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Mathematical Model for Pipeline Control Applying in-Line Robotic Device

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Abstract

Background/Objectives: This article focuses on the development of a mathematical model of the robotic device to evaluate the range of its movement along the pipeline of a complex shape. **Methods**: The mathematical model is based on the mathematical description of the robot path, formation of the model for the robot movement process; calculation of cable pulling tension in some parts of the pipeline and along the entire control route; determining the robot traction force; determining the maximum distance of the robot movement. The mathematical model is implemented using MatLab software environment; its operativity can be verified by entering conventional settings in the program. **Findings:** The simulation result for the mathematical model of the robotic device in the MatLab software environment is a graph which shows a diagram of the controlled pipeline with regard to its configuration; graph which shows the robotic device advancement in the pipeline with a marked area where the operation will be stopped. Thus using the developed mathematical model makes it possible to calculate the length of the pipeline achievable for control taking into account the influence of the pipeline geometry and receive the data on each section of the monitored pipeline. **Applications/Improvements:** This mathematical model provides solution for timely inspection of corrosion damage in subsurface pipelines of small-diameter heating networks (DN200, DN400) of housing and public utilities in places inaccessible for external inspection.

Keywords: In-Line Diagnostics, Mathematical Model, Pipeline, Robotic Device, Route of Movement (Distance of Movement)

1. Introduction

To ensure the reliable and safe operation of the pipelines the actual and objective information about the technical conditions of the pipes is required which is obtained in different ways, including that of the in-line diagnostics. The pipelines possess considerable lengths and complex configuration, including straight and vertical sections, elevations and downward inclinations, bends in different directions, which makes the in-line diagnostics difficult[1].

To perform the Nondestructive Testing (NDT), the measuring system has to be delivered to the section under control. This task can be solved with the help of a robotic complex. To ensure passing through the complicated sections, the modules of the RD are connected with each other by means of a universal joint group (gimbals). Robotic device is controlled and fed through the connecting cable line from the control system, positioned at the robot entry point.

To evaluate the distance of the RD travelling inside the pipeline, the mathematical model should be built, which makes the object of this study.

The basis for creating the mathematical model is represented by the spatial shape of the pipeline. For the purposes of developing the model, the route of the robot movement is split in separate sections. The principle of such splitting implies ensuring that at each section of the pipeline the conditions of interaction of the moving robot and of the feeding cable with the constructive elements of the pipe interior are preserved constant. As such, the following sections can be considered:

- Straight horizontal sections.
- Straight inclined sections.

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- Bends in the horizontal plane.
- Bends in the vertical plane.

All sections should be described in terms of a system of parameters, determining the conditions of the robot motion[2].

2. Example of Pipeline Configuration

As an example, let us consider the pipeline configuration represented in Figure 1.

The whole route of the robot motion is split in consecutive sections. For each section, the coordinates of the section beginning and of the section end are determined. The basic parameters of the considered pipeline configuration are given in Table 1.

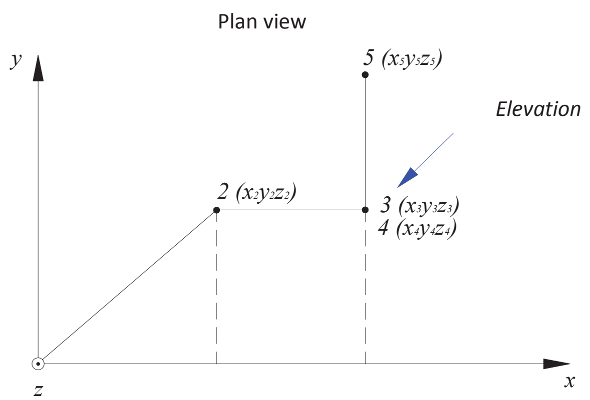


Figure 1. Example of pipeline configuration.

In Table 1, the coordinates of sections “Bend” are determined as one point. Actual schematic of section “Bend” is shown in Figure 2.

3. Mathematical Description of RD Movement Trajectory

The pipeline bend radii are determined, as a rule, in accordance with the documentation for the particular pipeline system. The rest of the parameters are to be calculated by the program based on the coordinates of the beginnings and of the ends of the sections in line with the formulae as follows.

- The angle of the section inclination in relation to the horizontal plane

$$\alpha = \arcsin\frac{z_2 - z_1}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}} \qquad (1)$$

Where (x_1, y_1, z_1) and (x_2, y_2, z_2) are the coordinates of the section beginning and of the section end according to the RD movement direction.

- The spatial angle between sections “a” and “в”

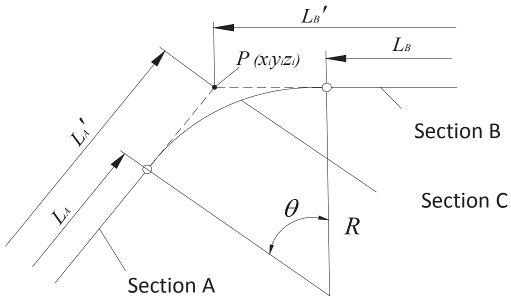
$$\theta = \arccos\frac{a_x 2_x + a_y 2_y + a_z 2_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}\cdot\sqrt{2_x^2 + 2_y^2 + 2_z^2}}, \qquad (2)$$

Where $a_x = x_2 - x_1$; $a_y = y_2 - y_1$; $a_z = z_2 - z_1$;
$\text{в}_x = x_3 - x_2$; $\text{в}_y = y_3 - y_2$; $\text{в}_z = z_3 - z_2$;

Coordinates with index “1” – the beginning of section “a”.

Table 1. Pipeline parameters

Section type	Section coordinates (m)						Section length (m)	Bend angle (°)	Bend radius, (m)
	Beginning			End					
	x	Y	z	x	y	z			
Straight	0	0	0	10	17.3	0	20	-	-
Bend	10	17.3	0	10	17.3	0	-	120	0.6
Straight	10	17.3	0	10	17.3	0	40	-	-
Bend	10	17.3	0	10	17.3	0	-	90	0.45
Straight (vertical)	10	17.3	0	10	17.3	1	1	-	-
Bend	10	17.3	1	10	17.3	1	-	90	0.45
Straight	10	17.3	1	10	17.3	1	25	-	-



$P(x_iy_iz_i)$ – junction point of sections A and B;
L_A and L_B – lengths of straight sections;
L_A' and L_B' – lengths of sections up to the vertices of angles of bend;
Θ – pipeline bend angle;
R – bend radius.

Figure 2. Schematic of “Bend” section.

Coordinates with index “2” – the end of section “a” and the beginning of section “в”.

Coordinates with index “3” – the end of section “в”.

- Section length

$$L_{1,2} = L'_{1,2} - R_1 tg\frac{\theta_1}{2} - R_2 tg\frac{\theta_2}{2}, \tag{3}$$

where $L'_{1,2} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$;

Θ_1 is the angle between the preceding section and the section under consideration.

Θ_2 is the angle between the angle under consideration and the following section.

- The inclination angle of the pipeline bend plane in relation to the horizontal plane

$$\phi = \arccos\frac{\tilde{N}}{\sqrt{A^2+B^2+C^2}} = \begin{cases} \varphi, \text{ if } \varphi \le 90^{\circ} \\ 180 - \varphi, \text{ if } \varphi \ge 90^{\circ} \end{cases} \tag{4}$$

where A, B, C are the solutions to the equation of plane $A{\cdot}X + B{\cdot}Y + C{\cdot}Z + D = 0$, drawn through three points:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix}$$

4. Creating the Model for RD Movement Process

The boundaries of the sections, in which the RD travelling route is split, are the nodal points, where the cable pulling tension is determined, i.e., the force, which has to be applied to the cable in the direction of its movement to make the loading point move along the route with the preset velocity.

To analyze the possibility of such movement, it is necessary to determine the RD pulling force for each nodal point at such position in the pipeline, where the point of the cable connection to the RD is located in the specified nodal point.

Figure 3 shows the boundary of one of the sections (point “1”) and the position of the RD, where the RD pulling force is compared to the cable pulling tension in point “1”. In Figure 3, the position of RD is indicated in bold line.

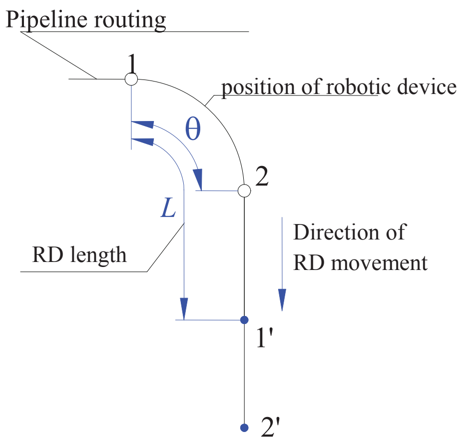


Figure 3. Boundary of the pipeline section.

The force of the cable pulling tension in a specified point of the movement trajectory is understood as a force, required for drawing a part of the cable, situated in the pipeline.

Under such arrangement the RD tail part coincides with point "1", and the RD head coincides with point "1'". The length of curve "1-1'" is equal to the RD length "L". Under these conditions T_1 (cable pulling tension in point "1") is compared to F_1 (the RD pulling force in point "1").

Point "1'" is assumed to be the boundary of an extra section "2–1'".

When the section of the pipeline ending in point "2" is considered, the model of the RD tail part is also placed in point "2", and the head part is placed in point "2'", and it is distanced from point "2" by the length of the RD. It is considered acceptable that the additional control points can be introduced to calculate the RD pulling force and the cable pulling tension.

It should be noted that the RD can be of a considerable length and thus it may occupy several sections in this position, including the nonlinear sections and the bends.

The stated principle is taken as the basis for comparing the cable pulling tension with the corresponding RD pulling force values.

The described comparison of the cable pulling tension "T" and the RD pulling force "F" is carried out for each nodal point of the RD travelling route. In the beginning of the route, the pulling force "F", as a rule, exceeds the cable pulling tension. As the RD moves along the route, the pulling tension alters (normally, increases), and, at a particular distance from the beginning of the route, the condition

$$F \geq T, \tag{5}$$

determining the possibility of movements, will not be met. This condition determines the maximum distance, where the pipeline control can be implemented.

5. Calculating the Cable Pulling Tension at Particular Pipeline Sections

In drawing the cable through each new section, the pulling tension of the cable, i.e., the force, required for further drawing of the cable, alters. In most cases, this force increases.

The RD passing through each pipeline section is associated with alteration in the pulling tension value. The pulling tension value at the beginning of *i* section is identified as T_i, and that at the end of the section is identified as T_{i+1}. The pulling tension value at the end of *i* section corresponds to the pulling tension at the beginning of the following section. Relationship of T_{i+1} and T_i is predetermined by the type of the section (according to Table 1.) and by its characteristics.

Relationship between T_{i+1} and T_i for different types of the pipeline sections is given below:

Straight horizontal section (Figure 4).

$$T_{i+1}(1.0) = T_i + \mu_k \cdot m_k \cdot g \cdot D, \tag{6}$$

Where T_i is the cable pulling tension at the beginning of the section.

T_{i+1} is the cable pulling tension at the end of the section.

μ_k is the cable friction quotient.

m_k is the cable bulk weight.

D is the section length.

i is the sequential number of the section (including the bend sections).

g is 9.8 m/sec^2.

Straight inclined section at the pipeline elevation ($\alpha < 0$) (Figure 5).

$$T_{i+1}(1.1) = T_i + m_k \cdot g \cdot D(\sin\alpha + \mu_k \cos\alpha), \tag{7}$$

Where α is the section inclination angle in relation to the horizontal plane.

At the vertical upward elevation ($\alpha = \frac{\pi}{2}$)

$$T_{i+1}(1.1) = T_i + m_k \cdot g \cdot D \tag{8}$$

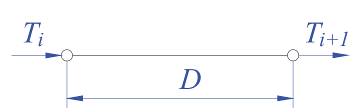


Figure 4. Straight horizontal section.

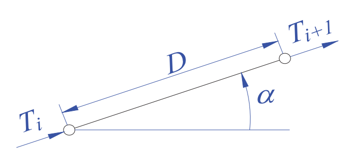


Figure 5. Straight inclined section at pipeline elevation ($\alpha > 0$).

Straight inclined section at the pipeline downward inclination (α < 0) (Figure 6).

$$T_{i+1}(1.2)=T_i+m_k\cdot g\cdot L(\sin\alpha+\mu_k\cos\alpha). \qquad (9)$$

At the vertical downward inclination ($\alpha=-\frac{\pi}{2}$)

$$T_{i+1}(1.2)=T_i+m_k\cdot g\cdot D. \qquad (10)$$

Note: If angle α is given with a sign, then Formula (9) coincides with Formula (7).

Pipeline bend in the horizontal plane (Figure 7).

$$T_{i+1}(2.0)=T_i ch(\mu_k\theta)+\sqrt{T_i^2+(m_k\cdot g\cdot R)^2}\,sh(\mu_k\theta) \quad ,$$

Where

$$\left.\begin{aligned} sh(x)&=\frac{e^x-e^{-x}}{2};\\ ch(x)&=\frac{e^x+e^{-x}}{2}.\end{aligned}\right\} \qquad (11)$$

Here and elsewhere below Θ is given in radians.

The following can be assumed approximately:

$$T_{i+1}=T_i\times e^{\mu_{\cdot}\theta} \qquad (12)$$

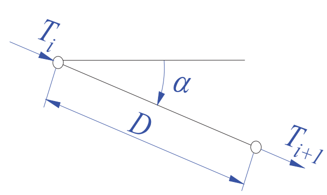


Figure 6. Straight inclined section at pipeline coming down (α < 0).

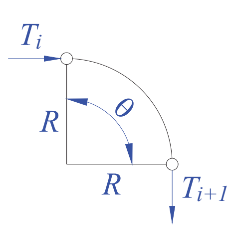


Figure 7. Pipeline bend in the horizontal plane.

The pipeline bend in the vertical plane (at convex upward movement) (Figure 8).

$$T_{i+1}(3.1)=T_i e^{\mu_k\theta}+\frac{m_k gR}{(1+\mu_k^2)}[2\mu_k e^{\mu_k\theta}\sin\theta+(1-\mu_k^2)(1-e^{\mu_k\theta}\cos\theta)] \qquad (13)$$

The pipeline bend in the vertical plane (at convex downward movement) (Figure 9).

$$T_{i+1}(3.2)=T_i e^{\mu_k\theta}+\frac{m_k gR}{(1+\mu_k^2)}[2\mu_k\sin\theta-(1-\mu_k^2)(e^{\mu_k\theta}-\cos\theta)] \qquad (14)$$

The pipeline bend in the vertical plane (at concave upward movement) (Figure 10).

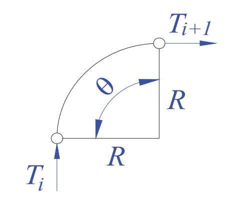


Figure 8. Pipeline bend in the vertical plane (at convex upward movement).

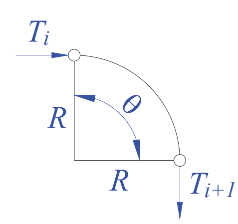


Figure 9. Pipeline bend in the vertical plane (at convex downward movement).

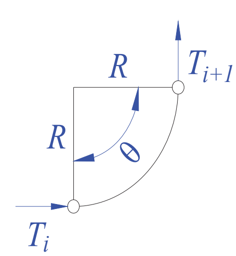


Figure 10. Pipeline bend in the vertical plane (at concave upward movement).

$$T_{i+1}(3.3)=T_i e^{\mu_k \theta}-\frac{m_k gR}{(1+\mu_k^2)}[2\mu_k \sin\theta-(1-\mu_k^2)(e^{\mu_k \theta}-\cos\theta)] \quad (15)$$

The pipeline bend in the vertical plane (at concave downward movement) (Figure 11).

$$T_{i+1}(3.4)=T_i e^{\mu_k \theta}-\frac{m_k gR}{(1+\mu_k^2)}[2\mu_k e^{\mu_k \theta} \sin\theta+(1-\mu_k^2)(1-e^{\mu_k \theta} \cos\theta)] \quad (16)$$

In formulating the algorithm of the model consisting of a set of several sections, in the case when the cable is charged freely into the pipeline inlet, T_0 can be assumed to be equal to zero. The obtained value of T_{i+1} for each section is used as T_i for the subsequent section.

In the case when angle Θ is displaced from the vertical axis by some angle β, the calculations should be done according to the combined formulae.

Exemplified by the pulling tension at convex upward movement, the following is true (Figure 12).

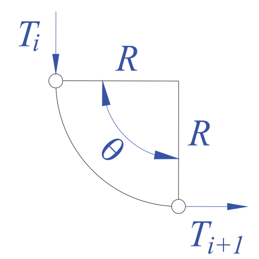


Figure 11. Pipeline bend in the vertical plane (at concave downward movement).

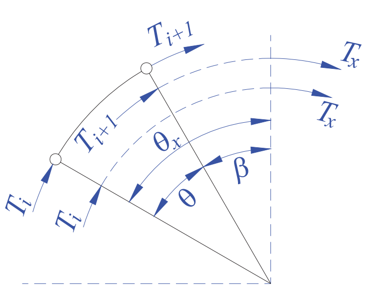


Figure 12. Pipeline bend at convex upward movement.

$$\left.\begin{aligned}
&\theta_x=\theta+\beta\\
&T_x=T_i e^{\mu\theta_x}+\frac{m\cdot g\cdot R}{1+\mu^2}\cdot[2\mu e^{\mu\theta_x}\sin\theta_x+(1-\mu^2)\cdot(1-e^{\mu\theta_x}\cos\theta_x)]\\
&T_{i+1}=\frac{T_x+\frac{m\cdot g\cdot R}{1+\mu^2}\cdot[2\mu e^{\mu\beta}\sin\beta+(1-\mu^2)\cdot(1-e^{\mu\beta}\cos\beta)]}{e^{\mu\beta}}
\end{aligned}\right\} \quad (17)$$

To calculate the pulling tension with Formulae (6) – (17) the information about the parameters of the sections, involved in these calculations, is required. Part of this information, pertaining to the pipeline geometry, is determined according to Formulae (1) – (4).

6. Determining the RD Pulling Force

To ensure the RD movement along the pipeline, the pulling force of its drives has to be higher than the cable pulling tension (see Condition (5).

The RD pulling force depends on the pipeline inclination and configuration. Normally, the cable pulling tension acquires maximum value at the end of the considered section. Therefore, to ensure the comparison of this value with the RD pulling force, its pulling force is also determined at the end of the considered pipeline section.

To undertake calculations of the RD pulling force, its equivalent model has to be built.

The RD is a sequence of mutually connected modules, including the driving (pulling) modules, the measuring and the auxiliary modules. Due to the use of flexible couplings between the modules, it is possible that a flexible chain with redistributed weight, redistributed pulling force and redistributed pipeline thrust strength, with the characteristics corresponding to the characteristics of the real device, can be used as the RD model.

To be more specific, assume that the force of the driving modules is distributed regularly along the RD length (this is, to a large extent, in line with reality). Sliding friction of the chain elements is replaced by the rolling friction of the centering wheels of the robot.

Given the fact that the RD can occupy several sections, the assumed model makes it possible to calculate the value of the RD pulling force according to the formula as follows:

$$F_{pull}=F_0+\sum_{i=1}^{m}\Delta F_i, \quad (18)$$

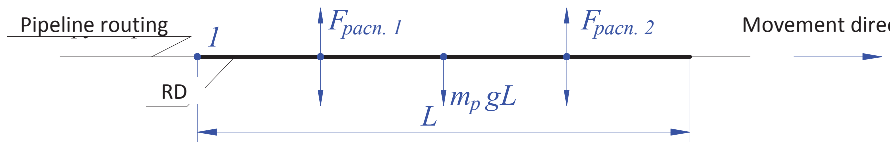


Figure 13. Straight horizontal section.

Where F_0 is the calculated value of the total force of the driving modules;

ΔF_i is the alteration of the pulling force occurring due to the friction and due to the inclination of *i* section of the pipeline;

m is the number of sections where RD is situated.

Below the methodology is given for calculating the value of the RD pulling force alteration in relation to the calculated value for all types of the pipeline sections under consideration. In this case, as compared to the sections, where the cable pulling tension is investigated, the sections related to the pipe bend have been united.

Straight horizontal section (Figure 13).

$$\Delta F(1.0) = -\mu_p L(m_p g + f_{thrust.1} + f_{thrust.2}), \tag{19}$$

where point "1" is the end point of the preceding pipeline section, for which the pulling tension is compared to the RD pulling force;

ΔF is variation of the RD pulling force occurring due to the friction forces, as compared to the calculated value;

L is the RD length;

m_p is the weight of the unit of the RD length,

$$m_p = \frac{\sum_{i=1}^{n} M_i}{L} \tag{20}$$

M_i is the weight of one component of the RD;

n is the number of the RD components;

$f_{thrust1}$ is the distributed thrust force of the wheel pairs and tracks of the driving modules per unit of the RD length;

$f_{thrust2}$ is the distributed thrust force of the rest of the modules per unit of the RD length.

Assume that the thrust force values $f_{thrust1}$ and $f_{thrust2}$ are distributed regularly along the RD length.

Then,

$$f_{thrust1} = \frac{\sum_{i=1}^{n_1} f_{i\ thrust1}}{\mathrm{L}};$$

$$f_{thrust2} = \frac{\sum_{i=1}^{n_2} f_{i\ thrust2}}{\mathrm{L}}$$

where n_1 and n_2 are the number of the driving and of the idle modules accordingly.

Straight inclined section at pipeline elevation ($\alpha > 0$) (Figure 14).

α is the inclination angle of the pipeline section.

$$\Delta F(1.1) = -m_p g L \sin\alpha - \mu_p L(m_p g \cos\alpha + f_{thrust1} + f_{thrust2}) \tag{21}$$

At the vertical elevation

$$\Delta F(1.1) = -m_p g L - \mu_p L(f_{thrust1} + f_{thrust2}) \tag{22}$$

Straight inclined section at the pipeline downward inclination ($\alpha < 0$) (Figure 15).

$$\Delta F(1.2) = -m_p g L \sin\alpha - \mu_p L(m_p g \cos\alpha + f_{thrust1} + f_{thrust2}) \tag{23}$$

At the vertical downward inclination:

$$\Delta F(1.2) = m_p g L - \mu_p L(f_{thrust1} + F_{thrust2}) \tag{24}$$

Note: If the section D length is shorter than the RD length L in Equations (19) – (27), value D should be used instead of value L.

The bend in the horizontal plane (Figure 16).

With regard to formula (12) the following is obtained:

$$\Delta F(2.0) = -T_1(e^{\mu_p \theta} - 1) - \mu_p R\theta(m_p g + f_{thrust1} + f_{thrust2}) \tag{25}$$

where θ is the bend angle.

R is the bend radius.

T_1 is the cable pulling tension in point "1".

The pipeline bend in the inclined plane at convex trajectory movement (Figure 17).

$$\Delta F(3.1-3.2)=-T_1(e^{\mu_p\theta}-1)-m_p gR[\cos(\theta-\alpha)-\cos\alpha]\cdot\sin\phi-$$
$$-\mu_p[m_p gR\cdot A\cdot(1+\frac{\theta-A}{A}\cdot\cos\phi)+(f_{thrust1}+f_{thrust2})\cdot R\theta], \qquad (26)$$

where $A=[\sin(\alpha-\theta)-\sin\alpha]$, $-90^{\circ} < \alpha < 90^{\circ}$;

φ is the inclination angle of the bend plane in relation to the horizontal plane, $0 < \varphi < 90^{\circ}$. At $\varphi = 0$ the results of the calculations in Formulae (26) and (25) coincide.

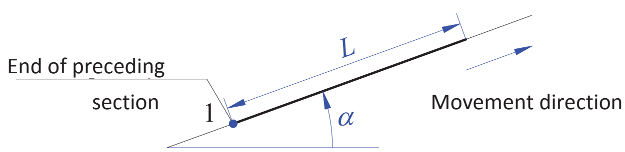


Figure 14. Straight inclined section at pipeline elevation (α > 0).

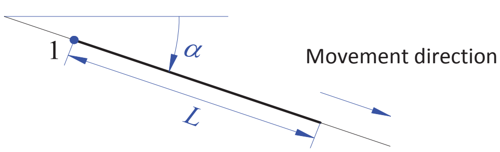


Figure 15. Straight inclined section at pipeline coming down (α < 0).

Angle Θ can take the values, at which points "1" and "2" are located above the axis of abscissas. Here and elsewhere below Θ is given in radians.

The pipeline bend in the inclined plane at the concave trajectory movement (Figure 18).

Points "1" and "2" should be located below the axis of abscissas.

$$\Delta F(3.3-3.4)=-T_1(e^{\mu_p\theta}-1)+m_p gR[\cos(\theta+\alpha)-\cos\alpha]\cdot\sin\phi-$$
$$-\mu_p\{m_p gRB(1+\frac{\theta-B}{B}\cdot\cos\phi)+(f_{thrust1}+f_{thrust2})\cdot R\theta\}, \qquad (27)$$

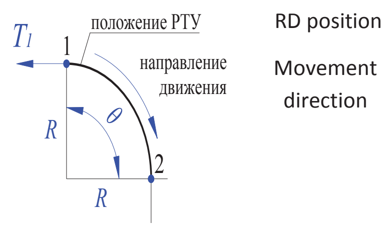
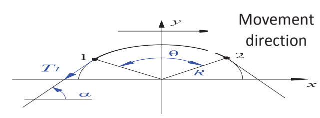
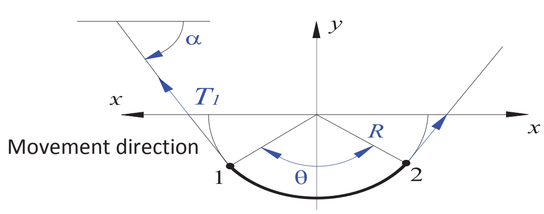


Figure 16. Bend in the horizontal plane.



1 –End of preceding section.
2 –Beginning of the following section.
R –Bend radius.
Θ –Bend angle (always positive).
α –Angle of bends of the preceding section.

Figure 17. Pipeline bend in inclined plane at convex trajectory movement.



1 – End of preceding section.
2 – Beginning of the following. Section.
R – Bend radius.
Θ – The bend angle.
α – Inclination angle of the preceding section, - 90° < α < 90°.

Figure 18. Pipeline bend in inclined plane at concave trajectory movement.

where $B = [\sin(\alpha+\theta) - \sin\alpha]$.

Considering here the spatial location of the pipeline, determined by the coordinates of the nodal points, the task of identifying the type of bend becomes urgent. The bend type depends on the position of the bend angle in the bend plane (convex or concave bend).

The inclination angle of the bend plane in relation to the horizontal plane is determined according to Formula (4). At inclination angle φ = 0 the bend takes place in the horizontal plane, and at φ = 90° it takes place in the vertical plane. φ alters from 0 to 90°.

The bend type (convex or concave) is determined by the relative spatial positions of the linear sections that make the bend angle.

The bend is considered convex if coordinate Z of the bend angle vertex is higher than the coordinate Z of the projection of this vertex on the line, connecting the beginning of the first and the end of the second linear sections.

Assume that the coordinates of the points at the beginning of the first section, at the angle vortex and at the end of the second section are designated respectively:

$$A\ (x_A, y_A, z_A);\ S\ (x_S, y_S, z_S);\ B\ (x_B, y_B, z_B),$$

Then the condition of the bend trajectory convexity will be as follows:

$$z_S \geq z_K , \tag{6.11}$$

where

$$z_K = z_A + (z_B - z_A)\cdot\frac{(x_S - x_A)\cdot(x_B - x_A) + (y_S - y_A)\cdot(y_B - y_A) + (z_S - z_A)\cdot(z_B - z_A)}{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

Otherwise, the vertical bend is considered concave.

7. Formulating the General Model for the Process of RD Motion Inside the Pipeline

7.1 Determining the Cable Pulling Tension along the Entire Control Route

In this example, the pipeline data from Table 1 will be used. The equations for determining the pulling tension values at all sections can be represented as follows:

$$\left.\begin{array}{l}
T0 = 0, \\
T_1(1.0) = V_{1.0}(T_0, m_k, \mu_k, D_1), \\
T_2(2.0) = V_{2.0}(T_1, m_k, \mu_k, R_2), \\
T_3(1.0) = V_{1.0}(T_2, m_k, \mu_k, D_3), \\
T_4(3.3) = V_{3.3}(T_3, m_k, \mu_k, R_4, \theta_4), \\
T_5(1.2) = V_{1.2}(T_4, m_k, D_5), \\
T_6(3.1) = V_{3.1}(T_5, \mu_k, m_k, R_6, \theta_6), \\
T_7(1.0) = V_{1.0}(T_6, \mu_k, m_k, D_1).
\end{array}\right\} \tag{28}$$

The following designations have been used:

T_0 is the cable pulling tension at the pipeline inlet;

T_i(x.x) is the cable pulling tension at the end of *i* section, the model of which is designated as x.x.

$V_{\hat{o}.\hat{o}}(T_{i-1}, m_k, \mu_k, R_4, \theta_4)$ is an example of the function expressing the dependency of T_i on T_{i-1} and on the parameters of section *i* according to the Formulae (19) – (27).

The value of tension at each of the following sections is calculated using the value of tension at the preceding section.

7.2 Determining the RD Pulling Force at the Pipeline Control Route

Equations for determining the RD pulling force in the pipeline can be represented as follows:

$$\left.\begin{array}{l}
F_1 = F_0 + \Delta F(2.0) = W_{1,2}(F_0, T_1, m_p, \mu_p, \theta_2, L, f_{thrust1}, f_{thr} \\
F_2 = F_0 + \Delta F(1.0) = W_{2,3}(F_0, \mu_p, m_p, L, f_{thrust1}, f_{thru} \\
F_3 = F_0 + \Delta F(3.3) = W_{3,4}(F_0, T_3, m_p, \mu_p, \theta_4, R_4, L, f_{thrust1}, f_{thru} \\
F_4 = F_0 + \Delta F(1.2) = W_{4,5}(F_0, m_p, \mu_p, \phi_5, L, f_{thrust1}, f_{thru} \\
F_5 = F_0 + \Delta F(3.1) = W_{5,6}(F_0, T_5, m_p, \mu_p, L, R_6, \theta_6, f_{thrust1}, f_{thru} \\
F_6 = F_0 + \Delta F(1.2) = W_{6,7}(F_0, \mu_p, m_p, L, \phi_7, f_{thrust1}, f_{thrust2}).
\end{array}\right\} \tag{29}$$

where F_0 is the calculated value of the RD pulling force;

$W_{i,(i+1)}(F_0, T_i, \mu_p, m_p, \theta, \phi, L, R, f_{thrust1}, f_{thrust2})$ is an example of the function, expressing the relationship between the RD pulling force at i section and the value of Ti and the parameters of section (i+1) and of parameters of the following sections, where the RD is situated, according to the Formulae (19) – (27).

In the given formulae the index designating the RD pulling force coincides with the index of the section, at the end of which the pulling tension is calculated. However, at this moment, the RD is located in the following section of the pipeline, and its pulling force depends on the characteristics of that particular section. Therefore, the parameter values of that section in which the RD is situated, are introduced into the formula.

The conditions for potential RD travelling up to and including the section with number "n" can be represented as follows:

$$F_i \geq T_i \,, \qquad (30)$$

For all 1 < i < n.

Failure to meet the Condition (27) at any section means that the RD will not be able to overcome this force, and its further movement in the same direction is impossible.

The suggested mathematical model of a robotic device for evaluating the distance of its travelling in the complex shape pipeline has been realized in MatLab software environment. The input data for the program are the coordinates of the control route sections and such RD parameters as module length, module weight, cable weight, friction quotients of the modules and of the cable at the pipe surface.

To verify the practicability of the mathematical model, the conditional parameters of the robotic device and the coordinates of the pipeline sections were entered into the program.

The results of the program performance are represented by two plots. In one of them, the schematic of the controlled pipeline is represented, taking into account its configuration. The second plot represents the RD movement in the pipeline, and the section is marked, where its operation stops, i.e., where the cable pulling tension becomes higher than the RD pulling force.

Thus, assisted by a mathematical model, it is possible to evaluate the maximum distance of control. Also, for

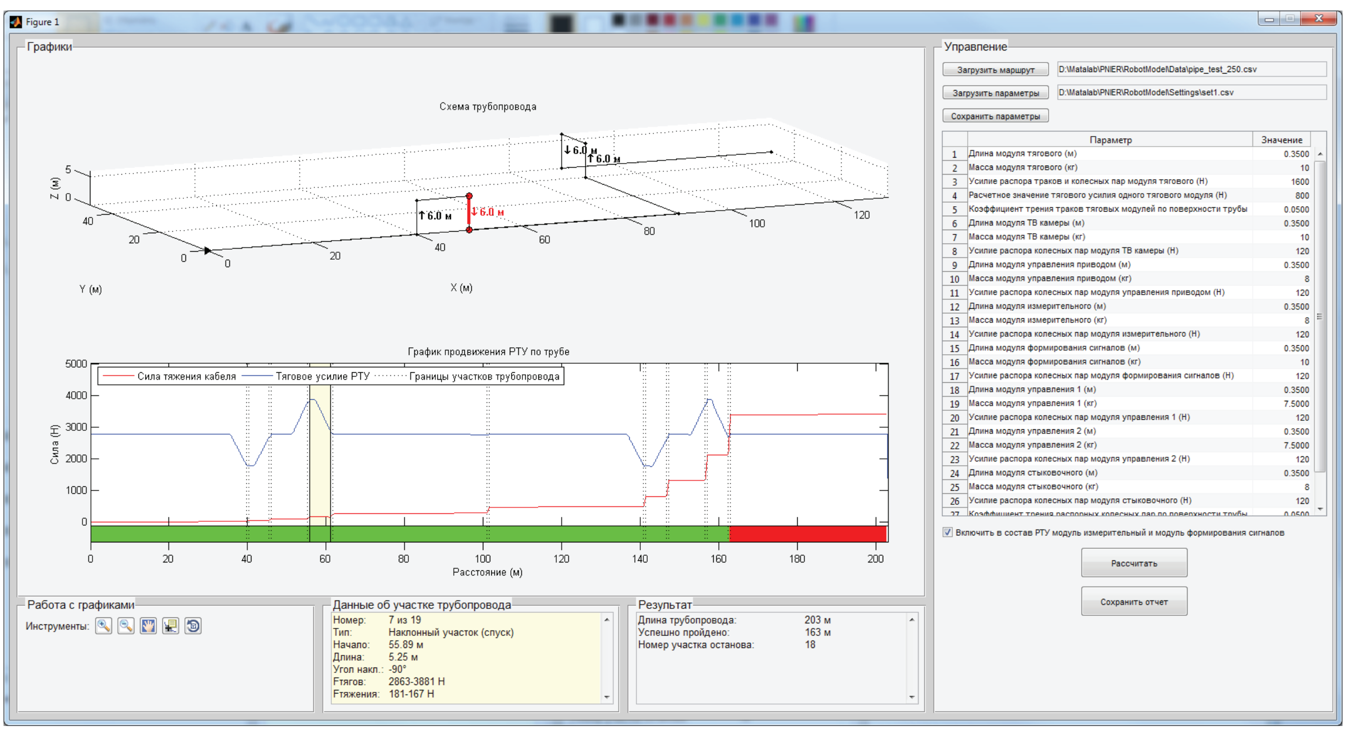


Figure 19. Results of building the RD mathematical model in the MatLab software environment.

each pipeline section it is possible to obtain the model results, namely: Cable pulling tension, robotic device pulling force; the number, the inclination angle and the coordinates of the sections.

8. Conclusion

Developing the methods and means of operative control fulfilling the determining function in the complex maintenance of the pipelines is a prerequisite for accident-free operation, a precondition for considerable savings of material and labor resources. In executing the in-line diagnostics such control can be carried out only with a robotic device, which enables delivering the measuring system to the controlled section of the pipe. To evaluate the travelling distance the mathematical model for the robot has been developed and represented in this study.

In the course of developing the mathematical model, the following results have been achieved:

- The mathematical description of the RD movement trajectory has been given.
- The model for the RD movement process has been created.
- The calculations of the cable pulling tension at particular sections of the pipeline have been carried out.
- The RD pulling force has been determined.
- The general model for the RD movement process in the pipeline have been formulated.
- The maximum distance of the RD travelling has been identified.

The suggested mathematical model for the robotic device has been realized in MatLab software environment. This model makes it possible to calculate the pipeline length accessible for control, taking into account the effects of the pipeline geometry, as well as to obtain the data for each of the pipeline section under control.

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10. References

1. In-line control of process pipelines; Available from: https://introscan.ru/
2. Pipeline sections; Available from: http://www.ngpedia.ru/
3. NS 130 specification for laying underground cables up to and including 11kV; 2013.