established by optimizing the parameters of a radial basis function (RBF) neural network. **Findings:** The method reduces the complexity in terms of time and space. In other words it reduces the informational complexity of the RBF network for the problem of curve reconstruction. It also allows for noise in the data by using the inherent capabilities of a RBF neural network. **Applications/Improvements: S**cenes generated by modeling and animation using multimedia techniques contain curves and surfaces and thus the presented approach is useful in efficient transmission of images and video sequences.

Keywords: Control Points, Curve Reconstruction, Feature Extraction, Informational Complexity, RBF Neural Networks

1. Introduction

Interpolation and curve fitting are the fundamental problems in many fields of computer graphics, multi-media applications and mathematical modelling. Storage, retrieval and transmission of all points on a curve require lot of space as well as time. Representing a curve using control points not only saves space but also leads to its systematic and mathematical representation. Apart from that it also reduces the bandwidth requirement for transmission which is a major bottleneck in wireless communication. The purpose is to represent the curve with a minimum number of control points. Lot of research is being done to minimize the number of control points at the transmitter end and an efficient and accurate reconstruction of the curve at the receiver end to reduce the bandwidth requirement. In this paper we have talked about an approach based on the reconstruction of curve using the extracted feature points at the transmitter end. We have used piecewise feature extraction of curves to find the control points of a given curve. The curve is reconstructed from the features by using an RBF neural network. The training set of RBF net is reduced to the set of feature points. So we have optimized the storage requirements of a curve by combining the capabilities of feature extraction and RBF neural networks. The approach is better than the approach based on taking arbitrary points as control points because arbitrary points may not be able to capture the peaks and valleys of a curve or may not be able to handle high variations in the curvature of the curve. RBF network is used rather than the traditional curve fitting or interpolation techniques because then

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the curve is forced to lie in to a family of polynomial functions and also these methods fail completely in the presence of noise. RBF neural networks are considered to be an efficient tool for interpolation, curve fitting and function approximation leading to efficient modelling of curves and surfaces. A comprehensive survey of RBF neural networks for the solution of these problems is given in[1,2]. The problem of reconstruction of curves and surfaces has been solved in various ways using these networks. It has been concluded that RBF networks can approximate incomplete and noisy curves and surfaces. Also they are efficient in terms of learning speed and the accuracy of computations[3,4]. RBF networks perform better than back propagation and generalized regression neural networks as given in[5]. These networks are superior in terms of precision, error and convergence speed[6].

There are other approaches to curve reconstruction which are based on the concepts computational geometry. These approaches make use of Delaunay Triangulations and their duals Voronnoi diagrams. The crust algorithm given in[7], NN-crust in[8], Conservative Crust in[9] and Alplha shapes in[10] are some of the benchmarks in the field of curve reconstruction. The drawback associated with these approaches is that they are not able to handle noise in the data and also in case of a dense sample the complexity of these methods become very high. Moreover they assume certain sampling condition on the data for the curve reconstruction.

None of the approach used in literature is based on extracting the control points rather all the approaches are based on considering the arbitrary points thus incurring the complexity in terms of number of points and entire reconstruction process. Approach used in the current work is based on the extraction of features of the curve and then reconstruction is carried out with the extracted feature points using a RBF neural network. The features that are considered here are the features of a curve which are geometric invariant i.e. the features which are not affected by a particular group of transformations. The optimization is in terms of storage space and the learning process of RBF neural network. RBF neural networks have universal approximation properties. In order to obtain the results we need to solve a system of equations involving the training sample data. So, if the number of samples is large then it becomes hard to solve the equations and obtain the weights. In other words the presented approach reduces the informational complexity of the neural network for the reconstruction of curves. We can define informational complexity of a neural network as the minimum amount of information needed for the neural network to learn the relationship in the data.

The method is applied to smooth, continuous and simple curves which have lot of variations in the curvature. The resultant curve is a good approximation to the original curve. Effectiveness of the method is shown with the help of reconstruction of various curves and the mean square error in each of the case. The detailed algorithm is given in section 4, implementation details are given in section 5 and the theoretical aspects of the problem are given in section 2. Mathematical formulation of the problem is given in section 3. Conclusion and the future work are in section 6.

2. Theoretical Concepts

2.1 Function Approximation, Interpolation and Curve Fitting

Function Approximation: As the name suggests function approximation is to approximate a given

function $f(x)$ by another function $f1(x)$ such that

$$|f(x) - f1(x)| < e \tag{1}$$

where e is the error which is very small and $f1(x)$ is a linear combination of some simple functions. But many times the function $f(x)$ is not presented completely; rather its value is given at a specified number of discrete points. To find the function passing through all the given points is known as interpolation. So with interpolation, in case of 2-d the curve is expected to pass through all the given set of data points. The major drawbacks of interpolation are the use of fixed degrees of polynomials and the degradation of results in the presence of noise. Another approach for the reconstruction of curves is curve fitting, in which the reconstructed curve may not pass through the given set of data points. Principle of least square fitting is used so that the sum of the squares of the distance between the original curve and the reconstructed curve is minimum. But with the traditional curve fitting techniques like polynomial fitting or spline fitting, the curve is forced to have a polynomial of fixed degree. The next milestone in the theory of curve fitting was achieved with the advent of artificial neural networks. These are the networks designed artificially to mimic the functioning of the human brain specifically the learning tasks carried by human beings with examples. These networks have been used in several practical applications including face recognition[11], weather and storm analysis[12], prediction of stock market[13], prediction of pressure drop in heat exchangers[14] and tourism arrival forecasting[15]. Also the neural networks can perform curve fitting from a given set of data points with desired amount of accuracy. These networks can work for noisy data sets as well. The neural network used in the current work is an RBF neural network.

2.2 RBF Neural Networks

RBF neural networks designed by Moody and Darken in 1989 are a class of feed-forward networks which follow supervised learning paradigm. They are known to be the general function approximators as they are able to achieve the universal approximation property[16]. The structure of a radial basis network consists of three layers- the input layer, the hidden layer and the output layer. Input layer is a simple layer responsible for obtaining input from the environment. It is connected to the hidden layer in which each node is a radial basis function with the center and width as parameters. The hidden layer is in-turn connected to a linear output layer which gives weighted sum of the outputs of the hidden layer. Many basis functions could be used for the hidden layer but the most common is a Gaussian function. It is a bell shaped curve given by the following equation:

$$f_j = \exp\left(-\frac{|x - c_j|}{2\sigma^2}\right) \tag{2}$$

where fj represents output of the j^{th} hidden neuron/radial basis neuron, x is the input vector, c_j is the center of the j^{th} hidden neuron and also known as spread is the smoothness parameter that needs to be optimized for the problem; higher values of leads to over smoothness whereas lower values lead to insufficient smoothness.

Initially RBFs were used for exact interpolation having a fixed number of centers, same as the number of input neurons. But this made an RBF network computationally expensive and applicable only to exact modelling of the given data. Appropriate centers have to be chosen to deal with the noise in the data and thus to have better approximation capabilities. There are many ways in which RBF centers could be chosen which enables it to deal with the problems like function approximation and fitting of curves. The centers could be a subset of the given training set or could be suitably chosen as a part of training. Cluster centers obtained by supervised clustering are also used as an efficient representation for centers in RBF networks. Various center selection method and their impact on the results is given in[3, 17]. We have used a greedy way of addition of centers so that the RBF fits the data to a desired accuracy.

3. Mathematical Formulations

3.1 Universal Approximation of Neural Networks

Universal Approximation Theorem[18] forms the basis for the approximation theory of neural networks. It states that every continuous function on compact subsets on Rn could be approximated by a single feed forward network which is having one hidden layer with finite number of neurons. It assumes certain restrictions on the activation function used in the network. Mathematically the theorem states that given a continuous function $h \in [0,1]^m$ then we can always find a function H such that:

$$|H(x) - h(x)| < e \tag{3}$$

$\forall x$ in $[0, 1]^n$, and $\exists\ \alpha \in R^n$, $b \in R^m$ and $W \in R^{n\times m}$ such that

$$H(x) = \alpha.\ f(Wx + b) \tag{4}$$

where f is a monotonically increasing, continuous, bounded and a non-constant function.

So RBF networks with suitable radial basis functions at the hidden node can approximate any continuous function. Schoenberg Interpolation theorem[19], Micchelli interpolation theorem[20] and the theorem given by Park and Sandberg[16] are major contributions in the field providing suitable conditions on radial basis functions. The condition given by Park and Sandberg for RBF networks to be universal approximators was that the function should be integrable, bounded, continuous almost everywhere and its integration should not be zero. The condition of integrability of the functions was relaxed by Lio proving that if the radial basis function used in the network is bounded, piecewise continuous and not a polynomial then the network can approximate any continuous function.

The activation function that we have used in this paper is the most common activation function- Gaussian function which is integrable and thus satisfies the conditions of Park and Sandberg.

3.2 Features

i) Local Maxima: It is a point on the curve where the first order derivative i.e the slope of the curve is zero and the derivative of the curve changes sign from positive to negative.

$dy/dx = 0$ gives the points of local maxima with dy/dx going from +ve to –ve.

ii) Local Minima: It is a point on the curve where the first order derivative is zero and changes sign from negative to positive.

$dy/dx = 0$ gives the points of local minima with dy/dx going from –ve to +ve to.

iii) Points of inflection: A point is said to be a point of inflection if the curve is increasing on one side and decreasing on the other side of curve or vice versa.

$d^2y/dx^2 = 0$ gives the points of inflection with d^2y/dx^2 changing sign from +ve to –ve or from –ve to +ve.

iv) x and y intercepts: These are the points of a curve that intersect with either of the x or y axis.

As discussed earlier that the transmission of all the points of a curve incurs lot complexity in terms of storage and computational time. The practical approach is to use few points rather than the entire data set to approximate a given curve. The problem is to decide which critical points or control points to select so that the reconstructed curve is a good approximation to the given curve.

Freeman (1978) has included the following points in his definition of critical points:

- Curvature maxima
- Curvature minima
- End points
- Points of intersection

Hoffman and Richards (1982) state that critical points found by first finding the maxima, minima, and zeroes of curvature are invariant under rotations, translations, and uniform scaling.

The features that are used in the current work are – local maxima, local minima, starting point, end point and the centroid of the curve. The reason for taking centroid as one of the feature point or control point is that RBF networks give good approximations with cluster centers as the centers for RBF networks.

4. Algorithm

The problem of effectively storing and then reconstructing the curves is carried out using the following approach:

- The initial input data is a point set $P = \{x_i\} \quad R^2$. Initially all the points of the input data lie on the curve.
- Divide the curve in to pieces ie the input points are segmented and from each of the pieces, features are extracted. Based on the discussion in section 3.2 the following geometric invariant features are considered:
  - Start and end point of the curve.
  - Maxima, minima, and centroid of each of the pieces.
  - A RBF neural network is created as follows
  - The training set comprises of the feature points extracted in the previous step.
  - A simple greedy algorithm is used for the selection of centers[21]. Initially a subset from the given set of nodes is taken as center
    - Error is evaluated at all the nodes
    - If the error is less than the desired error then stop
    - Otherwise add new centers towards the points where the error is large
    - Repeat the fitting process of RBF and re-evaluate the error; the loop begins again
  - A Gaussian basis function is used in the hidden layer as:
  - $f(||x-c_j||) = \exp(-||x- c_j||^2 / 2\sigma^2)$
  - The output of the RBF is computed as a weighted sum of the output of the Gaussian basis neurons i.e.

$$F(x) = \sum_{n=1}^{C} (w_i \, f(|| x - c_j ||)$$

  - Adjust the parameter spread for the network. Same spread is taken for all the neurons.
- Noise is added to the feature set.
- A Radial basis function is trained for this noisy input set in a similar way as done for the un-noisy data.
- Reconstruct the curve using the trained RBF neural network and the control points (noisy and un-noisy input feature points).
- Results of the reconstructed curve (for both noisy and un-noisy input) are compared to the original curve by evaluating the mean square error term.

5. Simulation Results

The algorithm is applied to various smooth, continuous, open and non-intersecting curves. Some examples illustrating the effectiveness of the algorithm are shown in this section. Implementation of the algorithm is done in MAT LAB R2013a on Windows platform. The results of the following four curves are shown:

y = xsin(x),

y = sinc(x),

y = sin(4πx)exp(-5x) and

y = 2 sin(πexp(-2x)).

Results of the first curve are shown in Figure 1(a)-1(e), Initial curve is shown in Figure 1(a), extracted features are shown in Figure 1(b), Figure 1(c) shows the reconstructed curve and the original curve. Figure 1(d) shows the feature points with a noise of .01 added to each one of them. The reconstructed curve from the noisy points and the original curve are shown in Figure 1(e). The curve is divided in to 7 pieces and thus the number of control points is 23. The parameter spread is taken to be 2. Similarly the results of second, third and fourth curves are shown in Figure 2(a)-2(e), 3(a)-3(e) and 4(a)-4(e) respectively.

5.1 Comparative Analysis

Performance of each of the curve with feature points without noise and with noise is shown in Table 1, along with the spread and the number of feature points taken in each curve.

It is clear from the figures and the table that the reconstructed curve with the control points as the training set and RBF neural networks is a good approximation to the original curve. The performance in terms of mean square error is shown in the table for each of the curves. Also it can be observed that the number of control points in each of the curve is very less. The results are also good when the control points are contaminated with noise.

Yi liao has given an example of curve approximation for a simple function with two peaks and one valley using RBF neural networks[22]. The value of the function lies in the range-1 to 1 and the training set taken contains 21 points. Whereas in the first example considered in this paper the value of the function is in the range 0 to 30 with 5 peaks and 5 valleys; and the number of training points is just 23 which is approximately same as in[22] but the range in which the curve is spread is 15 times more than that and it has lot of variations in the curvature.

The simulation example given by C Enachescu[23] for approximating different functions using neural networks contain 50 training points in the range 0 to 1 with 5 peaks and 4 valleys.

The above mentioned results are the results of reconstruction when the sample is without noise. B Walezak[3] has used a noisy data set with 50 samples as the training set used in approximating a given function which lies in the range-1 to 1 with 1 peak only. A comparison of the various basis functions for RBF neural networks is given by[24] and the functions which are reconstructed also use a large training data.

The surfaces reconstructed in[4] and[5] also use a large data set for reconstruction using RBF networks. So the technique presented in the current work optimizes the storage and transmission requirements of the curves and facilitates for an efficient retrieval.

Table1. Performance of the reconstructed curves

Given Curve	No. of feature points	Spread	Performance Without Noise	Performance With Noise
Curve 1 y = xsin(x)	23	2	1.0318 e-28	1.82259 e-27
Curve 2 y = sinc(x);	20	0.7	9.25271 e-33	4.61918 e-32
Curve 3 y = sin(4πx) exp(-5x)	11	0.3	1.6239 e-25	1.91886 e-23
Curve 4 y = 2 sin(πexp (-2x))	11	1.5	2.52047 e-11	3.22954 e-08

6. Conclusion

An approach based on extracting the control points of a curve and then reconstructing it using the same points as a training set to a RBF neural network is presented. The feature points which are invariant under a set of transformations are selected as the training set for RBF network. The results show that instead of storing the entire curve it is effective to store only the control points. The reconstructed curve is a good approximation to the original curve. The method deals with the noise in the data as well by utilizing the capabilities of a RBF net. So, with this method we are able to reduce the memory requirement and evaluation time without loss of accuracy. The method can interpolate the curve effectively but the results are not good for extrapolation of the curve. It is tested on simple non intersecting curves with lot of variations in the curvature. Ongoing research includes the identification of discontinuities in the curve thus leading to intersections. Other neural network techniques like the wavelet neural networks or hyper basis function (HBF) neural networks can be used to reconstruct such types of curves.

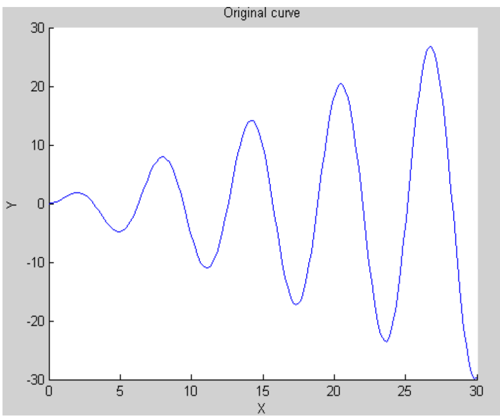

Figure 1(a). Original curve.

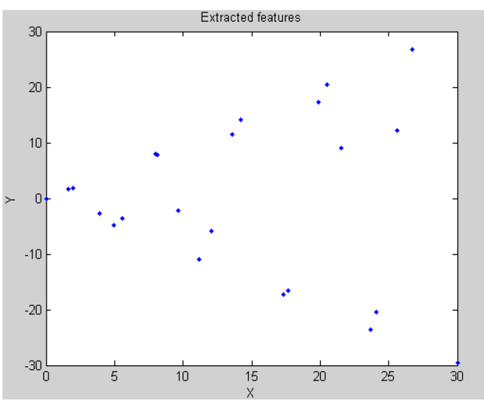

Figure 1(b). Extracted features.

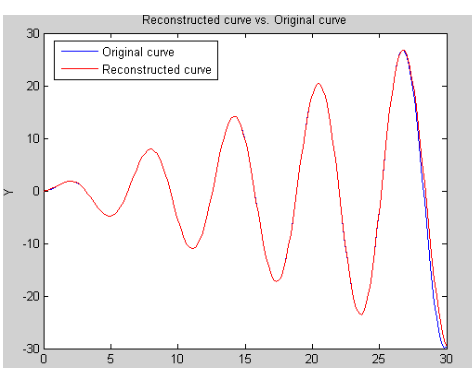

Figure 1(c). Reconstructed curve vs. Original curve with

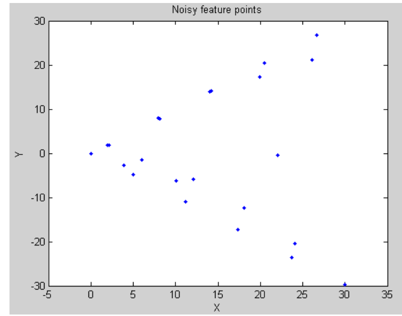

Figure 1(d). Noise added to features (Noisy feature points). spread = 2 and no. of points =23.

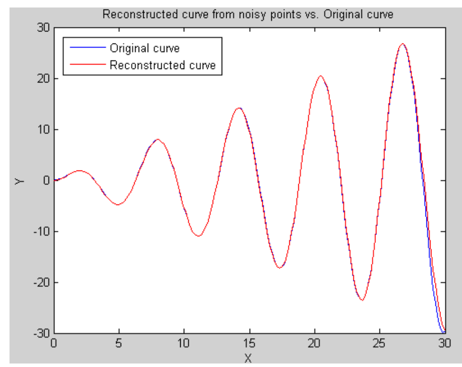

Figure 1(e). Reconstructed curve from noisy points vs. original curve.

Figure 1. Steps of the curve given by y = xsin(x).

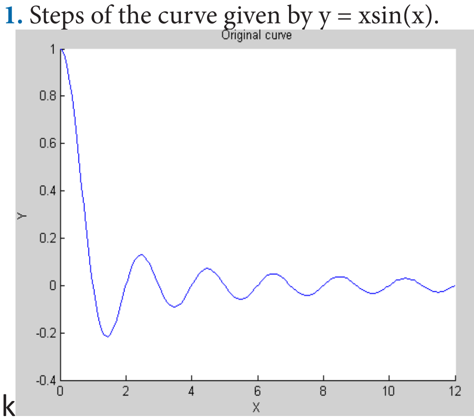

Figure 2(a). Original curve.

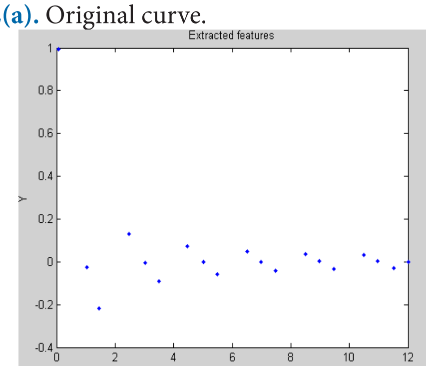

Figure 2(b). Extracted features.

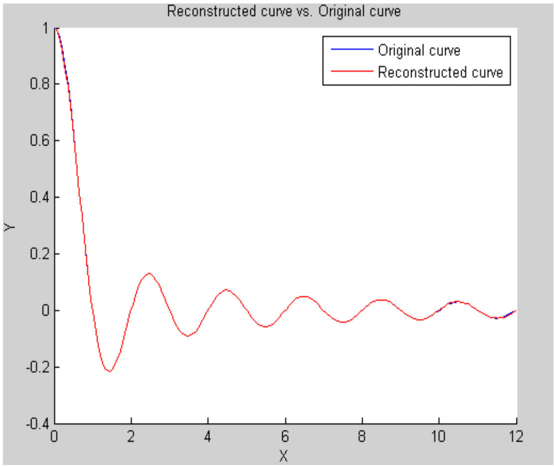

Figure 2(c). Reconstructed curve vs. Original curve with

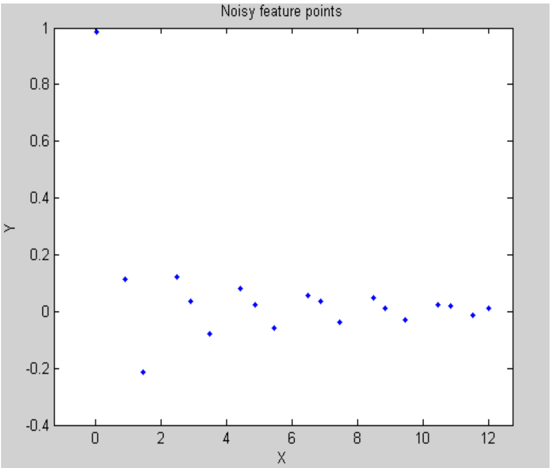

Figure 2(d). Noise added to features (Noisy feature points). spread = .7 and no. of points =20.

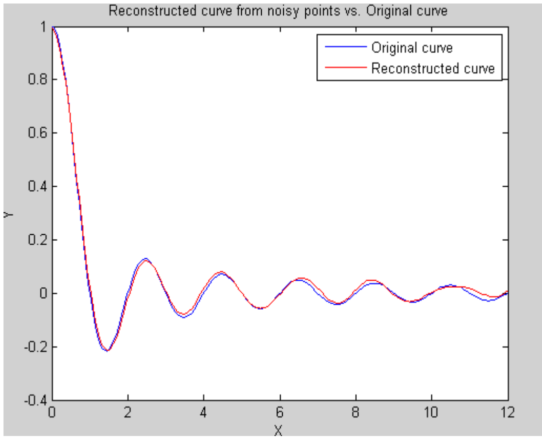

Figure 2(e). Reconstructed curve from noisy points vs. original curve.

Figure 2. Steps of the curve given by y = sinc(x).

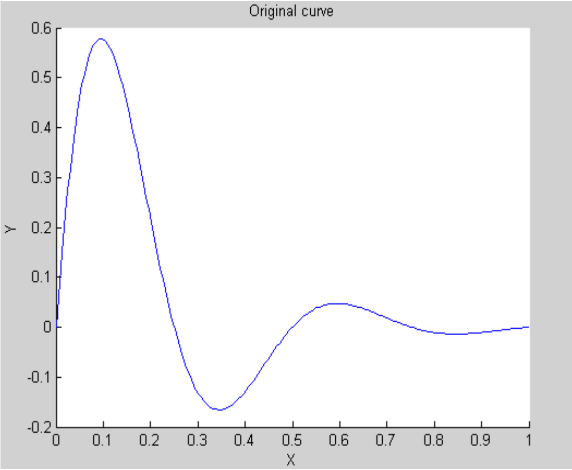

Figure 3(a). Original curve

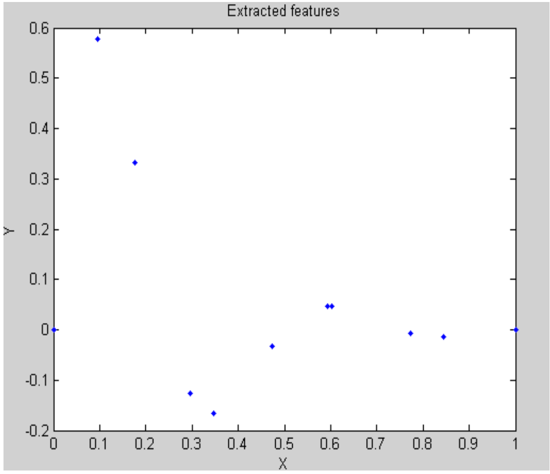

Figure 3(b). Extracted features.

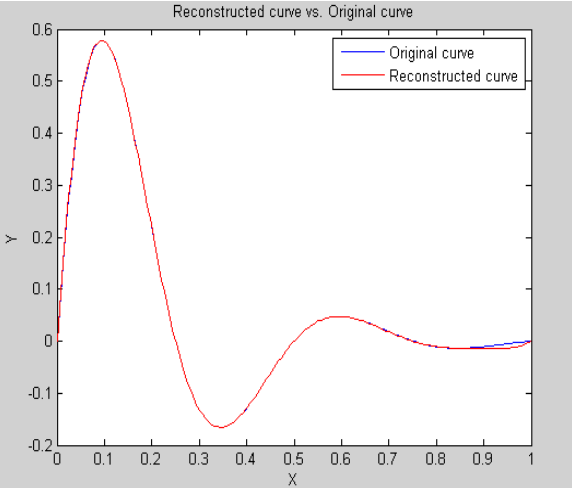

Figure 3(c). Reconstructed curve vs. Original curve with

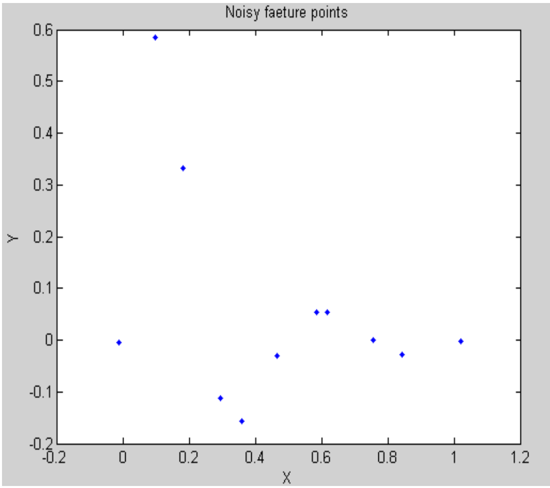

Figure 3(d). Noise added to features (Noisy feature points). spread = .3 and no. of points =11.

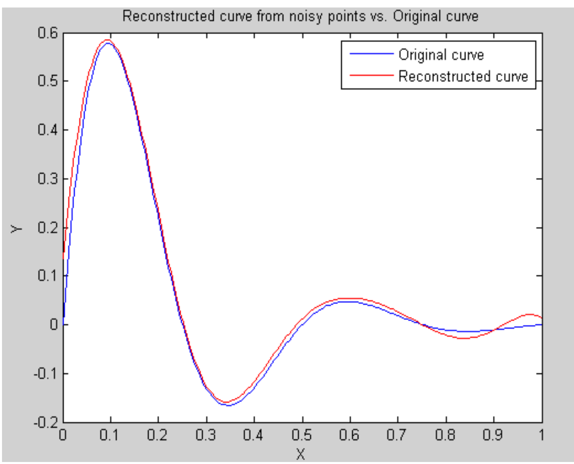

Figure 3(e). Reconstructed curve from noisy points vs. original curve.

Figure 3. Steps of the curve given by y= sin(4πx)exp(-5x).

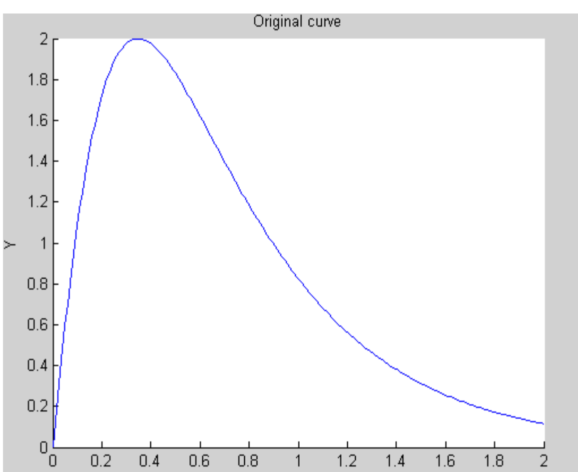

Figure 4(a). Original curve.

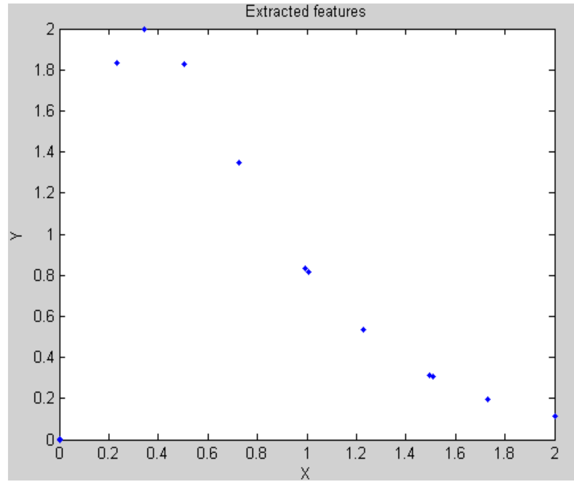

Figure 4(b). Extracted features.

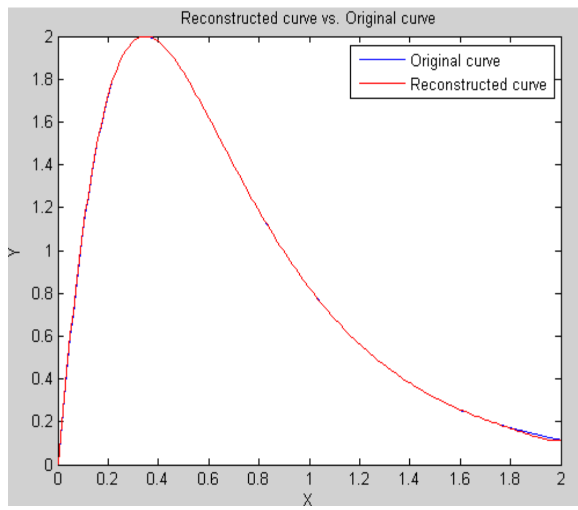

Figure 4(c). Reconstructed curve vs. Original curve with

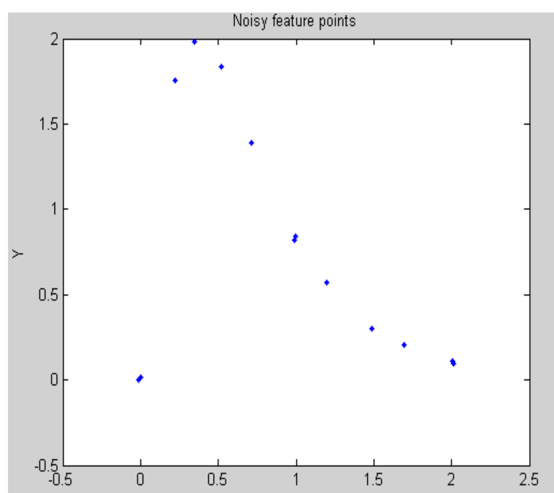

Figure 4(d). Noise added to features (Noisy feature points). spread = 1.5 and no. of points =11.

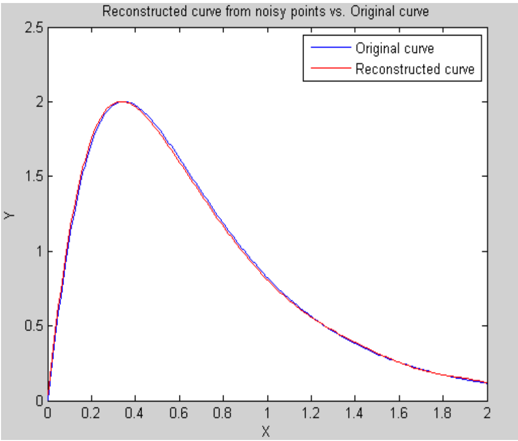

Figure 4(e). Reconstructed curve from noisy points vs. original curve.

Figure 4. Steps of the curve given by y= 2 sin(πexp(-2x)).

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