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The Comparison of the Performance of Prandtl| Mixing Length, Turbulence Kinetic Energy, K-e, RNG and LES Turbulence Models in Simulation of the Positive Wave Motion Caused by Dam Break on the Erodible Bed

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Abstract

Background/Objectives: Dam-break problem study has significant importance in the hydraulic study of the sudden floods. The aim of this research was a prediction of movement pattern of the positive wave. **Methods/Statistical Analysis:** This research was based on numerical technique of Volume of Fluid (VOF). Flow-3D software was used for simulation. For the first time, the performance of all turbulence models was evaluated in the calculation of free flow surface and bed profiles. Two statistical methods of the regression and relative error were used for determining of accuracy of simulation results. The Spine wine (2005) experimental results were used for validation of numerical simulation. **Findings:** The results of two statistical methods of regression and normal relative error in comparison between experimental and numerical data showed that the LES turbulence model has the highest efficiency in predicting the free flow surface and sedimentary bed profiles and dimension-less profiles of free surface of the flow, distance- time, velocity-time and acceleration-time of progress of positive wave front. Two last dimensions-less profiles, for the first time were introduced in this research. Based on statistical results the K-e, RNG, Prandtle Mixing Length and Turbulence Kinetic Energy have less efficiency than the LES model, respectively. None of the models were able to predict changes of sedimentary bed appropriately. Therefore, the LES model is most accurate turbulence model in predicting of a positive wave caused by a dam break in mobile bed. The results of this research were compatible with numerical researches on dam-break problem based on VOF technique. **Applications:** In numerical solving of dam-break hydraulic problem, selection of turbulence model has a significant effect on accuracy of simulation. The VOF technique is fast and powerful method for this purpose.

Keywords: Dam Break, Flow-3D, Mobile Bed, Turbulence Models, VOF

1. Introduction

Dam break in its ideal form is a hydraulic problem that includes the formation of an unsteady flow as a waveform which is causes due to sudden placing or removing an obstacle such as a gate valve in the path of a channel. The flow appears as the propagation of positive and negative flows respectively in the downstream and upstream of its formation point. The main reason for studying the waves caused by sudden dam break, is prediction of behavior of sudden floods produced by its. These floods are much larger than the floods created by heavy rainfall or snowmelt runoff in terms of size and created damages. Today, this point has great importance in the fields of exploiting water resources, environmental protection, ecological management, etc. This importance stems from

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the catastrophic nature of sudden dam failures and loss of life and property caused by it[1]. Some the causes of dam failure are weaknesses in structural designing and measures related to maintenance, overflowing from the dam crest, leakage from the walls and foundation and earthquake[2].

On the other hand, erosion and sediment movement in unsteady flows in both forms of suspended and bed load is very important on the hydraulic structures design, water resource and river morphology, etc[3-8].

The first study on the dam failure was conducted by Ritter[9]. Ritter solution was an ideal analytical solution based on the shallow water Saint-Venant equations. In 1952[10], presented a two-dimensional solution by adding Chezy resistance to nonlinear shallow water equations for the dam failure problem. Also, some researcher's[11-14] proposed theoretical solutions based on the Saint-Venant equations to simulate shallow water flow caused by the dam failure. In general, studies on the dam break are divided to two groups of laboratory studies[15-24] and mathematical studies including analytical and numerical solutions.

Compared the approximate models of flood routing caused by dam break, based on the Saint-Venant equation[25]. Town son and Al-salihi[26] modeled the movement of positive wave caused by dam break with criteria approach, and solving shallow water equations, Used a one-dimensional numerical model for solving shallow water equations[27]. The effect of viscosity in the flow which is caused by the dam break on a horizontal bed[28]. Studied the dam failure on steep slopes by providing an exact solution for shallow water equations[29], used a Lagrangian particle-based approach in the simulation of dam break flow[30]. Conducted an experimental– numerical research, and studied the effects of the abrupt change in slope and channel topography on the dam break flow by using shallow water models and averaged Reynolds equations[31].

Used Godunov approach in numerical solution of shallow water equations to simulate the dam failure[32,33]. Using finite volume technique to solve Navier-Stokes averaged Reynolds equations for simulation of flow of dam break on uneven beds[34]. Used an experimental-numerical approach for modeling the encounter of shock waves caused by the dam break, on the vertical wall located in the lower end of the experimental model[35].

As it can be seen, in the numerical approaches which were used by previous researchers to solve dam failure problem, no comprehensive-comparative study is conducted on turbulence models, as an important parameter which is effective on the simulation results in the dam break phenomenon on a mobile bed.

Therefore, in this study, the efficiency of zero-equation turbulence models of Prandtl mixing length, turbulent kinetic energy (one equation), K-e, Renormalized Groups (RNG) and Large Eddy Simulation (LES) were compared in simulation of positive wave movement created by dam break on erodible sand bed. Flow-3D software version 10.0.1 was used in simulation.

The results of the model were compared with results of an experimental model of dam break on mobile bed[36].

2. Materials and Methods

The study is based on computer modeling of dam break on mobile bed by using Flow-3D software. Simulations in the software were based on the governing equations of RANS on the movement. Further, the efficiency of turbulence models as effective factors on the accuracy of numerical model results were compared.

Table 2. Experimental mixing length model

Experimental						Model					
x(m)	H(m)	(H-.15)	H*	X	X-3	x(m)	H(m)	(H-.15)	H*	X	X-3
2.0667	0.3758	0.2258	0.6452	3.0630	0.0630	2.0694	0.3844	0.2344	0.6697	3.0670	0.0670
2.1084	0.3735	0.2235	0.6387	3.1249	0.1249	2.1006	0.3821	0.2321	0.6630	3.1133	0.1133
2.1474	0.3713	0.2213	0.6323	3.1826	0.1826	2.1469	0.3786	0.2286	0.6530	3.1819	0.1819
2.1892	0.3690	0.2190	0.6258	3.2445	0.2445	2.1926	0.3751	0.2251	0.6430	3.2496	0.2496
2.2226	0.3662	0.2162	0.6177	3.2940	0.2940	2.2227	0.3726	0.2226	0.6361	3.2942	0.2942
2.2616	0.3623	0.2123	0.6065	3.3518	0.3518	2.2672	0.3691	0.2191	0.6260	3.3602	0.3602
2.3033	0.3600	0.2100	0.6000	3.4137	0.4137	2.3111	0.3656	0.2156	0.6160	3.4252	0.4252
2.3395	0.3566	0.2066	0.5903	3.4673	0.4673	2.3399	0.3633	0.2133	0.6093	3.4679	0.4679
2.3785	0.3538	0.2038	0.5823	3.5251	0.5251	2.3684	0.3609	0.2109	0.6027	3.5101	0.5101
2.4174	0.3510	0.2010	0.5742	3.5828	0.5828	2.4104	0.3575	0.2075	0.5928	3.5724	0.5724
2.4536	0.3465	0.1965	0.5613	3.6365	0.6365	2.4517	0.3541	0.2041	0.5832	3.6336	0.6336

2.4898	0.3419	0.1919	0.5484	3.6901	0.6901	2.4921	0.3508	0.2008	0.5737	3.6935	0.6935
2.5260	0.3391	0.1891	0.5403	3.7437	0.7437	2.5316	0.3475	0.1975	0.5643	3.7520	0.7520
2.5622	0.3369	0.1869	0.5339	3.7973	0.7973	2.5702	0.3450	0.1950	0.5572	3.8093	0.8093
2.6039	0.3346	0.1846	0.5274	3.8592	0.8592	2.6079	0.3412	0.1912	0.5462	3.8651	0.8651
2.6401	0.3318	0.1818	0.5194	3.9129	0.9129	2.6446	0.3381	0.1881	0.5376	3.9195	0.9195
2.6791	0.3278	0.1778	0.5081	3.9706	0.9706	2.6804	0.3352	0.1852	0.5291	3.9725	0.9725
2.7209	0.3250	0.1750	0.5000	4.0325	1.0325	2.7264	0.3314	0.1814	0.5182	4.0407	1.0407
2.7515	0.3227	0.1727	0.4935	4.0779	1.0779	2.7597	0.3286	0.1786	0.5103	4.0900	1.0900
2.8016	0.3177	0.1677	0.4790	4.1522	1.1522	2.8023	0.3251	0.1751	0.5003	4.1532	1.1532
2.8406	0.3160	0.1660	0.4742	4.2099	1.2099	2.8429	0.3218	0.1718	0.4908	4.2134	1.2134
2.8712	0.3137	0.1637	0.4677	4.2553	1.2553	2.8720	0.3194	0.1694	0.4840	4.2565	1.2565
2.9102	0.3103	0.1603	0.4581	4.3131	1.3131	2.9178	0.3156	0.1656	0.4732	4.3244	1.3244
2.9436	0.3069	0.1569	0.4484	4.3626	1.3626	2.9436	0.3135	0.1635	0.4671	4.3626	1.3626
2.9881	0.3047	0.1547	0.4419	4.4286	1.4286	2.9837	0.3103	0.1603	0.4579	4.4220	1.4220
3.0271	0.3024	0.1524	0.4355	4.4864	1.4864	3.0295	0.3065	0.1565	0.4472	4.4899	1.4899
3.0661	0.2996	0.1496	0.4274	4.5441	1.5441	3.0595	0.3041	0.1541	0.4402	4.5344	1.5344
3.1051	0.2996	0.1496	0.4274	4.6019	1.6019	3.1045	0.3002	0.1502	0.4291	4.6011	1.6011
3.1496	0.2990	0.1490	0.4258	4.6680	1.6680	3.1495	0.2967	0.1467	0.4193	4.6679	1.6679
3.1886	0.2979	0.1479	0.4226	4.7258	1.7258	3.1946	0.2932	0.1432	0.4091	4.7346	1.7346
3.2248	0.2973	0.1473	0.4210	4.7794	1.7794	3.2246	0.2908	0.1408	0.4023	4.7791	1.7791
3.2582	0.2934	0.1434	0.4097	4.8289	1.8289	3.2546	0.2885	0.1385	0.3957	4.8236	1.8236
3.3028	0.2917	0.1417	0.4048	4.8950	1.8950	3.3147	0.2835	0.1335	0.3815	4.9127	1.9127
3.3445	0.2889	0.1389	0.3968	4.9568	1.9568	3.3448	0.2815	0.1315	0.3758	4.9572	1.9572
3.3807	0.2855	0.1355	0.3871	5.0105	2.0105	3.3899	0.2785	0.1285	0.3671	5.0240	2.0240
3.4197	0.2838	0.1338	0.3823	5.0683	2.0683	3.4199	0.2764	0.1264	0.3611	5.0686	2.0686
3.4615	0.2815	0.1315	0.3758	5.1301	2.1301	3.4650	0.2733	0.1233	0.3524	5.1354	2.1354
3.5004	0.2781	0.1281	0.3661	5.1879	2.1879	3.5101	0.2706	0.1206	0.3445	5.2022	2.2022
3.5366	0.2748	0.1248	0.3565	5.2415	2.2415	3.5402	0.2682	0.1182	0.3378	5.2468	2.2468
3.5728	0.2725	0.1225	0.3500	5.2952	2.2952	3.5702	0.2661	0.1161	0.3318	5.2914	2.2914
3.6146	0.2708	0.1208	0.3452	5.3571	2.3571	3.6153	0.2633	0.1133	0.3237	5.3582	2.3582
3.6536	0.2697	0.1197	0.3419	5.4149	2.4149	3.6605	0.2602	0.1102	0.3148	5.4251	2.4251
3.6953	0.2680	0.1180	0.3371	5.4768	2.4768	3.6906	0.2581	0.1081	0.3090	5.4697	2.4697
3.7343	0.2657	0.1157	0.3306	5.5345	2.5345	3.7357	0.2555	0.1055	0.3013	5.5366	2.5366
3.7761	0.2635	0.1135	0.3242	5.5964	2.5964	3.7808	0.2522	0.1022	0.2919	5.6035	2.6035
3.8123	0.2606	0.1106	0.3161	5.6501	2.6501	3.8109	0.2502	0.1002	0.2864	5.6481	2.6481
3.8512	0.2567	0.1067	0.3048	5.7078	2.7078	3.8561	0.2474	0.0974	0.2782	5.7150	2.7150
3.8874	0.2550	0.1050	0.3000	5.7615	2.7615	3.8862	0.2455	0.0955	0.2729	5.7596	2.7596
3.9320	0.2533	0.1033	0.2952	5.8275	2.8275	3.9313	0.2428	0.0928	0.2650	5.8265	2.8265
3.9738	0.2527	0.1027	0.2935	5.8894	2.8894	3.9765	0.2402	0.0902	0.2578	5.8934	2.8934
4.0100	0.2505	0.1005	0.2871	5.9431	2.9431	4.0066	0.2382	0.0882	0.2520	5.9380	2.9380
4.0517	0.2488	0.0988	0.2823	6.0050	3.0050	4.0517	0.2356	0.0856	0.2445	6.0050	3.0050
4.0907	0.2471	0.0971	0.2774	6.0627	3.0627	4.0969	0.2330	0.0830	0.2372	6.0719	3.0719
4.1325	0.2448	0.0948	0.2710	6.1246	3.1246	4.1270	0.2313	0.0813	0.2324	6.1165	3.1165
4.1687	0.2431	0.0931	0.2661	6.1783	3.1783	4.1571	0.2296	0.0796	0.2276	6.1612	3.1612
4.2132	0.2409	0.0909	0.2597	6.2443	3.2443	4.2174	0.2264	0.0764	0.2183	6.2504	3.2504
4.2522	0.2392	0.0892	0.2548	6.3021	3.3021	4.2475	0.2245	0.0745	0.2129	6.2951	3.2951
4.2856	0.2358	0.0858	0.2452	6.3516	3.3516	4.2776	0.2230	0.0730	0.2085	6.3397	3.3397
4.3274	0.2330	0.0830	0.2371	6.4135	3.4135	4.3228	0.2206	0.0706	0.2018	6.4067	3.4067
4.3691	0.2313	0.0813	0.2323	6.4754	3.4754	4.3680	0.2183	0.0683	0.1951	6.4736	3.4736
4.4081	0.2307	0.0807	0.2306	6.5332	3.5332	4.3981	0.2167	0.0667	0.1906	6.5183	3.5183
4.4499	0.2290	0.0790	0.2258	6.5951	3.5951	4.4433	0.2144	0.0644	0.1839	6.5852	3.5852

4.4889	0.2268	0.0768	0.2194	6.6529	3.6529	4.4884	0.2120	0.0620	0.1773	6.6522	3.6522
4.5306	0.2240	0.0740	0.2113	6.7148	3.7148	4.5336	0.2100	0.0600	0.1716	6.7192	3.7192
4.5613	0.2206	0.0706	0.2016	6.7601	3.7601	4.5637	0.2084	0.0584	0.1667	6.7638	3.7638
4.6058	0.2189	0.0689	0.1968	6.8262	3.8262	4.6089	0.2062	0.0562	0.1605	6.8308	3.8308
4.6448	0.2160	0.0660	0.1887	6.8839	3.8839	4.6390	0.2048	0.0548	0.1564	6.8754	3.8754
4.6754	0.2110	0.0610	0.1742	6.9293	3.9293	4.6692	0.2033	0.0533	0.1524	6.9200	3.9200
4.7144	0.2093	0.0593	0.1694	6.9870	3.9870	4.7143	0.2013	0.0513	0.1465	6.9870	3.9870
4.7534	0.2076	0.0576	0.1645	7.0448	4.0448	4.7445	0.2000	0.0500	0.1427	7.0316	4.0316
4.7923	0.2053	0.0553	0.1581	7.1026	4.1026	4.7896	0.1980	0.0480	0.1373	7.0986	4.0986
4.8369	0.2042	0.0542	0.1548	7.1686	4.1686	4.8348	0.1962	0.0462	0.1321	7.1655	4.1655
4.8731	0.2042	0.0542	0.1548	7.2223	4.2223	4.8649	0.1945	0.0445	0.1272	7.2102	4.2102
4.9177	0.2025	0.0525	0.1500	7.2883	4.2883	4.9101	0.1926	0.0426	0.1218	7.2771	4.2771
4.9567	0.2025	0.0525	0.1500	7.3461	4.3461	4.9553	0.1908	0.0408	0.1165	7.3441	4.3441
4.9984	0.2019	0.0519	0.1484	7.4080	4.4080	4.9854	0.1896	0.0396	0.1131	7.3887	4.3887
5.0374	0.2014	0.0514	0.1468	7.4659	4.4659	5.0305	0.1879	0.0379	0.1083	7.4556	4.4556
5.0764	0.2014	0.0514	0.1468	7.5236	4.5236	5.0757	0.1863	0.0363	0.1037	7.5225	4.5225
5.1182	0.1980	0.0480	0.1371	7.5855	4.5855	5.1058	0.1852	0.0352	0.1007	7.5672	4.5672
5.1544	0.1952	0.0452	0.1290	7.6392	4.6392	5.1509	0.1837	0.0337	0.0963	7.6341	4.6341
5.1934	0.1935	0.0435	0.1242	7.6969	4.6969	5.1961	0.1821	0.0321	0.0917	7.7010	4.7010
5.2324	0.1935	0.0435	0.1242	7.7547	4.7547	5.2262	0.1812	0.0312	0.0892	7.7456	4.7456
5.2769	0.1906	0.0406	0.1161	7.8208	4.8208	5.2713	0.1794	0.0294	0.0840	7.8125	4.8125
5.3103	0.1867	0.0367	0.1048	7.8702	4.8702	5.3164	0.1780	0.0280	0.0800	7.8794	4.8794
5.3521	0.1856	0.0356	0.1016	7.9322	4.9322	5.3465	0.1770	0.0270	0.0772	7.9240	4.9240
5.3910	0.1827	0.0327	0.0935	7.9899	4.9899	5.3917	0.1757	0.0257	0.0733	7.9908	4.9908
5.4300	0.1805	0.0305	0.0871	8.0477	5.0477	5.4368	0.1744	0.0244	0.0696	8.0577	5.0577
5.4718	0.1794	0.0294	0.0839	8.1096	5.1096	5.4668	0.1736	0.0236	0.0673	8.1023	5.1023
5.5108	0.1760	0.0260	0.0742	8.1674	5.1674	5.5119	0.1724	0.0224	0.0640	8.1691	5.1691
5.5497	0.1743	0.0243	0.0694	8.2251	5.2251	5.5420	0.1717	0.0217	0.0620	8.2137	5.2137
5.5915	0.1726	0.0226	0.0645	8.2870	5.2870	5.5871	0.1707	0.0207	0.0591	8.2805	5.2805
5.6305	0.1709	0.0209	0.0597	8.3448	5.3448	5.6322	0.1697	0.0197	0.0563	8.3473	5.3473
5.6583	0.1658	0.0158	0.0452	8.3860	5.3860	5.6472	0.1694	0.0194	0.0554	8.3696	5.3696
5.6889	0.1590	0.0090	0.0258	8.4314	5.4314	5.6772	0.1688	0.0188	0.0536	8.4141	5.4141
5.7139	0.1540	0.0040	0.0113	8.4685	5.4685	5.7073	0.1682	0.0182	0.0519	8.4586	5.4586

Table 3. Prandtle mixing length model

	t(sec)	x(m)	x revised(m)	g(ms^{-2})	H0(m)	(H0/g)^.5	X*	t*	U*	a*
	0.0000	3.0000	0.0000	9.8060	0.3500	0.1889	0.0000	0.0000	-	-
Prandtle Model	0.2500	3.1945	0.1945	9.8060	0.3500	0.1889	0.5557	1.3233	0.4200	0.3174
	0.5000	4.0969	1.0969	9.8060	0.3500	0.1889	3.1340	2.6466	1.1842	0.4474
	0.7500	5.0300	2.0300	9.8060	0.3500	0.1889	5.8000	3.9698	1.4610	0.3680
	1.0000	5.9620	2.9620	9.8060	0.3500	0.1889	8.4629	5.2931	1.5988	0.3021

Table 4. Prandtle mixing parameters

(Hexp)i	(Hmodel)i	(Hexp-Hmodel)i^2	(Hexp)^2
(m)	(m)		
0.37581	0.38439	0.00007	0.14123
0.37355	0.38206	0.00007	0.13954
0.37129	0.37855	0.00005	0.13786
0.36903	0.37507	0.00004	0.13618
0.36621	0.37263	0.00004	0.13411

0.36226	0.36910	0.00005	0.13123
0.36000	0.36559	0.00003	0.12960
0.35661	0.36326	0.00004	0.12717
0.35379	0.36095	0.00005	0.12517
0.35097	0.35749	0.00004	0.12318
0.34645	0.35412	0.00006	0.12003
0.34194	0.35079	0.00008	0.11692
0.33911	0.34752	0.00007	0.11500
0.33685	0.34502	0.00007	0.11347
0.33460	0.34119	0.00004	0.11196
0.33177	0.33814	0.00004	0.11007
0.32782	0.33519	0.00005	0.10747
0.32500	0.33139	0.00004	0.10562
0.32274	0.32859	0.00003	0.10416
0.31766	0.32510	0.00006	0.10091
0.31597	0.32178	0.00003	0.09984
0.31371	0.31939	0.00003	0.09841
0.31032	0.31563	0.00003	0.09630
0.30694	0.31349	0.00004	0.09421
0.30468	0.31025	0.00003	0.09283
0.30242	0.30651	0.00002	0.09146
0.29960	0.30407	0.00002	0.08976
0.29960	0.30017	0.00000	0.08976
0.29903	0.29674	0.00001	0.08942
0.29790	0.29317	0.00002	0.08875
0.29734	0.29081	0.00004	0.08841
0.29339	0.28848	0.00002	0.08608
0.29169	0.28353	0.00007	0.08509
0.28887	0.28154	0.00005	0.08345
0.28548	0.27848	0.00005	0.08150
0.28379	0.27640	0.00005	0.08054
0.28153	0.27333	0.00007	0.07926
0.27815	0.27058	0.00006	0.07736
0.27476	0.26824	0.00004	0.07549
0.27250	0.26612	0.00004	0.07426
0.27081	0.26329	0.00006	0.07334
0.26968	0.26019	0.00009	0.07273
0.26798	0.25814	0.00010	0.07182
0.26573	0.25547	0.00011	0.07061
0.26347	0.25217	0.00013	0.06942
0.26065	0.25022	0.00011	0.06794
0.25669	0.24738	0.00009	0.06589
0.25500	0.24552	0.00009	0.06502
0.25331	0.24275	0.00011	0.06416
0.25274	0.24024	0.00016	0.06388
0.25048	0.23818	0.00015	0.06274
0.24879	0.23557	0.00017	0.06190
0.24710	0.23300	0.00020	0.06106
0.24484	0.23133	0.00018	0.05995

0.24315	0.22965	0.00018	0.05912
0.24089	0.22640	0.00021	0.05803
0.23919	0.22453	0.00022	0.05721
0.23581	0.22299	0.00016	0.05560
0.23298	0.22063	0.00015	0.05428
0.23129	0.21828	0.00017	0.05350
0.23073	0.21671	0.00020	0.05323
0.22903	0.21438	0.00021	0.05246
0.22677	0.21204	0.00022	0.05143
0.22395	0.21005	0.00019	0.05015
0.22056	0.20835	0.00015	0.04865
0.21887	0.20618	0.00016	0.04790
0.21605	0.20475	0.00013	0.04668
0.21097	0.20333	0.00006	0.04451
0.20927	0.20127	0.00006	0.04380
0.20758	0.19996	0.00006	0.04309
0.20532	0.19805	0.00005	0.04216
0.20419	0.19624	0.00006	0.04170
0.20419	0.19452	0.00009	0.04170
0.20250	0.19263	0.00010	0.04101
0.20250	0.19078	0.00014	0.04101
0.20194	0.18960	0.00015	0.04078
0.20137	0.18791	0.00018	0.04055
0.20137	0.18629	0.00023	0.04055
0.19798	0.18525	0.00016	0.03920
0.19516	0.18372	0.00013	0.03809
0.19347	0.18210	0.00013	0.03743
0.19347	0.18121	0.00015	0.03743
0.19065	0.17939	0.00013	0.03635
0.18669	0.17799	0.00008	0.03485
0.18556	0.17703	0.00007	0.03443
0.18274	0.17567	0.00005	0.03339
0.18048	0.17437	0.00004	0.03257
0.17935	0.17355	0.00003	0.03217
0.17597	0.17241	0.00001	0.03096
0.17427	0.17169	0.00001	0.03037
0.17258	0.17068	0.00000	0.02978
0.17089	0.16972	0.00000	0.02920
0.16581	0.16940	0.00001	0.02749
0.15903	0.16877	0.00009	0.02529
0.15395	0.16815	0.00020	0.02370

Epsilon l = ((sig(Hex-Hmod)^2)/
(sig(Hex)^2))^0.5 =.03502

Governing relationships on the flow turbulence models are as follows. Its parameters are presented in Table 2 to 4.

2.1 Governing Equations

2.1.1 Prandtl Mixing Length Model

Zero-equation model is a model that assumes the viscosity of the fluid in areas with high shear increases due to turbulence mixing mechanism[37]. Simple turbulence equations in this case are as follows:

$$P_{1T}+P_{2T}=CK^{1.5}/1_{M} \qquad (1)$$

In the above relationship, P_{1T} and P_{2T} are turbulence generation functions, C is a constant coefficient (about 0.11), K is turbulence kinetic energy and l_m is the scale of turbulence length.

Table 5. Turbulence kinetic energy (one equation) experimental model

Experimental						model					
x(m)	H(m)	(H-.15)	H*	X	X-3	x(m)	H(m)	(H-.15)	H*	X	X-3
2.0667	0.3758	0.2258	0.6452	3.0630	0.0630	2.0694	0.3844	0.2344	0.6698	3.0670	0.0670
2.1084	0.3735	0.2235	0.6387	3.1249	0.1249	2.1006	0.3821	0.2321	0.6631	3.1133	0.1133
2.1474	0.3713	0.2213	0.6323	3.1826	0.1826	2.1469	0.3786	0.2286	0.6531	3.1819	0.1819
2.1892	0.3690	0.2190	0.6258	3.2445	0.2445	2.1926	0.3751	0.2251	0.6431	3.2496	0.2496
2.2226	0.3662	0.2162	0.6177	3.2940	0.2940	2.2227	0.3727	0.2227	0.6362	3.2942	0.2942
2.2616	0.3623	0.2123	0.6065	3.3518	0.3518	2.2672	0.3691	0.2191	0.6261	3.3602	0.3602
2.3033	0.3600	0.2100	0.6000	3.4137	0.4137	2.2965	0.3668	0.2168	0.6194	3.4036	0.4036
2.3395	0.3566	0.2066	0.5903	3.4673	0.4673	2.3399	0.3633	0.2133	0.6094	3.4679	0.4679
2.3785	0.3538	0.2038	0.5823	3.5251	0.5251	2.3825	0.3601	0.2101	0.6002	3.5310	0.5310
2.4174	0.3510	0.2010	0.5742	3.5828	0.5828	2.4104	0.3575	0.2075	0.5929	3.5724	0.5724
2.4536	0.3465	0.1965	0.5613	3.6365	0.6365	2.4517	0.3541	0.2041	0.5832	3.6336	0.6336
2.4898	0.3419	0.1919	0.5484	3.6901	0.6901	2.4921	0.3508	0.2008	0.5737	3.6935	0.6935
2.5260	0.3391	0.1891	0.5403	3.7437	0.7437	2.5185	0.3486	0.1986	0.5675	3.7327	0.7327
2.5622	0.3369	0.1869	0.5339	3.7973	0.7973	2.5575	0.3453	0.1953	0.5581	3.7903	0.7903
2.6039	0.3346	0.1846	0.5274	3.8592	0.8592	2.6079	0.3412	0.1912	0.5463	3.8651	0.8651
2.6401	0.3318	0.1818	0.5194	3.9129	0.9129	2.6446	0.3382	0.1882	0.5376	3.9195	0.9195
2.6791	0.3278	0.1778	0.5081	3.9706	0.9706	2.6686	0.3362	0.1862	0.5320	3.9550	0.9550
2.7209	0.3250	0.1750	0.5000	4.0325	1.0325	2.7264	0.3314	0.1814	0.5183	4.0407	1.0407
2.7515	0.3227	0.1727	0.4935	4.0779	1.0779	2.7597	0.3286	0.1786	0.5103	4.0900	1.0900
2.8016	0.3177	0.1677	0.4790	4.1522	1.1522	2.8023	0.3251	0.1751	0.5003	4.1532	1.1532
2.8406	0.3160	0.1660	0.4742	4.2099	1.2099	2.8429	0.3218	0.1718	0.4909	4.2134	1.2134
2.8712	0.3137	0.1637	0.4677	4.2553	1.2553	2.8720	0.3194	0.1694	0.4841	4.2565	1.2565
2.9102	0.3103	0.1603	0.4581	4.3131	1.3131	2.9178	0.3157	0.1657	0.4733	4.3244	1.3244
2.9436	0.3069	0.1569	0.4484	4.3626	1.3626	2.9436	0.3135	0.1635	0.4672	4.3626	1.3626
2.9881	0.3047	0.1547	0.4419	4.4286	1.4286	2.9837	0.3103	0.1603	0.4579	4.4220	1.4220
3.0271	0.3024	0.1524	0.4355	4.4864	1.4864	3.0295	0.3065	0.1565	0.4472	4.4899	1.4899
3.0661	0.2996	0.1496	0.4274	4.5441	1.5441	3.0595	0.3041	0.1541	0.4402	4.5344	1.5344
3.1051	0.2996	0.1496	0.4274	4.6019	1.6019	3.1045	0.3002	0.1502	0.4291	4.6011	1.6011
3.1496	0.2990	0.1490	0.4258	4.6680	1.6680	3.1495	0.2968	0.1468	0.4193	4.6679	1.6679
3.1886	0.2979	0.1479	0.4226	4.7258	1.7258	3.1796	0.2944	0.1444	0.4125	4.7124	1.7124
3.2248	0.2973	0.1473	0.4210	4.7794	1.7794	3.2246	0.2908	0.1408	0.4024	4.7791	1.7791
3.2582	0.2934	0.1434	0.4097	4.8289	1.8289	3.2546	0.2885	0.1385	0.3957	4.8236	1.8236
3.3028	0.2917	0.1417	0.4048	4.8950	1.8950	3.2997	0.2854	0.1354	0.3868	4.8904	1.8904
3.3445	0.2889	0.1389	0.3968	4.9568	1.9568	3.3448	0.2815	0.1315	0.3758	4.9572	1.9572
3.3807	0.2855	0.1355	0.3871	5.0105	2.0105	3.3899	0.2785	0.1285	0.3672	5.0240	2.0240
3.4197	0.2838	0.1338	0.3823	5.0683	2.0683	3.4199	0.2764	0.1264	0.3612	5.0686	2.0686
3.4615	0.2815	0.1315	0.3758	5.1301	2.1301	3.4650	0.2733	0.1233	0.3524	5.1354	2.1354

3.5004	0.2781	0.1281	0.3661	5.1879	2.1879	3.4951	0.2713	0.1213	0.3466	5.1799	2.1799
3.5366	0.2748	0.1248	0.3565	5.2415	2.2415	3.5251	0.2694	0.1194	0.3412	5.2245	2.2245
3.5728	0.2725	0.1225	0.3500	5.2952	2.2952	3.5702	0.2661	0.1161	0.3318	5.2914	2.2914
3.6146	0.2708	0.1208	0.3452	5.3571	2.3571	3.6153	0.2633	0.1133	0.3237	5.3582	2.3582
3.6536	0.2697	0.1197	0.3419	5.4149	2.4149	3.6454	0.2613	0.1113	0.3179	5.4028	2.4028
3.6953	0.2680	0.1180	0.3371	5.4768	2.4768	3.6906	0.2582	0.1082	0.3090	5.4697	2.4697
3.7343	0.2657	0.1157	0.3306	5.5345	2.5345	3.7357	0.2555	0.1055	0.3014	5.5366	2.5366
3.7761	0.2635	0.1135	0.3242	5.5964	2.5964	3.7658	0.2531	0.1031	0.2946	5.5812	2.5812
3.8123	0.2606	0.1106	0.3161	5.6501	2.6501	3.8109	0.2502	0.1002	0.2863	5.6481	2.6481
3.8512	0.2567	0.1067	0.3048	5.7078	2.7078	3.8561	0.2474	0.0974	0.2782	5.7150	2.7150
3.8874	0.2550	0.1050	0.3000	5.7615	2.7615	3.8862	0.2455	0.0955	0.2729	5.7596	2.7596
3.9320	0.2533	0.1033	0.2952	5.8275	2.8275	3.9313	0.2427	0.0927	0.2650	5.8265	2.8265
3.9738	0.2527	0.1027	0.2935	5.8894	2.8894	3.9765	0.2402	0.0902	0.2578	5.8934	2.8934
4.0100	0.2505	0.1005	0.2871	5.9431	2.9431	4.0066	0.2382	0.0882	0.2519	5.9380	2.9380
4.0517	0.2488	0.0988	0.2823	6.0050	3.0050	4.0517	0.2356	0.0856	0.2445	6.0050	3.0050
4.0907	0.2471	0.0971	0.2774	6.0627	3.0627	4.0969	0.2330	0.0830	0.2371	6.0719	3.0719
4.1325	0.2448	0.0948	0.2710	6.1246	3.1246	4.1270	0.2313	0.0813	0.2322	6.1165	3.1165
4.1687	0.2431	0.0931	0.2661	6.1783	3.1783	4.1571	0.2296	0.0796	0.2274	6.1612	3.1612
4.2132	0.2409	0.0909	0.2597	6.2443	3.2443	4.2174	0.2263	0.0763	0.2181	6.2504	3.2504
4.2522	0.2392	0.0892	0.2548	6.3021	3.3021	4.2475	0.2245	0.0745	0.2128	6.2951	3.2951
4.2856	0.2358	0.0858	0.2452	6.3516	3.3516	4.2776	0.2229	0.0729	0.2084	6.3397	3.3397
4.3274	0.2330	0.0830	0.2371	6.4135	3.4135	4.3228	0.2206	0.0706	0.2016	6.4067	3.4067
4.3691	0.2313	0.0813	0.2323	6.4754	3.4754	4.3680	0.2182	0.0682	0.1949	6.4736	3.4736
4.4081	0.2307	0.0807	0.2306	6.5332	3.5332	4.3981	0.2166	0.0666	0.1904	6.5183	3.5183
4.4499	0.2290	0.0790	0.2258	6.5951	3.5951	4.4433	0.2143	0.0643	0.1838	6.5852	3.5852
4.4889	0.2268	0.0768	0.2194	6.6529	3.6529	4.4884	0.2120	0.0620	0.1771	6.6522	3.6522
4.5306	0.2240	0.0740	0.2113	6.7148	3.7148	4.5336	0.2097	0.0597	0.1705	6.7192	3.7192
4.5613	0.2206	0.0706	0.2016	6.7601	3.7601	4.5637	0.2083	0.0583	0.1667	6.7638	3.7638
4.6058	0.2189	0.0689	0.1968	6.8262	3.8262	4.6089	0.2062	0.0562	0.1605	6.8308	3.8308
4.6448	0.2160	0.0660	0.1887	6.8839	3.8839	4.6390	0.2047	0.0547	0.1563	6.8754	3.8754
4.6754	0.2110	0.0610	0.1742	6.9293	3.9293	4.6692	0.2033	0.0533	0.1522	6.9200	3.9200
4.7144	0.2093	0.0593	0.1694	6.9870	3.9870	4.7143	0.2012	0.0512	0.1462	6.9870	3.9870
4.7534	0.2076	0.0576	0.1645	7.0448	4.0448	4.7595	0.1992	0.0492	0.1406	7.0540	4.0540
4.7923	0.2053	0.0553	0.1581	7.1026	4.1026	4.7896	0.1980	0.0480	0.1371	7.0986	4.0986
4.8369	0.2042	0.0542	0.1548	7.1686	4.1686	4.8348	0.1963	0.0463	0.1323	7.1655	4.1655
4.8731	0.2042	0.0542	0.1548	7.2223	4.2223	4.8649	0.1945	0.0445	0.1271	7.2102	4.2102
4.9177	0.2025	0.0525	0.1500	7.2883	4.2883	4.9101	0.1926	0.0426	0.1218	7.2771	4.2771
4.9567	0.2025	0.0525	0.1500	7.3461	4.3461	4.9553	0.1908	0.0408	0.1165	7.3441	4.3441
4.9984	0.2019	0.0519	0.1484	7.4080	4.4080	4.9854	0.1896	0.0396	0.1130	7.3887	4.3887
5.0374	0.2014	0.0514	0.1468	7.4659	4.4659	5.0305	0.1878	0.0378	0.1080	7.4556	4.4556
5.0764	0.2014	0.0514	0.1468	7.5236	4.5236	5.0757	0.1861	0.0361	0.1032	7.5225	4.5225
5.1182	0.1980	0.0480	0.1371	7.5855	4.5855	5.1058	0.1851	0.0351	0.1002	7.5672	4.5672
5.1544	0.1952	0.0452	0.1290	7.6392	4.6392	5.1509	0.1835	0.0335	0.0958	7.6341	4.6341
5.1934	0.1935	0.0435	0.1242	7.6969	4.6969	5.1961	0.1819	0.0319	0.0912	7.7010	4.7010
5.2324	0.1935	0.0435	0.1242	7.7547	4.7547	5.2262	0.1812	0.0312	0.0890	7.7456	4.7456
5.2769	0.1906	0.0406	0.1161	7.8208	4.8208	5.2713	0.1793	0.0293	0.0837	7.8125	4.8125
5.3103	0.1867	0.0367	0.1048	7.8702	4.8702	5.3164	0.1779	0.0279	0.0798	7.8794	4.8794
5.3521	0.1856	0.0356	0.1016	7.9322	4.9322	5.3465	0.1770	0.0270	0.0772	7.9240	4.9240
5.3910	0.1827	0.0327	0.0935	7.9899	4.9899	5.3917	0.1757	0.0257	0.0733	7.9908	4.9908
5.4300	0.1805	0.0305	0.0871	8.0477	5.0477	5.4368	0.1744	0.0244	0.0697	8.0577	5.0577
5.4718	0.1794	0.0294	0.0839	8.1096	5.1096	5.4668	0.1736	0.0236	0.0674	8.1023	5.1023
5.5108	0.1760	0.0260	0.0742	8.1674	5.1674	5.5119	0.1724	0.0224	0.0641	8.1691	5.1691

5.5497	0.1743	0.0243	0.0694	8.2251	5.2251	5.5420	0.1717	0.0217	0.0620	8.2137	5.2137
5.5915	0.1726	0.0226	0.0645	8.2870	5.2870	5.5871	0.1707	0.0207	0.0591	8.2805	5.2805
5.6305	0.1709	0.0209	0.0597	8.3448	5.3448	5.6322	0.1697	0.0197	0.0562	8.3473	5.3473
5.6583	0.1658	0.0158	0.0452	8.3860	5.3860	5.6472	0.1694	0.0194	0.0553	8.3696	5.3696
5.6889	0.1590	0.0090	0.0258	8.4314	5.4314	5.6772	0.1687	0.0187	0.0535	8.4141	5.4141
5.7139	0.1540	0.0040	0.0113	8.4685	5.4685	5.7073	0.1681	0.0181	0.0518	8.4586	5.4586

Table 6. Turbulence kinetic energy (one equation) model

	t(sec)	x(m)	x revised(m)	g(ms^-2)	H0(m)	(H0/g)^.5	X*	t*	U*	a*
Kinetic Energy Model	0.0000	0.0000	0.0000	9.8000	0.3500	0.1890	0.0000	0.0000	_	_
	0.2500	3.1946	0.1946	9.8060	0.3500	0.1889	0.5560	1.3233	0.4201	0.3175
	0.5000	4.0960	1.0960	9.8060	0.3500	0.1889	3.1314	2.6466	1.1832	0.4471
	0.7500	5.0300	2.0300	9.8060	0.3500	0.1889	5.8000	3.9698	1.4610	0.3680
	1.0000	5.9620	2.9620	9.8060	0.3500	0.1889	8.4629	5.2931	1.5988	0.3021

Table 7. Turbulence kinetic energy parameters

(Hex)i (m)	(Hmodel)i (m)	(Hex-Hmodel)i^2	(Hex)^2
0.37581	0.38443	0.00007	0.14123
0.37355	0.38210	0.00007	0.13954
0.37129	0.37859	0.00005	0.13786
0.36903	0.37507	0.00004	0.13618
0.36621	0.37267	0.00004	0.13411
0.36226	0.36914	0.00005	0.13123
0.36000	0.36679	0.00005	0.12960
0.35661	0.36445	0.00006	0.12717
0.35379	0.36097	0.00005	0.12517
0.35097	0.35751	0.00004	0.12318
0.34645	0.35413	0.00006	0.12003
0.34194	0.35191	0.00010	0.11692
0.33911	0.34862	0.00009	0.11500
0.33685	0.34535	0.00007	0.11347
0.33460	0.34225	0.00006	0.11196
0.33177	0.33817	0.00004	0.11007
0.32782	0.33522	0.00005	0.10747
0.32500	0.33235	0.00005	0.10562
0.32274	0.33004	0.00005	0.10416
0.31766	0.32512	0.00006	0.10091
0.31597	0.32180	0.00003	0.09984
0.31371	0.31942	0.00003	0.09841
0.31032	0.31640	0.00004	0.09630
0.30694	0.31351	0.00004	0.09421
0.30468	0.31026	0.00003	0.09283
0.30242	0.30775	0.00003	0.09146

0.29960	0.30408	0.00002	0.08976
0.29960	0.30017	0.00000	0.08976
0.29903	0.29676	0.00001	0.08942
0.29790	0.29438	0.00001	0.08875
0.29734	0.29083	0.00004	0.08841
0.29339	0.28850	0.00002	0.08608
0.29169	0.28537	0.00004	0.08509
0.28887	0.28154	0.00005	0.08345
0.28548	0.27853	0.00005	0.08150
0.28379	0.27644	0.00005	0.08054
0.28153	0.27333	0.00007	0.07926
0.27815	0.27129	0.00005	0.07736
0.27476	0.26831	0.00004	0.07549
0.27250	0.26614	0.00004	0.07426
0.27081	0.26330	0.00006	0.07334
0.26968	0.26020	0.00009	0.07273
0.26798	0.25702	0.00012	0.07182
0.26573	0.25549	0.00010	0.07061
0.26347	0.25216	0.00013	0.06942
0.26065	0.24925	0.00013	0.06794
0.25669	0.24736	0.00009	0.06589
0.25500	0.24458	0.00011	0.06502
0.25331	0.24172	0.00013	0.06416
0.25274	0.24024	0.00016	0.06388
0.25048	0.23817	0.00015	0.06274
0.24879	0.23556	0.00018	0.06190
0.24710	0.23297	0.00020	0.06106
0.24484	0.23043	0.00021	0.05995
0.24315	0.22794	0.00023	0.05912
0.24089	0.22634	0.00021	0.05803
0.23919	0.22373	0.00024	0.05721
0.23581	0.22136	0.00021	0.05560
0.23298	0.22058	0.00015	0.05428
0.23129	0.21821	0.00017	0.05350
0.23073	0.21586	0.00022	0.05323
0.22903	0.21432	0.00022	0.05246
0.22677	0.21200	0.00022	0.05143
0.22395	0.20969	0.00020	0.05015
0.22056	0.20834	0.00015	0.04865
0.21887	0.20617	0.00016	0.04790
0.21605	0.20472	0.00013	0.04668
0.21097	0.20328	0.00006	0.04451
0.20927	0.20117	0.00007	0.04380
0.20758	0.19920	0.00007	0.04309
0.20532	0.19797	0.00005	0.04216
0.20419	0.19631	0.00006	0.04170

0.20419	0.19388	0.00011	0.04170
0.20250	0.19263	0.00010	0.04101
0.20250	0.19077	0.00014	0.04101
0.20194	0.18957	0.00015	0.04078
0.20137	0.18781	0.00018	0.04055
0.20137	0.18613	0.00023	0.04055
0.19798	0.18506	0.00017	0.03920
0.19516	0.18352	0.00014	0.03809
0.19347	0.18192	0.00013	0.03743
0.19347	0.18116	0.00015	0.03743
0.19065	0.17930	0.00013	0.03635
0.18669	0.17793	0.00008	0.03485
0.18556	0.17700	0.00007	0.03443
0.18274	0.17567	0.00005	0.03339
0.18048	0.17439	0.00004	0.03257
0.17935	0.17358	0.00003	0.03217
0.17597	0.17243	0.00001	0.03096
0.17427	0.17170	0.00001	0.03037
0.17258	0.17067	0.00000	0.02978
0.17089	0.16969	0.00000	0.02920
0.16581	0.16937	0.00001	0.02749
0.15903	0.16874	0.00009	0.02529
0.15395	0.16812	0.00020	0.02370

Epsilon l = ((sig(Hex-Hmod)^2)/(sig(Hex)^2))^0.5 = 0.03590

Table 8. K-Epsilon model parameters

Experimental						Model					
x(m)	H(m)	(H-.15)	H*	X	X-3	x(m)	H(m)	(H-.15)	H*	X	X-3
2.0667	0.3758	0.2258	0.6452	3.0630	0.0630	2.0694	0.3842	0.2342	0.6692	3.0670	0.1648
2.1084	0.3735	0.2235	0.6387	3.1249	0.1249	2.1006	0.3819	0.2319	0.6625	3.1133	0.2110
2.1474	0.3713	0.2213	0.6323	3.1826	0.1826	2.1469	0.3784	0.2284	0.6525	3.1819	0.2796
2.1892	0.3690	0.2190	0.6258	3.2445	0.2445	2.1774	0.3760	0.2260	0.6458	3.2271	0.3249
2.2226	0.3662	0.2162	0.6177	3.2940	0.2940	2.2227	0.3725	0.2225	0.6356	3.2942	0.3919
2.2616	0.3623	0.2123	0.6065	3.3518	0.3518	2.2672	0.3689	0.2189	0.6256	3.3602	0.4580
2.3033	0.3600	0.2100	0.6000	3.4137	0.4137	2.2965	0.3666	0.2166	0.6189	3.4036	0.5014
2.3395	0.3566	0.2066	0.5903	3.4673	0.4673	2.3399	0.3631	0.2131	0.6089	3.4679	0.5656
2.3785	0.3538	0.2038	0.5823	3.5251	0.5251	2.3684	0.3608	0.2108	0.6023	3.5101	0.6078
2.4174	0.3510	0.2010	0.5742	3.5828	0.5828	2.4104	0.3574	0.2074	0.5925	3.5724	0.6702
2.4536	0.3465	0.1965	0.5613	3.6365	0.6365	2.4517	0.3540	0.2040	0.5828	3.6336	0.7313
2.4898	0.3419	0.1919	0.5484	3.6901	0.6901	2.4787	0.3518	0.2018	0.5765	3.6736	0.7714
2.5260	0.3391	0.1891	0.5403	3.7437	0.7437	2.5185	0.3485	0.1985	0.5671	3.7327	0.8304
2.5622	0.3369	0.1869	0.5339	3.7973	0.7973	2.5575	0.3452	0.1952	0.5577	3.7903	0.8881
2.6039	0.3346	0.1846	0.5274	3.8592	0.8592	2.6079	0.3411	0.1911	0.5460	3.8651	0.9629
2.6401	0.3318	0.1818	0.5194	3.9129	0.9129	2.6446	0.3381	0.1881	0.5373	3.9195	1.0173
2.6791	0.3278	0.1778	0.5081	3.9706	0.9706	2.6686	0.3361	0.1861	0.5317	3.9550	1.0528
2.7209	0.3250	0.1750	0.5000	4.0325	1.0325	2.7264	0.3313	0.1813	0.5181	4.0407	1.1384
2.7515	0.3227	0.1727	0.4935	4.0779	1.0779	2.7597	0.3286	0.1786	0.5102	4.0900	1.1878

2.8016	0.3177	0.1677	0.4790	4.1522	1.1522	2.8023	0.3251	0.1751	0.5002	4.1532	1.2510
2.8406	0.3160	0.1660	0.4742	4.2099	1.2099	2.8429	0.3218	0.1718	0.4908	4.2134	1.3112
2.8712	0.3137	0.1637	0.4677	4.2553	1.2553	2.8720	0.3194	0.1694	0.4840	4.2565	1.3543
2.9102	0.3103	0.1603	0.4581	4.3131	1.3131	2.9178	0.3157	0.1657	0.4733	4.3244	1.4221
2.9436	0.3069	0.1569	0.4484	4.3626	1.3626	2.9436	0.3135	0.1635	0.4672	4.3626	1.4603
2.9881	0.3047	0.1547	0.4419	4.4286	1.4286	2.9837	0.3103	0.1603	0.4580	4.4220	1.5198
3.0271	0.3024	0.1524	0.4355	4.4864	1.4864	3.0260	0.3069	0.1569	0.4482	4.4848	1.5826
3.0661	0.2996	0.1496	0.4274	4.5441	1.5441	3.0669	0.3036	0.1536	0.4388	4.5453	1.6431
3.1051	0.2996	0.1496	0.4274	4.6019	1.6019	3.1021	0.3007	0.1507	0.4306	4.5975	1.6953
3.1496	0.2990	0.1490	0.4258	4.6680	1.6680	3.1492	0.2969	0.1469	0.4199	4.6674	1.7651
3.1886	0.2979	0.1479	0.4226	4.7258	1.7258	3.1893	0.2938	0.1438	0.4108	4.7268	1.8245
3.2248	0.2973	0.1473	0.4210	4.7794	1.7794	3.2207	0.2913	0.1413	0.4038	4.7733	1.8711
3.2582	0.2934	0.1434	0.4097	4.8289	1.8289	3.2532	0.2888	0.1388	0.3966	4.8215	1.9193
3.3028	0.2917	0.1417	0.4048	4.8950	1.8950	3.3098	0.2845	0.1345	0.3841	4.9054	2.0032
3.3445	0.2889	0.1389	0.3968	4.9568	1.9568	3.3452	0.2819	0.1319	0.3768	4.9578	2.0556
3.3807	0.2855	0.1355	0.3871	5.0105	2.0105	3.3816	0.2792	0.1292	0.3692	5.0117	2.1095
3.4197	0.2838	0.1338	0.3823	5.0683	2.0683	3.4189	0.2767	0.1267	0.3621	5.0670	2.1648
3.4615	0.2815	0.1315	0.3758	5.1301	2.1301	3.4572	0.2741	0.1241	0.3545	5.1238	2.2215
3.5004	0.2781	0.1281	0.3661	5.1879	2.1879	3.5096	0.2709	0.1209	0.3454	5.2015	2.2993
3.5366	0.2748	0.1248	0.3565	5.2415	2.2415	3.5365	0.2689	0.1189	0.3396	5.2413	2.3390
3.5728	0.2725	0.1225	0.3500	5.2952	2.2952	3.5774	0.2661	0.1161	0.3317	5.3020	2.3997
3.6146	0.2708	0.1208	0.3452	5.3571	2.3571	3.6191	0.2634	0.1134	0.3240	5.3638	2.4616
3.6536	0.2697	0.1197	0.3419	5.4149	2.4149	3.6474	0.2615	0.1115	0.3186	5.4058	2.5035
3.6953	0.2680	0.1180	0.3371	5.4768	2.4768	3.6905	0.2586	0.1086	0.3103	5.4696	2.5673
3.7343	0.2657	0.1157	0.3306	5.5345	2.5345	3.7343	0.2563	0.1063	0.3037	5.5345	2.6322
3.7761	0.2635	0.1135	0.3242	5.5964	2.5964	3.7788	0.2528	0.1028	0.2936	5.6005	2.6982
3.8123	0.2606	0.1106	0.3161	5.6501	2.6501	3.8089	0.2508	0.1008	0.2881	5.6450	2.7428
3.8512	0.2567	0.1067	0.3048	5.7078	2.7078	3.8545	0.2479	0.0979	0.2797	5.7126	2.8104
3.8874	0.2550	0.1050	0.3000	5.7615	2.7615	3.8853	0.2460	0.0960	0.2743	5.7583	2.8560
3.9320	0.2533	0.1033	0.2952	5.8275	2.8275	3.9319	0.2432	0.0932	0.2662	5.8274	2.9252
3.9738	0.2527	0.1027	0.2935	5.8894	2.8894	3.9792	0.2407	0.0907	0.2590	5.8974	2.9952
4.0100	0.2505	0.1005	0.2871	5.9431	2.9431	4.0110	0.2384	0.0884	0.2525	5.9446	3.0423
4.0517	0.2488	0.0988	0.2823	6.0050	3.0050	4.0591	0.2356	0.0856	0.2446	6.0159	3.1136
4.0907	0.2471	0.0971	0.2774	6.0627	3.0627	4.0915	0.2338	0.0838	0.2393	6.0639	3.1616
4.1325	0.2448	0.0948	0.2710	6.1246	3.1246	4.1241	0.2319	0.0819	0.2340	6.1122	3.2099
4.1687	0.2431	0.0931	0.2661	6.1783	3.1783	4.1568	0.2300	0.0800	0.2286	6.1608	3.2585
4.2132	0.2409	0.0909	0.2597	6.2443	3.2443	4.2064	0.2271	0.0771	0.2204	6.2342	3.3319
4.2522	0.2392	0.0892	0.2548	6.3021	3.3021	4.2563	0.2243	0.0743	0.2124	6.3082	3.4059
4.2856	0.2358	0.0858	0.2452	6.3516	3.3516	4.2898	0.2226	0.0726	0.2073	6.3578	3.4556
4.3274	0.2330	0.0830	0.2371	6.4135	3.4135	4.3235	0.2208	0.0708	0.2022	6.4077	3.5055
4.3691	0.2313	0.0813	0.2323	6.4754	3.4754	4.3573	0.2190	0.0690	0.1971	6.4578	3.5556
4.4081	0.2307	0.0807	0.2306	6.5332	3.5332	4.4083	0.2164	0.0664	0.1896	6.5334	3.6311
4.4499	0.2290	0.0790	0.2258	6.5951	3.5951	4.4424	0.2146	0.0646	0.1846	6.5840	3.6817
4.4889	0.2268	0.0768	0.2194	6.6529	3.6529	4.4767	0.2128	0.0628	0.1795	6.6348	3.7325
4.5306	0.2240	0.0740	0.2113	6.7148	3.7148	4.5283	0.2105	0.0605	0.1729	6.7112	3.8090
4.5613	0.2206	0.0706	0.2016	6.7601	3.7601	4.5628	0.2084	0.0584	0.1668	6.7624	3.8601
4.6058	0.2189	0.0689	0.1968	6.8262	3.8262	4.5974	0.2067	0.0567	0.1621	6.8137	3.9114
4.6448	0.2160	0.0660	0.1887	6.8839	3.8839	4.6494	0.2043	0.0543	0.1552	6.8908	3.9885
4.6754	0.2110	0.0610	0.1742	6.9293	3.9293	4.6668	0.2035	0.0535	0.1529	6.9165	4.0143
4.7144	0.2093	0.0593	0.1694	6.9870	3.9870	4.7190	0.2011	0.0511	0.1460	6.9939	4.0917
4.7534	0.2076	0.0576	0.1645	7.0448	4.0448	4.7539	0.1995	0.0495	0.1415	7.0456	4.1433
4.7923	0.2053	0.0553	0.1581	7.1026	4.1026	4.7888	0.1980	0.0480	0.1371	7.0973	4.1951

4.8369	0.2042	0.0542	0.1548	7.1686	4.1686	4.8237	0.1966	0.0466	0.1331	7.1491	4.2468
4.8731	0.2042	0.0542	0.1548	7.2223	4.2223	4.8761	0.1940	0.0440	0.1256	7.2268	4.3245
4.9177	0.2025	0.0525	0.1500	7.2883	4.2883	4.9111	0.1926	0.0426	0.1216	7.2786	4.3763
4.9567	0.2025	0.0525	0.1500	7.3461	4.3461	4.9460	0.1911	0.0411	0.1176	7.3304	4.4281
4.9984	0.2019	0.0519	0.1484	7.4080	4.4080	4.9984	0.1891	0.0391	0.1117	7.4081	4.5058
5.0374	0.2014	0.0514	0.1468	7.4659	4.4659	5.0333	0.1878	0.0378	0.1079	7.4598	4.5575
5.0764	0.2014	0.0514	0.1468	7.5236	4.5236	5.0682	0.1865	0.0365	0.1042	7.5114	4.6092
5.1182	0.1980	0.0480	0.1371	7.5855	4.5855	5.1030	0.1852	0.0352	0.1007	7.5630	4.6608
5.1544	0.1952	0.0452	0.1290	7.6392	4.6392	5.1551	0.1834	0.0334	0.0956	7.6403	4.7380
5.1934	0.1935	0.0435	0.1242	7.6969	4.6969	5.1898	0.1822	0.0322	0.0920	7.6917	4.7894
5.2324	0.1935	0.0435	0.1242	7.7547	4.7547	5.2244	0.1812	0.0312	0.0892	7.7429	4.8406
5.2769	0.1906	0.0406	0.1161	7.8208	4.8208	5.2761	0.1792	0.0292	0.0835	7.8195	4.9173
5.3103	0.1867	0.0367	0.1048	7.8702	4.8702	5.3104	0.1782	0.0282	0.0805	7.8704	4.9682
5.3521	0.1856	0.0356	0.1016	7.9322	4.9322	5.3447	0.1771	0.0271	0.0775	7.9212	5.0189
5.3910	0.1827	0.0327	0.0935	7.9899	4.9899	5.3958	0.1756	0.0256	0.0733	7.9970	5.0947
5.4300	0.1805	0.0305	0.0871	8.0477	5.0477	5.4297	0.1747	0.0247	0.0705	8.0472	5.1450
5.4718	0.1794	0.0294	0.0839	8.1096	5.1096	5.4635	0.1738	0.0238	0.0679	8.0973	5.1951
5.5108	0.1760	0.0260	0.0742	8.1674	5.1674	5.5139	0.1724	0.0224	0.0641	8.1720	5.2697
5.5497	0.1743	0.0243	0.0694	8.2251	5.2251	5.5473	0.1716	0.0216	0.0617	8.2215	5.3192
5.5915	0.1726	0.0226	0.0645	8.2870	5.2870	5.5970	0.1704	0.0204	0.0583	8.2952	5.3930
5.6305	0.1709	0.0209	0.0597	8.3448	5.3448	5.6300	0.1697	0.0197	0.0562	8.3441	5.4418
5.6583	0.1658	0.0158	0.0452	8.3860	5.3860	5.6464	0.1693	0.0193	0.0552	8.3684	5.4661
5.6889	0.1590	0.0090	0.0258	8.4314	5.4314	5.6790	0.1686	0.0186	0.0532	8.4167	5.5145
5.7139	0.1540	0.0040	0.0113	8.4685	5.4685	5.7115	0.1680	0.0180	0.0514	8.4648	5.5626

Table 9. K-Epsilon experimental parameters model

	t (sec)	x(m)	x revised(m)	g(ms^-2)	H0(m)	(H0/g)^.5	X*	t*	U*	a*
	0.0000	3.0000	0.0000	9.8060	0.3500	0.1889	0.0000	0.0000	_	_
K-Epsilon Model	0.2500	3.1940	0.1940	9.8060	0.3500	0.1889	0.5543	1.3233	0.4189	0.3165
	0.5000	4.0960	1.0960	9.8060	0.3500	0.1889	3.1314	2.6466	1.1832	0.4471
	0.7500	5.0300	2.0300	9.8060	0.3500	0.1889	5.8000	3.9698	1.4610	0.3680
	1.0000	5.9620	2.9620	9.8060	0.3500	0.1889	8.4629	5.2931	1.5988	0.3021

Table 10. K-Epsilon parameters model

(Hex)i (m)	(Hmodel)i (m)	(Hex-Hmodel)i^2	(Hex)^2
0.3758	0.3842	0.0001	0.1412
0.3735	0.3819	0.0001	0.1395
0.3713	0.3784	0.0001	0.1379
0.3690	0.3760	0.0000	0.1362
0.3662	0.3725	0.0000	0.1341
0.3623	0.3689	0.0000	0.1312
0.3600	0.3666	0.0000	0.1296
0.3566	0.3631	0.0000	0.1272
0.3538	0.3608	0.0000	0.1252
0.3510	0.3574	0.0000	0.1232
0.3465	0.3540	0.0001	0.1200
0.3419	0.3518	0.0001	0.1169

0.3391	0.3485	0.0001	0.1150
0.3369	0.3452	0.0001	0.1135
0.3346	0.3411	0.0000	0.1120
0.3318	0.3381	0.0000	0.1101
0.3278	0.3351	0.0001	0.1075
0.3250	0.3313	0.0000	0.1056
0.3227	0.3286	0.0000	0.1042
0.3177	0.3260	0.0001	0.1009
0.3160	0.3218	0.0000	0.0998
0.3137	0.3186	0.0000	0.0984
0.3103	0.3151	0.0000	0.0963
0.3069	0.3122	0.0000	0.0942
0.3047	0.3103	0.0000	0.0928
0.3024	0.3069	0.0000	0.0915
0.2996	0.3036	0.0000	0.0898
0.2996	0.3007	0.0000	0.0898
0.2990	0.2969	0.0000	0.0894
0.2979	0.2938	0.0000	0.0887
0.2973	0.2913	0.0000	0.0884
0.2934	0.2879	0.0000	0.0861
0.2917	0.2845	0.0001	0.0851
0.2889	0.2819	0.0000	0.0834
0.2855	0.2792	0.0000	0.0815
0.2838	0.2767	0.0000	0.0805
0.2815	0.2732	0.0001	0.0793
0.2781	0.2709	0.0001	0.0774
0.2748	0.2696	0.0000	0.0755
0.2725	0.2670	0.0000	0.0743
0.2708	0.2643	0.0000	0.0733
0.2697	0.2615	0.0001	0.0727
0.2680	0.2586	0.0001	0.0718
0.2657	0.2563	0.0001	0.0706
0.2635	0.2528	0.0001	0.0694
0.2606	0.2508	0.0001	0.0679
0.2567	0.2479	0.0001	0.0659
0.2550	0.2460	0.0001	0.0650
0.2533	0.2432	0.0001	0.0642
0.2527	0.2415	0.0001	0.0639
0.2505	0.2384	0.0001	0.0627
0.2488	0.2356	0.0002	0.0619
0.2471	0.2338	0.0002	0.0611
0.2448	0.2319	0.0002	0.0599
0.2431	0.2300	0.0002	0.0591
0.2409	0.2271	0.0002	0.0580
0.2392	0.2243	0.0002	0.0572
0.2358	0.2226	0.0002	0.0556

0.2330	0.2208	0.0001	0.0543
0.2313	0.2190	0.0002	0.0535
0.2307	0.2164	0.0002	0.0532
0.2290	0.2146	0.0002	0.0525
0.2268	0.2128	0.0002	0.0514
0.2240	0.2105	0.0002	0.0502
0.2206	0.2084	0.0001	0.0486
0.2189	0.2067	0.0001	0.0479
0.2160	0.2043	0.0001	0.0467
0.2110	0.2027	0.0001	0.0445
0.2093	0.2011	0.0001	0.0438
0.2076	0.1995	0.0001	0.0431
0.2053	0.1971	0.0001	0.0422
0.2042	0.1964	0.0001	0.0417
0.2042	0.1933	0.0001	0.0417
0.2025	0.1918	0.0001	0.0410
0.2025	0.1904	0.0001	0.0410
0.2019	0.1891	0.0002	0.0408
0.2014	0.1878	0.0002	0.0406
0.2014	0.1865	0.0002	0.0406
0.1980	0.1852	0.0002	0.0392
0.1952	0.1834	0.0001	0.0381
0.1935	0.1822	0.0001	0.0374
0.1935	0.1812	0.0002	0.0374
0.1906	0.1792	0.0001	0.0363
0.1867	0.1782	0.0001	0.0349
0.1856	0.1771	0.0001	0.0344
0.1827	0.1756	0.0001	0.0334
0.1805	0.1747	0.0000	0.0326
0.1794	0.1738	0.0000	0.0322
0.1760	0.1724	0.0000	0.0310
0.1743	0.1716	0.0000	0.0304
0.1726	0.1704	0.0000	0.0298
0.1709	0.1697	0.0000	0.0292
0.1658	0.1693	0.0000	0.0275
0.1590	0.1686	0.0001	0.0253
0.1540	0.1680	0.0002	0.0237
Epsilon l = ((sig(Hex-Hmod)^2)/ (sig(Hex)^2))^0.5=.0346			

$P_{1T} + P_{2T} = CK^{1.5}/l_m$

is turbulence loss. Also, near the walls, the below equation is established:

$$\frac{\overline{(u'v')^2}}{\vartheta t} = \frac{CK^{1.5}}{\ell_m} \quad (2)$$

We can conclude from Equation (1) and Equation (2) that:

$$P_{1t} + P_{2t} = \frac{(u'v')^2}{\vartheta t} \quad (3)$$

2.1.2 Single Equation Turbulent Kinetic Energy Model

The model is a single-equation model for the scale of turbulence and it is based on the turbulence kinetic energy K in terms of Reynolds' normal stress. The amount of energy in isotropic turbulence is one and a half times and it is in non-isotropic a half times of the square velocity fluctuations[38-39]. Its parameters are presented in Table 5 to 7.

The below transport equation is used to calculate K (in the non-isotropic case):

$$\frac{\partial K}{\partial t} + C_1 u\frac{\partial K}{\partial x} + C_2 v\frac{\partial K}{\partial y} + C_3 w\frac{\partial K}{\partial z} = P_t - \frac{CK^{1.5}}{l_m} \quad (4)$$

$$C_1 = \frac{A_x}{\forall_f}, C_2 = \frac{A_y}{\forall_f}, C_3 = \frac{A_z}{\forall_f} \quad (5)$$

$$P_t = P_{1t} + P_{2t} + diff_t \quad (6)$$

$$å = \frac{CK^{1.5}}{l_m} \quad (7)$$

In Equation (4) to (7), A, $\forall_f$, P_t, $diff_t$, are respectively FAVOR area and volume functions, turbulence generation and diffusion.

2.1.3 Kinetic Energy- Turbulence Loss Model (k-ε)

Two-equation turbulence models are models that use independent transport equations for turbulence length

Table 11. \RNG experiment model

Experimental						Model					
x(m)	H(m)	(H-.15)	H*	X	X-3	x(m)	H(m)	(H-.15)	H*	X	X-3
2.0667	0.3758	0.2258	0.6452	3.0630	0.0630	2.0547	0.3847	0.2347	0.6707	3.0452	0.1576
2.1084	0.3735	0.2235	0.6387	3.1249	0.1249	2.1068	0.3809	0.2309	0.6597	3.1225	0.2348
2.1474	0.3713	0.2213	0.6323	3.1826	0.1826	2.1326	0.3790	0.2290	0.6542	3.1607	0.2730
2.1892	0.3690	0.2190	0.6258	3.2445	0.2445	2.1836	0.3751	0.2251	0.6430	3.2363	0.3486
2.2226	0.3662	0.2162	0.6177	3.2940	0.2940	2.2338	0.3711	0.2211	0.6318	3.3106	0.4229
2.2616	0.3623	0.2123	0.6065	3.3518	0.3518	2.2585	0.3692	0.2192	0.6263	3.3473	0.4596
2.3033	0.3600	0.2100	0.6000	3.4137	0.4137	2.3074	0.3653	0.2153	0.6152	3.4197	0.5321
2.3395	0.3566	0.2066	0.5903	3.4673	0.4673	2.3315	0.3634	0.2134	0.6097	3.4554	0.5678
2.3785	0.3538	0.2038	0.5823	3.5251	0.5251	2.3790	0.3596	0.2096	0.5989	3.5258	0.6381
2.4174	0.3510	0.2010	0.5742	3.5828	0.5828	2.4023	0.3578	0.2078	0.5936	3.5604	0.6728
2.4536	0.3465	0.1965	0.5613	3.6365	0.6365	2.4483	0.3540	0.2040	0.5830	3.6286	0.7409
2.4898	0.3419	0.1919	0.5484	3.6901	0.6901	2.4933	0.3502	0.2002	0.5721	3.6952	0.8075
2.5260	0.3391	0.1891	0.5403	3.7437	0.7437	2.5153	0.3500	0.2000	0.5714	3.7279	0.8402
2.5622	0.3369	0.1869	0.5339	3.7973	0.7973	2.5586	0.3450	0.1950	0.5572	3.7920	0.9044
2.6039	0.3346	0.1846	0.5274	3.8592	0.8592	2.6007	0.3416	0.1916	0.5474	3.8545	0.9668
2.6401	0.3318	0.1818	0.5194	3.9129	0.9129	2.6417	0.3383	0.1883	0.5380	3.9152	1.0275
2.6791	0.3278	0.1778	0.5081	3.9706	0.9706	2.6814	0.3351	0.1851	0.5289	3.9740	1.0863
2.7209	0.3250	0.1750	0.5000	4.0325	1.0325	2.7198	0.3320	0.1820	0.5200	4.0310	1.1433
2.7515	0.3227	0.1727	0.4935	4.0779	1.0779	2.7569	0.3290	0.1790	0.5114	4.0859	1.1983
2.8016	0.3177	0.1677	0.4790	4.1522	1.1522	2.8100	0.3251	0.1751	0.5002	4.1646	1.2769
2.8406	0.3160	0.1660	0.4742	4.2099	1.2099	2.8436	0.3219	0.1719	0.4912	4.2144	1.3268
2.8712	0.3137	0.1637	0.4677	4.2553	1.2553	2.8757	0.3194	0.1694	0.4840	4.2621	1.3744
2.9102	0.3103	0.1603	0.4581	4.3131	1.3131	2.9064	0.3169	0.1669	0.4770	4.3075	1.4198
2.9436	0.3069	0.1569	0.4484	4.3626	1.3626	2.9494	0.3135	0.1635	0.4673	4.3712	1.4835

2.9881	0.3047	0.1547	0.4419	4.4286	1.4286	2.9888	0.3104	0.1604	0.4584	4.4295	1.5419
3.0271	0.3024	0.1524	0.4355	4.4864	1.4864	3.0258	0.3075	0.1575	0.4500	4.4844	1.5967
3.0661	0.2996	0.1496	0.4274	4.5441	1.5441	3.0636	0.3045	0.1545	0.4414	4.5405	1.6529
3.1051	0.2996	0.1496	0.4274	4.6019	1.6019	3.1019	0.3015	0.1515	0.4328	4.5973	1.7096
3.1496	0.2990	0.1490	0.4258	4.6680	1.6680	3.1407	0.2983	0.1483	0.4238	4.6547	1.7670
3.1886	0.2979	0.1479	0.4226	4.7258	1.7258	3.1798	0.2952	0.1452	0.4150	4.7127	1.8250
3.2248	0.2973	0.1473	0.4210	4.7794	1.7794	3.2193	0.2922	0.1422	0.4062	4.7713	1.8836
3.2582	0.2934	0.1434	0.4097	4.8289	1.8289	3.2592	0.2891	0.1391	0.3974	4.8304	1.9428
3.3028	0.2917	0.1417	0.4048	4.8950	1.8950	3.2995	0.2861	0.1361	0.3888	4.8902	2.0025
3.3445	0.2889	0.1389	0.3968	4.9568	1.9568	3.3402	0.2830	0.1330	0.3799	4.9504	2.0628
3.3807	0.2855	0.1355	0.3871	5.0105	2.0105	3.3813	0.2802	0.1302	0.3721	5.0113	2.1236
3.4197	0.2838	0.1338	0.3823	5.0683	2.0683	3.4088	0.2784	0.1284	0.3668	5.0521	2.1644
3.4615	0.2815	0.1315	0.3758	5.1301	2.1301	3.4644	0.2744	0.1244	0.3555	5.1344	2.2468
3.5004	0.2781	0.1281	0.3661	5.1879	2.1879	3.5064	0.2715	0.1215	0.3472	5.1968	2.3091
3.5366	0.2748	0.1248	0.3565	5.2415	2.2415	3.5346	0.2695	0.1195	0.3415	5.2386	2.3509
3.5728	0.2725	0.1225	0.3500	5.2952	2.2952	3.5772	0.2667	0.1167	0.3333	5.3017	2.4140
3.6146	0.2708	0.1208	0.3452	5.3571	2.3571	3.6058	0.2647	0.1147	0.3277	5.3441	2.4564
3.6536	0.2697	0.1197	0.3419	5.4149	2.4149	3.6489	0.2617	0.1117	0.3191	5.4079	2.5202
3.6953	0.2680	0.1180	0.3371	5.4768	2.4768	3.6923	0.2586	0.1086	0.3103	5.4722	2.5845
3.7343	0.2657	0.1157	0.3306	5.5345	2.5345	3.7359	0.2560	0.1060	0.3030	5.5369	2.6492
3.7761	0.2635	0.1135	0.3242	5.5964	2.5964	3.7798	0.2527	0.1027	0.2934	5.6020	2.7143
3.8123	0.2606	0.1106	0.3161	5.6501	2.6501	3.8093	0.2508	0.1008	0.2880	5.6456	2.7579
3.8512	0.2567	0.1067	0.3048	5.7078	2.7078	3.8536	0.2480	0.0980	0.2799	5.7114	2.8237
3.8874	0.2550	0.1050	0.3000	5.7615	2.7615	3.8833	0.2460	0.0960	0.2744	5.7554	2.8677
3.9320	0.2533	0.1033	0.2952	5.8275	2.8275	3.9281	0.2432	0.0932	0.2663	5.8217	2.9340
3.9738	0.2527	0.1027	0.2935	5.8894	2.8894	3.9730	0.2407	0.0907	0.2591	5.8883	3.0007
4.0100	0.2505	0.1005	0.2871	5.9431	2.9431	4.0182	0.2376	0.0876	0.2504	5.9553	3.0676
4.0517	0.2488	0.0988	0.2823	6.0050	3.0050	4.0485	0.2359	0.0859	0.2454	6.0001	3.1124
4.0907	0.2471	0.0971	0.2774	6.0627	3.0627	4.0940	0.2333	0.0833	0.2381	6.0676	3.1799
4.1325	0.2448	0.0948	0.2710	6.1246	3.1246	4.1397	0.2308	0.0808	0.2308	6.1353	3.2476
4.1687	0.2431	0.0931	0.2661	6.1783	3.1783	4.1703	0.2291	0.0791	0.2259	6.1806	3.2930
4.2132	0.2409	0.0909	0.2597	6.2443	3.2443	4.2163	0.2265	0.0765	0.2186	6.2488	3.3611
4.2522	0.2392	0.0892	0.2548	6.3021	3.3021	4.2470	0.2246	0.0746	0.2132	6.2944	3.4067
4.2856	0.2358	0.0858	0.2452	6.3516	3.3516	4.2778	0.2230	0.0730	0.2086	6.3401	3.4524
4.3274	0.2330	0.0830	0.2371	6.4135	3.4135	4.3242	0.2206	0.0706	0.2017	6.4088	3.5211
4.3691	0.2313	0.0813	0.2323	6.4754	3.4754	4.3552	0.2190	0.0690	0.1971	6.4547	3.5670
4.4081	0.2307	0.0807	0.2306	6.5332	3.5332	4.4018	0.2166	0.0666	0.1902	6.5238	3.6361
4.4499	0.2290	0.0790	0.2258	6.5951	3.5951	4.4485	0.2142	0.0642	0.1833	6.5930	3.7053
4.4889	0.2268	0.0768	0.2194	6.6529	3.6529	4.4797	0.2125	0.0625	0.1786	6.6392	3.7515
4.5306	0.2240	0.0740	0.2113	6.7148	3.7148	4.5266	0.2104	0.0604	0.1726	6.7087	3.8210
4.5613	0.2206	0.0706	0.2016	6.7601	3.7601	4.5579	0.2085	0.0585	0.1672	6.7551	3.8674
4.6058	0.2189	0.0689	0.1968	6.8262	3.8262	4.6049	0.2062	0.0562	0.1607	6.8248	3.9371
4.6448	0.2160	0.0660	0.1887	6.8839	3.8839	4.6363	0.2048	0.0548	0.1564	6.8714	3.9837
4.6754	0.2110	0.0610	0.1742	6.9293	3.9293	4.6678	0.2033	0.0533	0.1522	6.9180	4.0303
4.7144	0.2093	0.0593	0.1694	6.9870	3.9870	4.7150	0.2011	0.0511	0.1460	6.9879	4.1002

4.7534	0.2076	0.0576	0.1645	7.0448	4.0448	4.7465	0.1996	0.0496	0.1418	7.0346	4.1469
4.7923	0.2053	0.0553	0.1581	7.1026	4.1026	4.7938	0.1976	0.0476	0.1359	7.1047	4.2170
4.8369	0.2042	0.0542	0.1548	7.1686	4.1686	4.8253	0.1963	0.0463	0.1324	7.1515	4.2638
4.8731	0.2042	0.0542	0.1548	7.2223	4.2223	4.8727	0.1941	0.0441	0.1259	7.2216	4.3340
4.9177	0.2025	0.0525	0.1500	7.2883	4.2883	4.9042	0.1928	0.0428	0.1223	7.2684	4.3808
4.9567	0.2025	0.0525	0.1500	7.3461	4.3461	4.9516	0.1909	0.0409	0.1169	7.3387	4.4510
4.9984	0.2019	0.0519	0.1484	7.4080	4.4080	4.9990	0.1891	0.0391	0.1116	7.4089	4.5212
5.0374	0.2014	0.0514	0.1468	7.4659	4.4659	5.0306	0.1879	0.0379	0.1082	7.4557	4.5680
5.0764	0.2014	0.0514	0.1468	7.5236	4.5236	5.0780	0.1861	0.0361	0.1032	7.5259	4.6382
5.1182	0.1980	0.0480	0.1371	7.5855	4.5855	5.1095	0.1850	0.0350	0.1000	7.5727	4.6850
5.1544	0.1952	0.0452	0.1290	7.6392	4.6392	5.1568	0.1834	0.0334	0.0953	7.6428	4.7552
5.1934	0.1935	0.0435	0.1242	7.6969	4.6969	5.1884	0.1823	0.0323	0.0923	7.6896	4.8019
5.2324	0.1935	0.0435	0.1242	7.7547	4.7547	5.2356	0.1812	0.0312	0.0892	7.7596	4.8719
5.2769	0.1906	0.0406	0.1161	7.8208	4.8208	5.2671	0.1795	0.0295	0.0843	7.8063	4.9186
5.3103	0.1867	0.0367	0.1048	7.8702	4.8702	5.3143	0.1781	0.0281	0.0803	7.8762	4.9885
5.3521	0.1856	0.0356	0.1016	7.9322	4.9322	5.3457	0.1772	0.0272	0.0776	7.9227	5.0350
5.3910	0.1827	0.0327	0.0935	7.9899	4.9899	5.3927	0.1758	0.0258	0.0737	7.9924	5.1047
5.4300	0.1805	0.0305	0.0871	8.0477	5.0477	5.4397	0.1745	0.0245	0.0700	8.0620	5.1743
5.4718	0.1794	0.0294	0.0839	8.1096	5.1096	5.4709	0.1737	0.0237	0.0677	8.1083	5.2206
5.5108	0.1760	0.0260	0.0742	8.1674	5.1674	5.5177	0.1725	0.0225	0.0644	8.1776	5.2899
5.5497	0.1743	0.0243	0.0694	8.2251	5.2251	5.5488	0.1718	0.0218	0.0623	8.2237	5.3361
5.5915	0.1726	0.0226	0.0645	8.2870	5.2870	5.5954	0.1707	0.0207	0.0593	8.2928	5.4051
5.6305	0.1709	0.0209	0.0597	8.3448	5.3448	5.6109	0.1704	0.0204	0.0583	8.3157	5.4281
5.6583	0.1658	0.0158	0.0452	8.3860	5.3860	5.6573	0.1693	0.0193	0.0553	8.3845	5.4968
5.6889	0.1590	0.0090	0.0258	8.4314	5.4314	5.6881	0.1687	0.0187	0.0533	8.4302	5.5426
5.7139	0.1540	0.0040	0.0113	8.4685	5.4685	5.7189	0.1680	0.0180	0.0513	8.4759	5.5882

Table 12. RNG model

t (sec)	x(m)	x revised(m)	g(ms^-2)	H0(m)	(H0/g)^.5	X*	t*	U*	a*
0.0000	3.0000	0.0000	9.8060	0.3500	0.1889	0.0000	0.0000	_	_
0.2500	3.1940	0.1940	9.8060	0.3500	0.1889	0.5543	1.3233	0.4189	0.3165
0.5000	4.0960	1.0960	9.8060	0.3500	0.1889	3.1314	2.6466	1.1832	0.4471
0.7500	5.0300	2.0300	9.8060	0.3500	0.1889	5.8000	3.9698	1.4610	0.3680
1.0000	5.9600	2.9600	9.8060	0.3500	0.1889	8.4571	5.2931	1.5978	0.3019

Table 13. RNG parameters

(Hexp)i	(Hmodel)i	(Hexp-Hmodel)i^2	(Hexp)^2
0.3758	0.3847	0.0001	0.1412
0.3735	0.3809	0.0001	0.1395
0.3713	0.3770	0.0000	0.1379
0.3690	0.3751	0.0000	0.1362
0.3662	0.3711	0.0000	0.1341
0.3623	0.3692	0.0000	0.1312
0.3600	0.3653	0.0000	0.1296
0.3566	0.3634	0.0000	0.1272
0.3538	0.3596	0.0000	0.1252

0.3510	0.3559	0.0000	0.1232
0.3465	0.3540	0.0001	0.1200
0.3419	0.3502	0.0001	0.1169
0.3391	0.3500	0.0001	0.1150
0.3369	0.3450	0.0001	0.1135
0.3346	0.3416	0.0000	0.1120
0.3318	0.3383	0.0000	0.1101
0.3278	0.3351	0.0001	0.1075
0.3250	0.3320	0.0000	0.1056
0.3227	0.3290	0.0000	0.1042
0.3177	0.3251	0.0001	0.1009
0.3160	0.3219	0.0000	0.0998
0.3137	0.3194	0.0000	0.0984
0.3103	0.3169	0.0000	0.0963
0.3069	0.3135	0.0000	0.0942
0.3047	0.3104	0.0000	0.0928
0.3024	0.3075	0.0000	0.0915
0.2996	0.3045	0.0000	0.0898
0.2996	0.3015	0.0000	0.0898
0.2990	0.2983	0.0000	0.0894
0.2979	0.2952	0.0000	0.0887
0.2973	0.2922	0.0000	0.0884
0.2934	0.2891	0.0000	0.0861
0.2917	0.2861	0.0000	0.0851
0.2889	0.2830	0.0000	0.0834
0.2855	0.2802	0.0000	0.0815
0.2838	0.2784	0.0000	0.0805
0.2815	0.2754	0.0000	0.0793
0.2781	0.2715	0.0000	0.0774
0.2748	0.2695	0.0000	0.0755
0.2725	0.2667	0.0000	0.0743
0.2708	0.2647	0.0000	0.0733
0.2697	0.2617	0.0001	0.0727
0.2680	0.2586	0.0001	0.0718
0.2657	0.2560	0.0001	0.0706
0.2635	0.2527	0.0001	0.0694
0.2606	0.2508	0.0001	0.0679
0.2567	0.2480	0.0001	0.0659
0.2550	0.2451	0.0001	0.0650
0.2533	0.2432	0.0001	0.0642
0.2527	0.2407	0.0001	0.0639
0.2505	0.2385	0.0001	0.0627
0.2488	0.2359	0.0002	0.0619
0.2471	0.2333	0.0002	0.0611
0.2448	0.2308	0.0002	0.0599

0.2431	0.2291	0.0002	0.0591
0.2409	0.2265	0.0002	0.0580
0.2392	0.2246	0.0002	0.0572
0.2358	0.2222	0.0002	0.0556
0.2330	0.2206	0.0002	0.0543
0.2313	0.2190	0.0002	0.0535
0.2307	0.2166	0.0002	0.0532
0.2290	0.2142	0.0002	0.0525
0.2268	0.2125	0.0002	0.0514
0.2240	0.2104	0.0002	0.0502
0.2206	0.2085	0.0001	0.0486
0.2189	0.2062	0.0002	0.0479
0.2160	0.2048	0.0001	0.0467
0.2110	0.2033	0.0001	0.0445
0.2093	0.2011	0.0001	0.0438
0.2076	0.1996	0.0001	0.0431
0.2053	0.1976	0.0001	0.0422
0.2042	0.1963	0.0001	0.0417
0.2042	0.1941	0.0001	0.0417
0.2025	0.1928	0.0001	0.0410
0.2025	0.1909	0.0001	0.0410
0.2019	0.1891	0.0002	0.0408
0.2014	0.1879	0.0002	0.0406
0.2014	0.1861	0.0002	0.0406
0.1980	0.1850	0.0002	0.0392
0.1952	0.1834	0.0001	0.0381
0.1935	0.1823	0.0001	0.0374
0.1935	0.1812	0.0001	0.0374
0.1906	0.1795	0.0001	0.0363
0.1867	0.1781	0.0001	0.0349
0.1856	0.1772	0.0001	0.0344
0.1827	0.1758	0.0000	0.0334
0.1805	0.1745	0.0000	0.0326
0.1794	0.1737	0.0000	0.0322
0.1760	0.1725	0.0000	0.0310
0.1743	0.1718	0.0000	0.0304
0.1726	0.1707	0.0000	0.0298
0.1709	0.1700	0.0000	0.0292
0.1658	0.1693	0.0000	0.0275
0.1590	0.1687	0.0001	0.0253
0.1540	0.1680	0.0002	0.0237
		Epsilon l = ((sig(Hex-Hmod)^2)/ (sig(Hex)^2))^0.5=.0344	

scales or turbulence parameters (such as kinetic energy and loss). These models are based on two fundamental assumptions. First, velocity fluctuations are locally isotropic and Reynolds normal stresses are the same at a point in the flow field. Second, the production terms (turbulence kinetic energy) and turbulence loss are locally in balance. In a k-ε model, it is assumed that turbulence viscosity is proportional with the ratio of the square of turbulence energy divided by turbulence loss[38]. Its parameters are presented in Tables 8 to 10.

$$\mu t \; \alpha \; \frac{\rho k^2}{\varepsilon} \tag{8}$$

Turbulence generation-loss equations in a conservative form are as follows[40]:

$$\frac{\partial k}{\partial t} + \vec{U}.\nabla - \nabla.(D_k \nabla_k) + \gamma_k k = F_k \tag{9}$$

$$D_k = \frac{\nu_T}{\sigma_k} + \nabla_L, D_\varepsilon = \frac{\nu_T}{\sigma_\varepsilon} + \nabla_L \tag{10}$$

$$\frac{\partial k}{\partial t} + \vec{U}.\nabla - \nabla.(D_k \nabla_k) + \gamma_k k = F_k \tag{11}$$

$$\gamma_k = \frac{\varepsilon}{k}, \gamma_\varepsilon = \frac{\varepsilon}{k} \tag{12}$$

$$F_k = \frac{\nu_T}{2} | \nabla\vec{U} + \nabla\vec{U}^T |^2, F_\varepsilon = \frac{C_1 k}{2} | \nabla\vec{U} + \nabla\vec{U}^T |^2 \tag{13}$$

In equations (8) to (13), U is average Reynolds velocity, is turbulence viscosity, K is turbulence kinetic energy, D_k and D_ε are diffusion coefficient of turbulence production and loss, γ_k and γ_ε are respectively reflection coefficient of turbulence production and loss and F_k and F_ε are spring terms. Also, $C_1 = 0.126$, $C_2 = 1.92$, $\sigma_k = 1$, $\sigma_\varepsilon = 1.3$. It should be noted that diffusion and reflection coefficients and spring terms have non-negative values for physical solutions.

2.1.4 RNG Model

This model is a kind of two-equation k-ε model which was introduced for the first time by [41] as Renormalized Group (RNG). The main differences compared to the standard k-ε model, including the presence of an additional term in turbulence losses equation, the effect of rotation on turbulence, the use of an analytical equation to determine the turbulence Prandtl number and using a differential equation to determine the effective viscosity[42]. Applying RNG method on turbulent incompressible flow produces the renormalized motion equation in the non-conservative form[43]. Its parameters are presented in Tables 11 to 13.

$$\frac{\partial \bar{u}_i}{\partial_t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{\partial \bar{P}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[(\upsilon + \upsilon_T)(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i})\right] \tag{14}$$

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \tag{15}$$

In Equations (14) and (15) $\bar{u}_i$ is average velocity, $\bar{P}$ is average pressure, ν is molecular viscosity and ν_T is eddy viscosity. In the above Reynolds numbers, eddy viscosity is in the form of Equation (9).

$$\upsilon_T = C\frac{k^2}{\varepsilon} \tag{16}$$

$$k = \frac{1}{2}\overline{u_i' u_i'}, \varepsilon = \upsilon(\overline{\frac{\partial u_i}{\partial x_j}})^2 \tag{17}$$

Table 14. LES model experimental

Experimental						Model					
x(m)	H(m)	(H-.15)	H*	X	X-3	x(m)	H(m)	(H-.15)	H*	X	X-3
2.0667	0.3758	0.2258	0.6452	3.0630	0.0630	2.0694	0.3847	0.2347	0.6707	3.0670	0.0670
2.1084	0.3735	0.2235	0.6387	3.1249	0.1249	2.1006	0.3824	0.2324	0.6641	3.1133	0.1133
2.1474	0.3713	0.2213	0.6323	3.1826	0.1826	2.1469	0.3789	0.2289	0.6541	3.1819	0.1819
2.1892	0.3690	0.2190	0.6258	3.2445	0.2445	2.1774	0.3766	0.2266	0.6474	3.2271	0.2271
2.2226	0.3662	0.2162	0.6177	3.2940	0.2940	2.2227	0.3730	0.2230	0.6373	3.2942	0.2942
2.2616	0.3623	0.2123	0.6065	3.3518	0.3518	2.2672	0.3695	0.2195	0.6272	3.3602	0.3602
2.3033	0.3600	0.2100	0.6000	3.4137	0.4137	2.2965	0.3672	0.2172	0.6205	3.4036	0.4036
2.3395	0.3566	0.2066	0.5903	3.4673	0.4673	2.3399	0.3637	0.2137	0.6106	3.4679	0.4679
2.3785	0.3538	0.2038	0.5823	3.5251	0.5251	2.3684	0.3614	0.2080	0.5942	3.5724	0.5724
2.4174	0.3510	0.2010	0.5742	3.5828	0.5828	2.4104	0.3580	0.2046	0.5846	3.6336	0.6336
2.4536	0.3465	0.1965	0.5613	3.6365	0.6365	2.4517	0.3546	0.2024	0.5782	3.6736	0.6736

2.4898	0.3419	0.1919	0.5484	3.6901	0.6901	2.4787	0.3524	0.1991	0.5689	3.7327	0.7327
2.5260	0.3391	0.1891	0.5403	3.7437	0.7437	2.5185	0.3491	0.1959	0.5598	3.7903	0.7903
2.5622	0.3369	0.1869	0.5339	3.7973	0.7973	2.5575	0.3459	0.1917	0.5478	3.8651	0.8651
2.6039	0.3346	0.1846	0.5274	3.8592	0.8592	2.6079	0.3417	0.1887	0.5392	3.9195	0.9195
2.6401	0.3318	0.1818	0.5194	3.9129	0.9129	2.6446	0.3387	0.1868	0.5336	3.9550	0.9550
2.6791	0.3278	0.1778	0.5081	3.9706	0.9706	2.6686	0.3368	0.1820	0.5200	4.0407	1.0407
2.7209	0.3250	0.1750	0.5000	4.0325	1.0325	2.7264	0.3320	0.1801	0.5147	4.0738	1.0738
2.7515	0.3227	0.1727	0.4935	4.0779	1.0779	2.7487	0.3301	0.1758	0.5022	4.1532	1.1532
2.8016	0.3177	0.1677	0.4790	4.1522	1.1522	2.8023	0.3258	0.1725	0.4927	4.2134	1.2134
2.8406	0.3160	0.1660	0.4742	4.2099	1.2099	2.8429	0.3225	0.1701	0.4860	4.2565	1.2565
2.8712	0.3137	0.1637	0.4677	4.2553	1.2553	2.8720	0.3201	0.1664	0.4754	4.3244	1.3244
2.9102	0.3103	0.1603	0.4581	4.3131	1.3131	2.9178	0.3164	0.1650	0.4714	4.3626	1.3626
2.9436	0.3069	0.1569	0.4484	4.3626	1.3626	2.9436	0.3150	0.1610	0.4601	4.4220	1.4220
2.9881	0.3047	0.1547	0.4419	4.4286	1.4286	2.9837	0.3110	0.1576	0.4504	4.4848	1.4848
3.0271	0.3024	0.1524	0.4355	4.4864	1.4864	3.0260	0.3076	0.1544	0.4411	4.5453	1.5453
3.0661	0.2996	0.1496	0.4274	4.5441	1.5441	3.0669	0.3044	0.1523	0.4351	4.5842	1.5842
3.1051	0.2996	0.1496	0.4274	4.6019	1.6019	3.0931	0.3023	0.1516	0.4330	4.5975	1.5975
3.1496	0.2990	0.1490	0.4258	4.6680	1.6680	3.1492	0.2978	0.1478	0.4224	4.6674	1.6674
3.1886	0.2979	0.1479	0.4226	4.7258	1.7258	3.1893	0.2947	0.1447	0.4135	4.7268	1.7268
3.2248	0.2973	0.1473	0.4210	4.7794	1.7794	3.2207	0.2923	0.1423	0.4066	4.7733	1.7733
3.2582	0.2934	0.1434	0.4097	4.8289	1.8289	3.2532	0.2899	0.1399	0.3996	4.8215	1.8215
3.3028	0.2917	0.1417	0.4048	4.8950	1.8950	3.3098	0.2859	0.1359	0.3883	4.9054	1.9054
3.3445	0.2889	0.1389	0.3968	4.9568	1.9568	3.3452	0.2831	0.1331	0.3804	4.9578	1.9578
3.3807	0.2855	0.1355	0.3871	5.0105	2.0105	3.3816	0.2807	0.1307	0.3734	5.0117	2.0117
3.4197	0.2838	0.1338	0.3823	5.0683	2.0683	3.4189	0.2781	0.1281	0.3661	5.0670	2.0670
3.4615	0.2815	0.1315	0.3758	5.1301	2.1301	3.4572	0.2756	0.1256	0.3590	5.1238	2.1238
3.5004	0.2781	0.1281	0.3661	5.1879	2.1879	3.5096	0.2722	0.1222	0.3493	5.2015	2.2015
3.5366	0.2748	0.1248	0.3565	5.2415	2.2415	3.5365	0.2708	0.1208	0.3453	5.2413	2.2413
3.5728	0.2725	0.1225	0.3500	5.2952	2.2952	3.5774	0.2679	0.1179	0.3368	5.3020	2.3020
3.6146	0.2708	0.1208	0.3452	5.3571	2.3571	3.6191	0.2653	0.1153	0.3295	5.3638	2.3638
3.6536	0.2697	0.1197	0.3419	5.4149	2.4149	3.6474	0.2636	0.1136	0.3246	5.4058	2.4058
3.6953	0.2680	0.1180	0.3371	5.4768	2.4768	3.6905	0.2610	0.1110	0.3172	5.4696	2.4696
3.7343	0.2657	0.1157	0.3306	5.5345	2.5345	3.7343	0.2585	0.1085	0.3100	5.5345	2.5345
3.7761	0.2635	0.1135	0.3242	5.5964	2.5964	3.7788	0.2564	0.1064	0.3039	5.6005	2.6005
3.8123	0.2606	0.1106	0.3161	5.6501	2.6501	3.8089	0.2545	0.1045	0.2987	5.6450	2.6450
3.8512	0.2567	0.1067	0.3048	5.7078	2.7078	3.8545	0.2534	0.1034	0.2955	5.7126	2.7126
3.8874	0.2550	0.1050	0.3000	5.7615	2.7615	3.8853	0.2530	0.1030	0.2943	5.7583	2.7583
3.9320	0.2533	0.1033	0.2952	5.8275	2.8275	3.9319	0.2528	0.1028	0.2936	5.8274	2.8274
3.9738	0.2527	0.1027	0.2935	5.8894	2.8894	3.9792	0.2523	0.1023	0.2922	5.8974	2.8974
4.0100	0.2505	0.1005	0.2871	5.9431	2.9431	4.0110	0.2513	0.1013	0.2895	5.9446	2.9446
4.0517	0.2488	0.0988	0.2823	6.0050	3.0050	4.0591	0.2488	0.0988	0.2822	6.0159	3.0159
4.0907	0.2471	0.0971	0.2774	6.0627	3.0627	4.0915	0.2466	0.0966	0.2761	6.0639	3.0639
4.1325	0.2448	0.0948	0.2710	6.1246	3.1246	4.1404	0.2433	0.0933	0.2665	6.1364	3.1364
4.1687	0.2431	0.0931	0.2661	6.1783	3.1783	4.1568	0.2421	0.0921	0.2630	6.1608	3.1608
4.2132	0.2409	0.0909	0.2597	6.2443	3.2443	4.2064	0.2387	0.0887	0.2533	6.2342	3.2342
4.2522	0.2392	0.0892	0.2548	6.3021	3.3021	4.2563	0.2348	0.0848	0.2423	6.3082	3.3082

4.2856	0.2358	0.0858	0.2452	6.3516	3.3516	4.2898	0.2324	0.0824	0.2355	6.3578	3.3578
4.3274	0.2330	0.0830	0.2371	6.4135	3.4135	4.3235	0.2302	0.0802	0.2291	6.4077	3.4077
4.3691	0.2313	0.0813	0.2323	6.4754	3.4754	4.3573	0.2280	0.0780	0.2229	6.4578	3.4578
4.4081	0.2307	0.0807	0.2306	6.5332	3.5332	4.4083	0.2256	0.0756	0.2159	6.5334	3.5334
4.4499	0.2290	0.0790	0.2258	6.5951	3.5951	4.4424	0.2243	0.0743	0.2122	6.5840	3.5840
4.4889	0.2268	0.0768	0.2194	6.6529	3.6529	4.4767	0.2228	0.0728	0.2080	6.6348	3.6348
4.5306	0.2240	0.0740	0.2113	6.7148	3.7148	4.5283	0.2207	0.0707	0.2019	6.7112	3.7112
4.5613	0.2206	0.0706	0.2016	6.7601	3.7601	4.5628	0.2197	0.0697	0.1991	6.7624	3.7624
4.6058	0.2189	0.0689	0.1968	6.8262	3.8262	4.5974	0.2190	0.0690	0.1971	6.8137	3.8137
4.6448	0.2160	0.0660	0.1887	6.8839	3.8839	4.6494	0.2178	0.0678	0.1936	6.8908	3.8908
4.6754	0.2110	0.0610	0.1742	6.9293	3.9293	4.6668	0.2172	0.0672	0.1921	6.9165	3.9165
4.7144	0.2093	0.0593	0.1694	6.9870	3.9870	4.7190	0.2153	0.0653	0.1866	6.9939	3.9939
4.7534	0.2076	0.0576	0.1645	7.0448	4.0448	4.7539	0.2140	0.0640	0.1830	7.0456	4.0456
4.7923	0.2053	0.0553	0.1581	7.1026	4.1026	4.7888	0.2130	0.0630	0.1800	7.0973	4.0973
4.8369	0.2042	0.0542	0.1548	7.1686	4.1686	4.8237	0.2123	0.0623	0.1780	7.1491	4.1491
4.8731	0.2042	0.0542	0.1548	7.2223	4.2223	4.8761	0.2087	0.0587	0.1676	7.2268	4.2268
4.9177	0.2025	0.0525	0.1500	7.2883	4.2883	4.9111	0.2077	0.0577	0.1648	7.2786	4.2786
4.9567	0.2025	0.0525	0.1500	7.3461	4.3461	4.9460	0.2073	0.0573	0.1638	7.3304	4.3304
4.9984	0.2019	0.0519	0.1484	7.4080	4.4080	4.9984	0.2066	0.0566	0.1617	7.4081	4.4081
5.0374	0.2014	0.0514	0.1468	7.4659	4.4659	5.0333	0.2057	0.0557	0.1590	7.4598	4.4598
5.0764	0.2014	0.0514	0.1468	7.5236	4.5236	5.0682	0.2042	0.0542	0.1548	7.5114	4.5114
5.1182	0.1980	0.0480	0.1371	7.5855	4.5855	5.1204	0.2007	0.0507	0.1449	7.5888	4.5888
5.1544	0.1952	0.0452	0.1290	7.6392	4.6392	5.1551	0.1980	0.0480	0.1372	7.6403	4.6403
5.1934	0.1935	0.0435	0.1242	7.6969	4.6969	5.1898	0.1953	0.0453	0.1294	7.6917	4.6917
5.2324	0.1935	0.0435	0.1242	7.7547	4.7547	5.2244	0.1930	0.0430	0.1228	7.7429	4.7429
5.2769	0.1906	0.0406	0.1161	7.8208	4.8208	5.2761	0.1900	0.0400	0.1143	7.8195	4.8195
5.3103	0.1867	0.0367	0.1048	7.8702	4.8702	5.3104	0.1872	0.0372	0.1063	7.8704	4.8704
5.3521	0.1856	0.0356	0.1016	7.9322	4.9322	5.3447	0.1855	0.0355	0.1015	7.9212	4.9212
5.3910	0.1827	0.0327	0.0935	7.9899	4.9899	5.3958	0.1835	0.0335	0.0958	7.9970	4.9970
5.4300	0.1805	0.0305	0.0871	8.0477	5.0477	5.4297	0.1825	0.0325	0.0929	8.0472	5.0472
5.4718	0.1794	0.0294	0.0839	8.1096	5.1096	5.4635	0.1833	0.0333	0.0953	8.0973	5.0973
5.5108	0.1760	0.0260	0.0742	8.1674	5.1674	5.5139	0.1708	0.0208	0.0596	8.1720	5.1720
5.5497	0.1743	0.0243	0.0694	8.2251	5.2251	5.5473	0.1628	0.0128	0.0364	8.2215	5.2215
5.5915	0.1726	0.0226	0.0645	8.2870	5.2870	5.5970	0.1500	0.0000	0.0000	8.2952	5.2952
5.6305	0.1709	0.0209	0.0597	8.3448	5.3448	5.6300	0.1500	0.0000	0.0000	8.3441	5.3441
5.6583	0.1658	0.0158	0.0452	8.3860	5.3860	5.6464	0.1500	0.0000	0.0000	8.3684	5.3684
5.6889	0.1590	0.0090	0.0258	8.4314	5.4314	5.6790	0.1500	0.0000	0.0000	8.4167	5.4167
5.7139	0.1540	0.0040	0.0113	8.4685	5.4685	5.7115	0.1500	0.0000	0.0000	8.4648	5.4648

Table 15. LES model

	t (sec)	x(m)	x revised(m)	g(ms^-2)	H0(m)	(H0/g)^.5	X*	t*	U*	a*
	0.0000	3.0000	0.0000	9.8060	0.3500	0.1889	0.0000	0.0000	_	_
	0.2500	3.1940	0.1940	9.8060	0.3500	0.1889	0.5543	1.3233	0.4189	0.3165
LES Model	0.5000	4.0000	1.0000	9.8060	0.3500	0.1889	2.8571	2.6466	1.0796	0.4079
	0.7500	4.5480	1.5480	9.8060	0.3500	0.1889	4.4229	3.9698	1.1141	0.2806
	1.0000	5.0450	2.0450	9.8060	0.3500	0.1889	5.8429	5.2931	1.1039	0.2085
	1.2500	5.5400	2.5400	9.8060	0.3500	0.1889	7.2571	6.6164	1.0968	0.1658

Table 16. LES model parameters

(Hex)i	(Hmod)i	(Hex-Hmod)i^2	(Hex)^2
0.3758	0.3847	0.0001	0.1412
0.3735	0.3824	0.0001	0.1395
0.3713	0.3789	0.0001	0.1379
0.3690	0.3754	0.0000	0.1362
0.3662	0.3730	0.0000	0.1341
0.3623	0.3695	0.0001	0.1312
0.3600	0.3672	0.0001	0.1296
0.3566	0.3637	0.0001	0.1272
0.3538	0.3614	0.0001	0.1252
0.3510	0.3580	0.0000	0.1232
0.3465	0.3546	0.0001	0.1200
0.3419	0.3524	0.0001	0.1169
0.3391	0.3491	0.0001	0.1150
0.3369	0.3459	0.0001	0.1135
0.3346	0.3417	0.0001	0.1120
0.3318	0.3387	0.0000	0.1101
0.3278	0.3358	0.0001	0.1075
0.3250	0.3329	0.0001	0.1056
0.3227	0.3300	0.0001	0.1042
0.3177	0.3258	0.0001	0.1009
0.3160	0.3225	0.0000	0.0998
0.3137	0.3201	0.0000	0.0984
0.3103	0.3171	0.0000	0.0963
0.3069	0.3150	0.0001	0.0942
0.3047	0.3110	0.0000	0.0928
0.3024	0.3076	0.0000	0.0915
0.2996	0.3044	0.0000	0.0898
0.2996	0.3016	0.0000	0.0898
0.2990	0.2978	0.0000	0.0894
0.2979	0.2947	0.0000	0.0887
0.2973	0.2923	0.0000	0.0884
0.2934	0.2899	0.0000	0.0861
0.2917	0.2859	0.0000	0.0851
0.2889	0.2831	0.0000	0.0834
0.2855	0.2807	0.0000	0.0815

0.2838	0.2781	0.0000	0.0805
0.2815	0.2756	0.0000	0.0793
0.2781	0.2722	0.0000	0.0774
0.2748	0.2708	0.0000	0.0755
0.2725	0.2679	0.0000	0.0743
0.2708	0.2653	0.0000	0.0733
0.2697	0.2636	0.0000	0.0727
0.2680	0.2610	0.0000	0.0718
0.2657	0.2585	0.0001	0.0706
0.2635	0.2564	0.0001	0.0694
0.2606	0.2545	0.0000	0.0679
0.2567	0.2534	0.0000	0.0659
0.2550	0.2530	0.0000	0.0650
0.2533	0.2528	0.0000	0.0642
0.2527	0.2523	0.0000	0.0639
0.2505	0.2513	0.0000	0.0627
0.2488	0.2488	0.0000	0.0619
0.2471	0.2466	0.0000	0.0611
0.2448	0.2444	0.0000	0.0599
0.2431	0.2421	0.0000	0.0591
0.2409	0.2387	0.0000	0.0580
0.2392	0.2348	0.0000	0.0572
0.2358	0.2324	0.0000	0.0556
0.2330	0.2302	0.0000	0.0543
0.2313	0.2280	0.0000	0.0535
0.2307	0.2256	0.0000	0.0532
0.2290	0.2243	0.0000	0.0525
0.2268	0.2228	0.0000	0.0514
0.2240	0.2207	0.0000	0.0502
0.2206	0.2197	0.0000	0.0486
0.2189	0.2190	0.0000	0.0479
0.2160	0.2178	0.0000	0.0467
0.2110	0.2166	0.0000	0.0445
0.2093	0.2153	0.0000	0.0438
0.2076	0.2140	0.0000	0.0431
0.2053	0.2130	0.0001	0.0422
0.2042	0.2123	0.0001	0.0417
0.2042	0.2087	0.0000	0.0417
0.2025	0.2077	0.0000	0.0410
0.2025	0.2073	0.0000	0.0410
0.2019	0.2066	0.0000	0.0408
0.2014	0.2057	0.0000	0.0406
0.2014	0.2042	0.0000	0.0406
0.1980	0.2020	0.0000	0.0392
0.1952	0.1980	0.0000	0.0381

0.1935	0.1953	0.0000	0.0374
0.1935	0.1930	0.0000	0.0374
0.1906	0.1900	0.0000	0.0363
0.1867	0.1872	0.0000	0.0349
0.1856	0.1855	0.0000	0.0344
0.1827	0.1835	0.0000	0.0334
0.1805	0.1825	0.0000	0.0326
0.1794	0.1833	0.0000	0.0322
0.1760	0.1708	0.0000	0.0310
0.1743	0.1628	0.0001	0.0304
0.1726	0.1500	0.0005	0.0298
0.1709	0.1500	0.0004	0.0292
0.1658	0.1500	0.0002	0.0275
0.1590	0.1500	0.0001	0.0253
0.1540	0.1500	0.0000	0.0237

Epsilon l = ((sig(Hex-Hmod)^2)/
(sig(Hex)^2))^0.5=.0245

RNG is a dimensionless coefficient which its value is 0.85 in C method. Kinetic energy and turbulence loss ratio is derived from rewritten transfer equations in the above Reynolds numbers as followings:

$$\frac{\partial k}{\partial t}+\bar{u}_i\frac{\partial k}{\partial x_i}=P-\varepsilon+\frac{\partial}{\partial x_i}\left(\frac{\upsilon_T}{\sigma_k}\frac{\partial k}{\partial x_i}\right) \quad (18)$$

$$\frac{\partial k}{\partial t}+\bar{u}_i\frac{\partial k}{\partial x_i}=P-\varepsilon+\frac{\partial}{\partial x_i}\left(\frac{\upsilon_T}{\sigma_k}\frac{\partial k}{\partial x_i}\right) \quad (19)$$

P is turbulence production and it is calculated by the following equations:

$$P=2\upsilon+\bar{S}_{ij}^2 \quad (20)$$

$$\bar{S}_{ij}=\frac{1}{2}\left(\frac{\partial\bar{u}_i}{\partial x_j}+\frac{\partial\bar{u}_i}{\partial x_i}\right)\frac{n!}{r!(n-r)!} \quad (21)$$

The constant coefficients of RNG equations are similar to constant coefficients of standard k-ε equations, but they have different values:

$$c=0.085, c_1=1.063, c_2=1.72, \sigma_k=0.7179, \sigma_\varepsilon=0.7179 \quad (22)$$

2.1.5 Large Eddy Simulation (LES)

This model was first introduced in the field of atmospheric modeling[44]. Since the direct numerical simulation of Navier–Stokes equations are very complicated and expensive in practice, so LES model is used. The model provides the possibility to solve directly Navier-Stokes equations through making smaller temporal and spatial scales. The solution procedure is so that short path filters are applied to the Navier–Stokes equations and temporal and spatial, small data scale (such as small eddies) are omitted from numerical solution. This information needs to further modeling[45-46]. There are two basic assumptions in using this model in Flow-3D software. First: The model is three-dimensional, and second: it is time-dependent[39]. Its parameters are presented in Tables 14 to 16.

Navier-Stokes Equation for incompressible flow in terms of mean flow in non-conservative form is as follows:

$$\frac{\partial U_i}{\partial t}+\frac{\partial U_iU_j}{\partial x_j}=-\frac{1}{\rho}\frac{\partial P}{\partial x_i}+\upsilon V^2U_i \quad (23)$$

By applying Leibniz law on the Equation (23) and filtering each of the terms and considering $<\phi>=\bar{\phi}$ as spatial averaging, Equation (23) changes to Equation (24):

$$\frac{\overline{\partial U_i}}{\partial t}+\frac{\partial <U_iU_j>}{\partial x_j}=-\frac{1}{\rho}\frac{\partial\bar{P}}{\partial x_i}+\upsilon V^2\bar{U}_i \quad (24)$$

In equation (24) nonlinear terms of $<U_iU_j>$, makes difficult solving the equation, to solve this problem, the Reynolds analysis is used:

$$U_i=\bar{U}_i+u_i' \quad (25)$$

In Equation (25) $\bar{U}_i$ represents the large-scale component of the velocity field and it is measurable and u_i' represents the small-scale component and needs to be filtered. Thus:

$$< U_i U_j > = < \bar{U}_i \bar{U}_j > + < \bar{U}_i u_j' > < u_i' \bar{U}_j > + < u_i' u_j' >$$

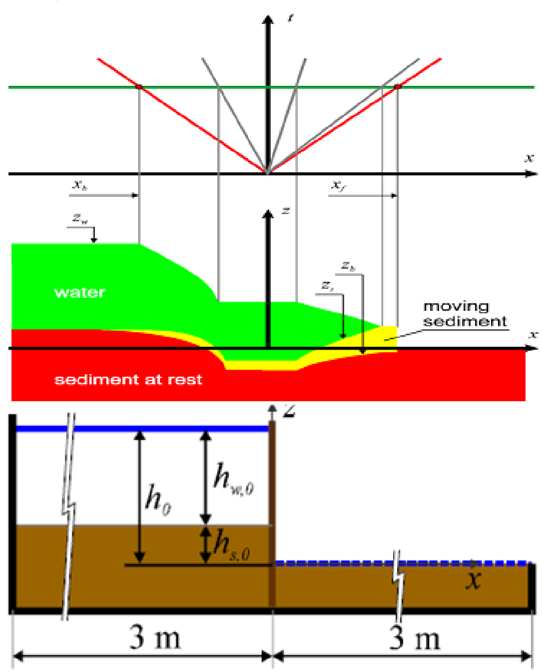
By substitution, the below equation is obtained:

$$\frac{\partial \bar{U}_i}{\partial t} + \frac{\partial \bar{U}_i \bar{U}_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_i} + \upsilon \nabla^2 \bar{U}_i \qquad (27)$$

In equation (27), stress term of τ_{ij} is defined as follows:

$$\tau_{ij} = \underbrace{< \bar{U}_i u_j' > + < u_i' \bar{U}_j >}_{C_{ij}} + \underbrace{< u_i' u_j' >}_{R_{ij}} \qquad (28)$$

C_{ij} are called cross terms, which represents the average created random forces of the average scales of sub grid on a large scale. R_{ij} terms are called Reynolds stress of subgrid which is the same as turbulence stresses created by averaging Reynolds in Navier-Stokes equations. Schematic of dam break on the mobile bed is illustrated in Figure 1.



.1

Figure 1. Schematic figure of the dam break on the mobile bed (left)33, profile of positive wave caused by dam failure with measured variables (right)17

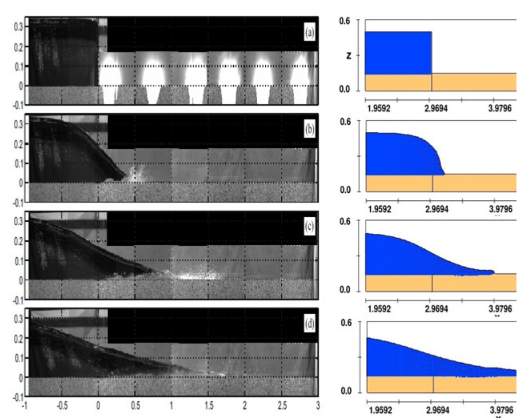
To conduct simulations in Flow-3D simulation software, first the geometry and physical condition of the problem was defined. In determining the geometric conditions, sensitivity analysis was performed on the size of grid cells. Three sizes of 0.005, 0.015 and 0.025 meter were selected for cells. In the simulation by using Prandtl mixing length and turbulence kinetic energy models, more accurate results were obtained in grids cells of 0.005, but because in more complicated turbulence models such as RNG, the calculations were cut in grid cell size of 0.005 m, the cell size of 0.015 m was selected as standard cell size to compare the performance of turbulence models.

After conducting the simulations by using turbulence models in Flow-3D software, the results were presented as graphs of the free surface flow profile, dimensionless profile off low's free surface, dimensionless profile of distance-time of progressing wave front, the dimensionless profile of velocity-time of progressing wave front and the dimensionless profile of acceleration-time of progressing wave front. It should be noted that below relations were used in preparation of graphs:

$$\frac{x}{t\sqrt{gH_0}} = X, \quad \frac{x}{H_0} = X^* \qquad (29)$$

$$\frac{t}{\sqrt{\frac{H}{g}}} = t^*, \quad \frac{H}{H_0} = H^* \qquad (30)$$

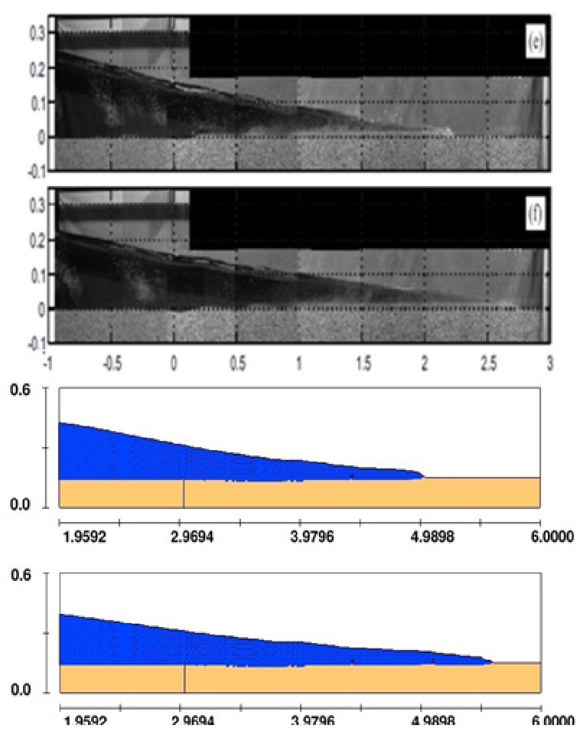
$$\frac{X^*}{t^{*2}} = a^*, \quad \frac{X^*}{t^*} = U^* \qquad (31)$$



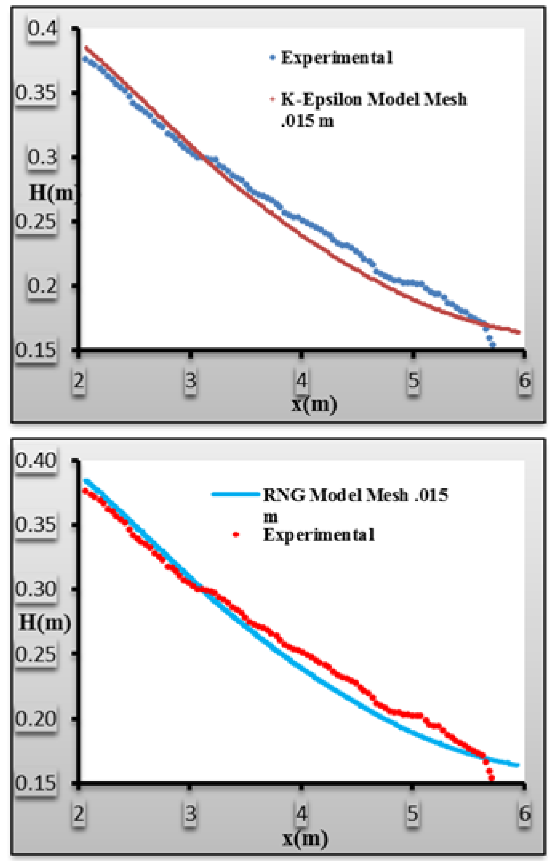
Figuer 2. Formation and motion of the positive wave after pulling the valve in time t = 0, 0.25, 0.5, 0.75 second in the experimental model (left) and in the Flow-3D (right), the results of the simulation by LES turbulence model

In the above relations, x, t, H_0, H, X, H^*, t^*, U^*, a^* are respectively the horizontal distance, the spent time after the experiment's initiation, the initial depth of the water behind the valve (in the upstream), H, is height of the free surface at time t, dimensionless variables of horizontal distance, dimensionless variable of the height of the free surface, dimensionless variable of progressing velocity of wave front and dimensionless variable of progressing acceleration of positive wave front shown in Figure 2.

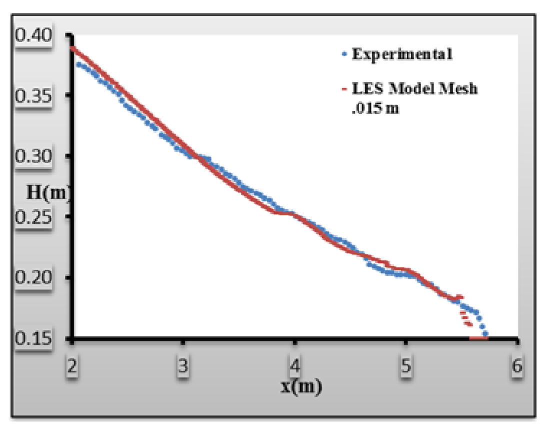
The results of modeling by turbulence models were compared with the results of the experimental model in time t=1.25s, then the pattern of flow's free surface and dimensionless Figures 3 to 7 were presented and the accuracy of models in simulations was evaluated by using statistical relations.



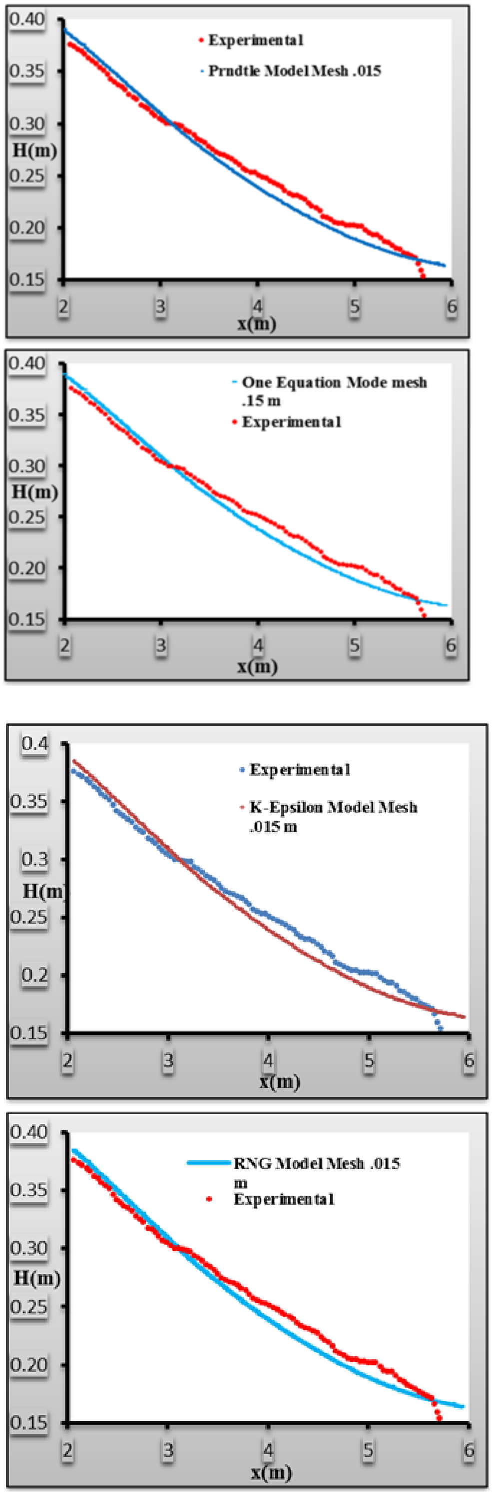
Figuer 3. The movement of positive wave after pulling the valve in time t = 1, 1.25 seconds in the experimental model (left) and in the Flow-3D (right), the results of the simulation by LES turbulence modelturbulence model



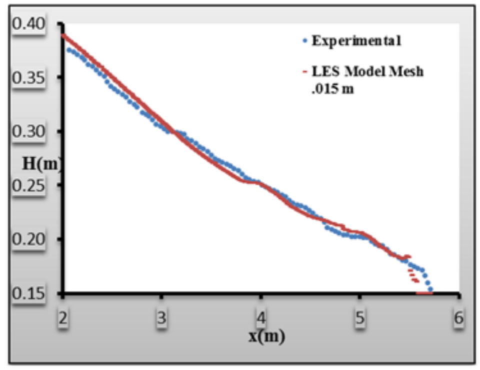
Figuer 4. Water free surface profiles in models of Prandtl mixing length, turbulence kinetic energy, k-ε and RNG (t =1.25 s)modelturbulence model



Figuer 5. Water free surface profiles in LES turbulence model and the experimental results (t =1.25 s)

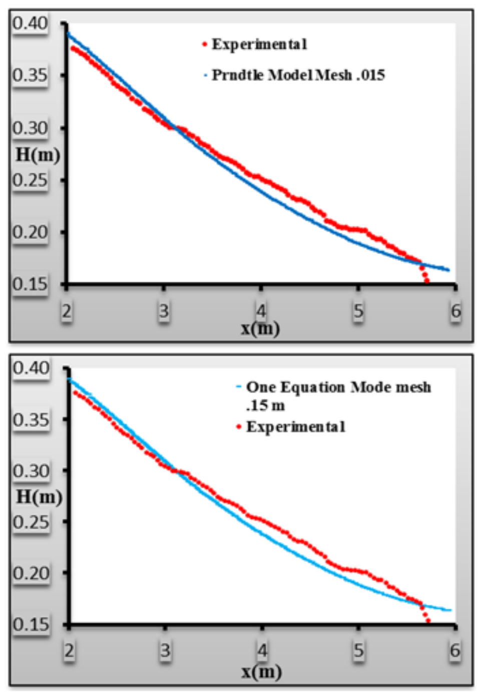


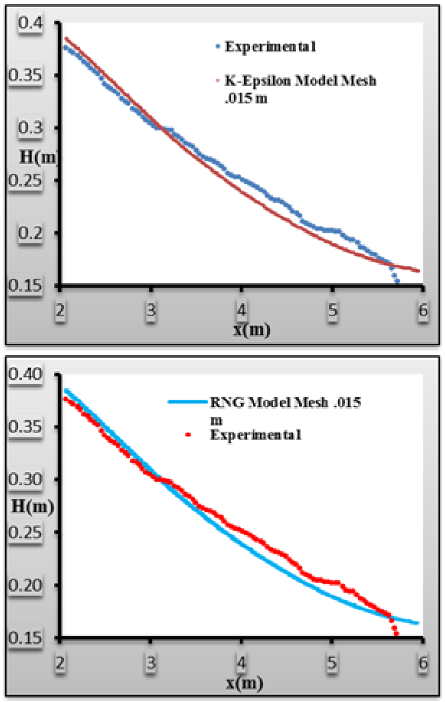
Figuer 6. Water free surface profiles in models of Prandtl mixing length, turbulence kinetic energy, k-ε and RNG (t =1.25 s)



Figuer 7. Water free surface profiles in LES turbulence model and the experimental results (t =1.25 s)

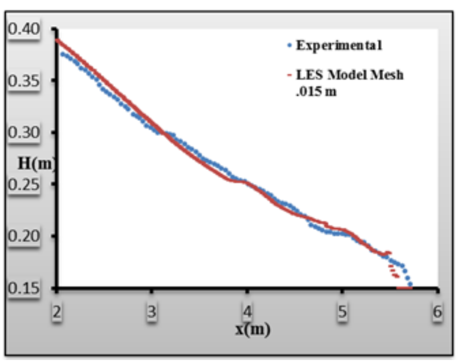
Due to the large number of graphs, just LES turbulence model graphs are presented in the followings shown in Figures 8 to 12. The figures include dimensionless profiles of water free surface, distance- time, velocity - time, acceleration - time of progress of positive wave front and the profile of the sedimentary bed.





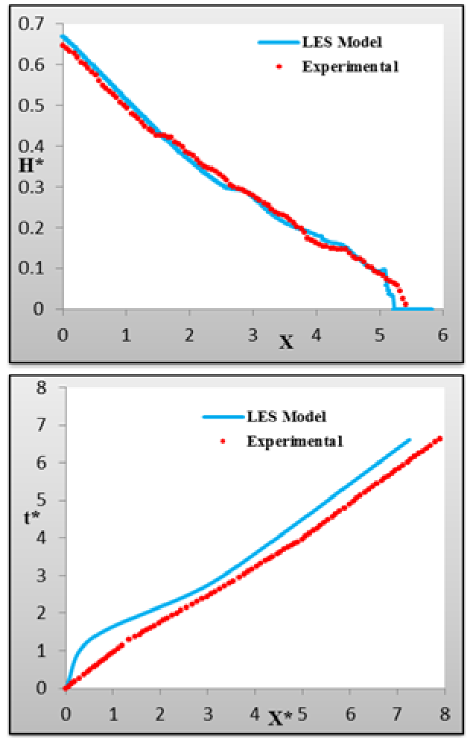
Figuer 8. Water free surface profiles in models of Prandtl mixing length, turbulence kinetic energy, k-ε and RNG (t =1.25 s)

Figure 8. Dimensionless profiles of free surface (right) and distance-time (left) of progress of a positive wave front in LES model and experimental results.

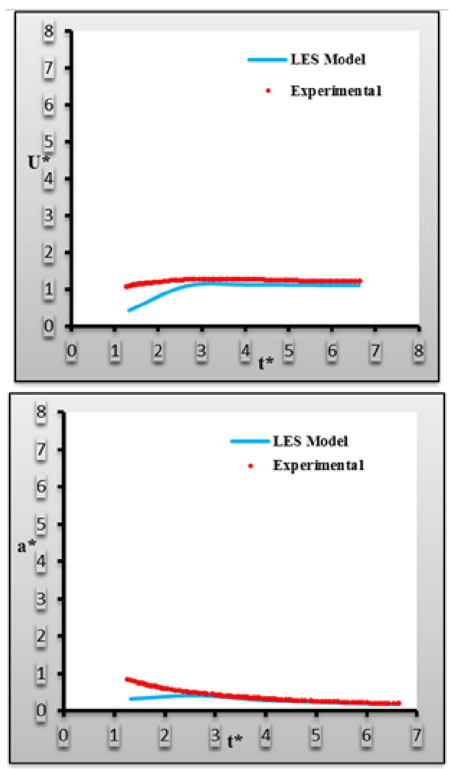


Figuer 9. Water free surface profiles in LES turbulence model and the experimental results (t =1.25 s)

Figure 9. Dimensionless profiles of velocity-time (right) and acceleration-time (left) of progress of a positive wave front in LES model and experimental results.

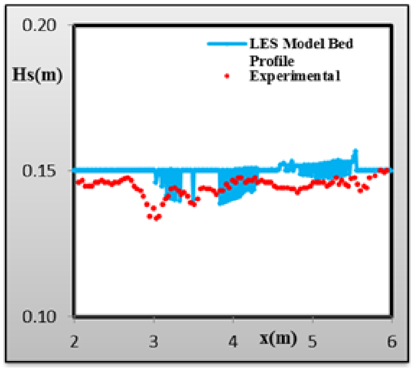


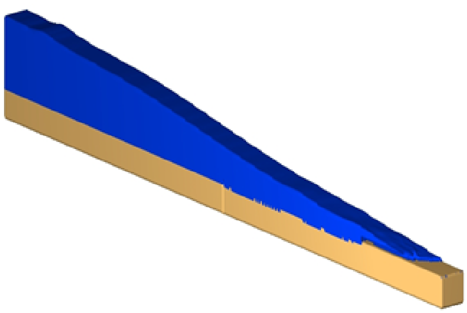
Figuer 10. Dimensionless profiles of free surface (right) and distance-time (left) of progress of a positive wave front in LES model and experimental results



Figuer 11. Dimensionless profiles of velocity-time (right) and acceleration-time (left) of progress of a positive wave front in LES model and experimental results

Figure 10. Sand bed profile and experimental results (right) and its simulation in Flow-3D software (left) at time t = 1.25 s.





Figuer 12. Sand bed profile and experimental results (right) and its simulation in Flow-3D software (left) at time t = 1.25 s

The accuracy of models in simulations and predictions about water free surface profiles, progressing velocity and acceleration of wave front was evaluated based on R^2criteria and percentage of relative normal error according to the following relationship:

$$\mathring{a}_L = \sqrt{\frac{\sum_{i=1}^{n}(\varnothing_{\exp_i} - \varnothing_{\mathrm{mod}_i})^2}{\sum_{i=1}^{n}(\varnothing_{\exp_i}{}^2)}} \qquad (32)$$

In equation (32), ε_L is relative normal error and ϕ_{exi} is the obtained variable from experimental results and ϕ_{modi} is the obtained variable from modeling results shown in Table 1.

Table 1. Normal error values and calculated for Figures 4-7 in turbulence models

Table 1. Normal error values and calculated for Figures 4-7 in turbulence models

α*	U*	Free Surface Flow profile	Accuracy Criteria	Model Name
---	---	0.913	R^2	Prandtl Mixing
0.782	0.326	0.035	ε_L	Length
---	---	0.886	R^2	Turbulence
0.782	0.326	0.036	ε_L	Kinetic Energy
---	---	0.906	R^2	K-Epsilon
0.783	0.327	0.034	ε_L	
---	--	0.900	R^2	RNG
0.783	0.327	0.034	ε_L	
---	---	---	R^2	LES
0.492	0.289	0.024	ε_L	

3. Research Findings

Based on the statistical calculations which are presented in Table 2, by considering ε_L as a basis for performance comparison of turbulence models, LES model has the lowest relative error in the calculation of free surface flow profile and therefore it is the most accurate turbulence model for simulating the dam failure phenomenon on the mobile bed. After LES model, the order of accuracy of models were respectively k-ε, RNG, Prandtl mixing length and turbulence kinetic energy. In calculating the progress velocity profile of positive wave front and progress acceleration profile of positive wave front, the LES model has the highest accuracy and then, other models have the same accuracy. In the simulation of profile of sedimentary sand bed, just LES model predicted changes in sedimentary bed profile.

4. Discussion and Conclusion

In this study, the performance of the turbulent flow models are compared in the Flow-3D Software in the simulation of the positive wave caused dam break on the mobile bed. The simulation results were compared using statistical calculations and analysis. The research findings showed that the performance of numerical model in predicting the behavior of a positive wave caused by dam break depends on the correct selection of turbulence model. LES model has the highest accuracy in simulating free flow surface profile and dimensionless profiles of velocity and acceleration progress of wave front. In simulation of motion of sedimentary sanded, just LES model could predict changes in sand bed. But the prediction didn't have appropriate accuracy and in general none of turbulence models could show the motion of sedimentary bed profile with reasonable accuracy.

5. References

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