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Nonlinear Analysis of Torsion in Reinforced Concrete Members after Developing Initial Crack

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Abstract

Background/Objectives: Nonlinear analysis of torsion in reinforced concrete members after developing initial crack is a significant effect on piece life time. **Methods/Statistical Analysis:** As a result of integration and cohesion of members in the reinforced concrete structures Torsion typically occurs. Reduced torsional stiffness problem was presented after semi-brittle break out in concrete due to presence of initial cracks in outer surfaces of concrete element in determining torsional stiffness. **Findings:** The idea was that reduced torsional stiffness is approximately one-tenth of its value before cracking and this is very effective on the distribution of anchors in the structure. After these studies, different torsion theories were presented which measured the reduced stiffness by using different methods. In this paper, by using these theories, a code was written for torsional analysis of reinforced concrete after cracking called "nlarctm" and 23 rectangular cross-sections were controlled and compared by this program. Comparisons of anchor changes was based on the torsion angle and results indicated that cross-section number (18) has higher torsional strength than similar cross-sections. Applications/ Improvements: The results of this research shows that by comparing three effects the resistance of reinforcements, compressive strength and the distance of collars, it's concluded that all of them are very effective in increase or decrease.

Keywords: Beam, Concrete, Initial Crack, nlarctm Code, Torsional Stiffness

1. Introduction

Torsion usually occurs due to integration and cohesion of members in the reinforced concrete structures. For this reason, this problem has less interested the designers of the armed concrete structures. In fact, torsion was considered as a secondary issue and as a result, its effect in designing was only seen as relying on large reliability coefficient which were used in the computational methods. During recent years, the effect of torsion and design for it has been emphasized and its certain criteria form a significant part of regulations. There is two basic reason for this change. First, design methods are based on more realistic concept of loads and the behavior of the structure than older methods and as a result, smaller reliability coefficients govern the design, and second, number and diversity of the structures in which torsion is the axis of the structure behavior-not a secondary effect-is developing. Therefore, widespread researches has been conducted about the designing concrete structures under the effect of torsion in the few last years and now we can claim that the results of these researchers take the form of regulation criteria which can lead to secure and economic designs. In the structural systems, torsion is usually divided into two groups: Determined static torsion which sometimes is called balance torsion and undetermined static torsion which is called compatibility torsion. In the first type, i.e., anchor torsion, torsional anchor is not possible only by using the balance of forces or in other words, the static of the problem, but using torsion compatibility of connected members like circumference beam, slab or column is necessary. It is clear that for this case, value of the torsional anchor along with the beam which is equal with the bending anchor in the slab,

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is the function of slab rotation in the connected end with the beam or in other words, it is a function of slab and beam stiffness. Generally, in stable undetermined structures, torsional anchor can be measured only when the torsional stiffness of reinforced concrete members is known. In a stable determined structure, like balcony cantilever slab, torsional anchor can be obtained from (static) balance equations and regulations' criteria can be used without problem; but the calculation of torsional anchor in a stable undetermined structure like a beam, needs using compatibility conditions, in addition to balance equations. In other words, torsional stiffness of beam should be considered.

Torsional stiffness of reinforced concrete cross-section is calculated with acceptable accuracy by Saint-Venan using elastic theory. But despite researches about torsion or a combination of torsion, bending and shear, our knowledge of torsional stiffness after cracking is limited and the need to this knowledge is highlighted by this fact that torsional members often crack under service loads and regulations for designing concrete structure remained silent about the torsional stiffness of cracked members. Besides, torsional stiffness after cracking is only a minor fraction of its value before cracking (approximately %10). In a fixed undetermined structure, this effective reduction in torsional stiffness after cracking will influence the distribution of anchors. In this paper, first we study the torsion theory. For this purpose, first the behavior of prismatic members under torsion will be evaluated by the inverse and semi-inverse methods; then, membrane analogy analysis for solving torsion problems was will be introduced and by using it, torsion problem and calculation of maximum shear stress caused by rectangular cross-sections will be analyzed. In following, plastic torsion will be calculated for rectangular cross-sections using plastic theory and sand pile method.

Then, torsion theories for concrete members will be studied and compared for concrete members like simple concrete, reinforced concrete, pre-stressed concrete and different loading conditions like torsion, combination of torsion and bending and general condition of combination and bending and shear. In the torsion of reinforced concrete member, the basic problem is finding the thickness of shear flow zone or (t_d) which is similar to the depth of compressive zone or (c) in bending condition. Rausch classic theory calculates the torsional strength of reinforced concrete members as unrealistically higher than real value. Error in this theory is caused by error in determining the position of shear flow zone which directly depends upon the thickness of this zone i.e., (t_d). In this paper, a simple method is presented to calculate shear flow zone (t_d). This method is based on the truss model theory and balance equations and compatibility conditions. Nonlinear analysis of reinforced concrete members under torsion is conducted by using truss model theory. Based on this assumption, 16 basic equations have been obtained for non-prestressed concrete members based on balance equations, compatibility conditions and aggregates' stress-strain dependencies (steel and concrete). Using these 16 basic equations, a method is presented to draw T - θ or τ - γ diagram for reinforced concrete members under torsion after cracking.

In following, 23 rectangular cross-section of the reinforced concrete with different thicknesses were analyzed by help pf nlarctm program. This program is written based on the nonlinear analysis of reinforced concrete members which is presented in the appendix. By referring to the results obtained from nonlinear analysis of 23 rectangular cross-sections, first the thickness of shear flow zone and its changes based on the total percent of longitudinal and transversal steel reinforcements will be discussed. This study leads to equations that by help of them, we can calculate the thickness value of shear zone with good accuracy. In addition, after comparing the parameter for shear flow zone thickness by using Whitney equivalent stress block height in bending, a relation will be presented in order to calculate this parameter which has high accuracy and significantly simplifies the calculation of shear flow zone thickness parameter. Then, cracks 'angle, torsional strength and cross-section torsional stiffness will be evaluated based on the changes in different quantities.

2. Torsion Theories

2.1 inverse and Semi-Inverse Methods

Often, elasticity general equations with border conditions cannot be solved by direct method. In order to solve these problems, inverse and semi-inverse methods are useful and efficient methods. In inverse method, every function which held in relevant differential equations, will be considered as a solution for it. In an indirect method which was first introduced[1], simplifying assumptions about the stress components or displacements were considered which converts high-order differential equations to equations that their mathematical solution is not complex as previous equations. In prismatic members torsion, it is assumed that displacement functions u,v,w have determined form. By imposing this assumption, governing equations convert to a differential equation. Consider prismatic bar with longitude L which is pinned in one end in plate and id under torsional moment around axis z (Figure 1).

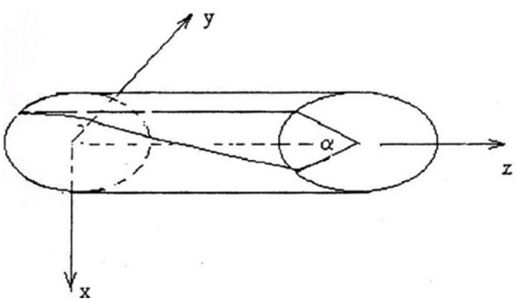

Figure 1. Prismatic bar under torsion[1].

Rotation of member in the pinned end is prevented. The amount of rotation in each cross-section depends on the distance of cross-section to its pinned end. Because deformation are slight, it is reasonable that torsion angle α in each cross-section is corresponding to the distance from cross-section to pinned end. In other words:

$$\alpha = \theta z$$

where θ is torsion angle to length unit of member. It is assumed that torsion angle α is small. Consider a cross-section with distance z from pinned end of member. After deformation, P' with coordinates (x, y, z) will displace to P'_1 with coordinate (x+u, y+v, z+w). Figure 2 shows this condition clearly[1].

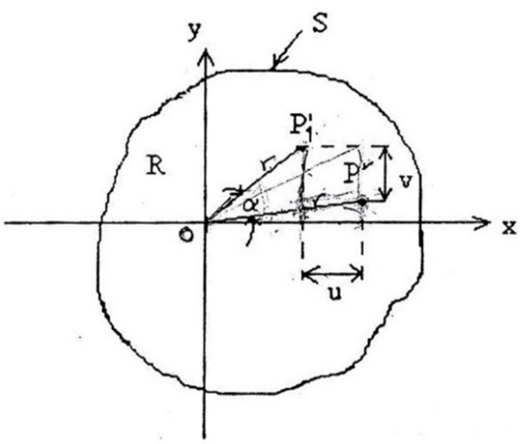

Figure 2. Displacement caused by torsion[1].

By imposing small displacements (α~0), we have:

$$\alpha = \theta z \tag{1}$$

Where in Equation (2), is torsion function. In following, by assuming that prismatic member is under centralized torsion without any external loading, stress equations can be expressed as Equation (3):

$$\begin{cases} u = -\theta yz \\ v = -\theta xz \\ w = \theta\varphi(x, y) \end{cases} \tag{2}$$

By imposing balance equations and ignoring body volume forces, torsion function should hold in the differential Equation (4). In fact, torsion function should be harmonic or compatible.

$$\begin{cases} \tau_{zy} = G\theta(\partial\varphi/\partial y + x) \\ \tau_{zx} = G\theta(\partial\varphi/\partial x - y) \\ \sigma_x = \sigma_y = \sigma_z = \tau_{xz} = 0 \end{cases} \tag{3}$$

By considering infinitely small element on the outer surface of the beam and regarding that stresses are zero on outer surfaces and considering beam stress tensor and cosines of angles for S-shape cross-section from Figure 2 with coordinate axis, we will have:

$$\nabla^2\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0 \tag{4}$$

From balance equations and Equation (3), we can conclude that:

$$\tau_{zx}\cos S_x + \tau_{zy}\cos S_y = 0 \tag{5}$$

$$\tau_{zx}\frac{dy}{ds} - \tau_{zy}\frac{dx}{ds} = 0$$

By imposing border condition of Equation (5) and shear stress Equations (6), we will have. $\frac{\partial\varphi}{\partial s} = 0$

$$\begin{cases} \tau_{zx}\dfrac{\partial\varphi}{\partial y} = G\theta(\partial\varphi/\partial x - y) \\ \tau_{zy} = -\dfrac{\partial\varphi}{\partial x} = G\theta(\partial\varphi/\partial y + x) \end{cases} \tag{6}$$

$$\rightarrow \nabla^2\varphi = -2G\theta$$

This equation shows that Prandtl stress function (torsion) should have fixed value along with surface border. In following, we can show that torsional anchor M_t that its half is caused by shear stress and the other half by is equal with:

$$M_t = 2\iint \varphi\, dx\, dy \tag{7}$$

2.2 Membrane Analogy Analysis

Membrane analogy analysis method was first used by L. Prandtl in solving torsion problems[1]. Consider a homogenous film which has support on its edges and its circumference is similar to the cross-section of twisted member. In addition, assumed film is placed under monotonic tension in its edges and monotonic lateral pressure is imposed to it (Figure 3).

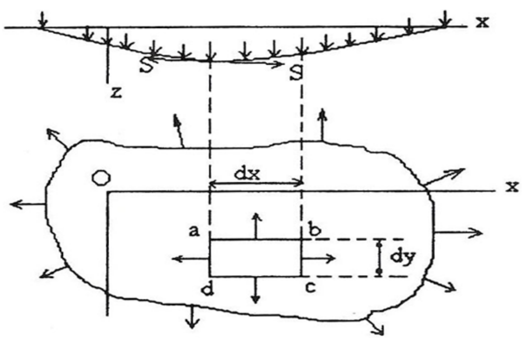

Figure 3. Homogenous film[2].

If the pressure on the surface of membrane is q and tension on circumference is s, result of tension forces imposed to ad and dc sides from small component *abcd* by assuming minor deformations is:

$$-s\frac{\partial^2 z}{\partial x^2}(dx\,dy) \tag{8}$$

Similarly, the result of tension forces imposed on two other sides of element is:

$$-s\frac{\partial^2 z}{\partial y^2}(dx\,dy) \tag{9}$$

By imposing forces' balance equation for element *abcd* from above membrane, we have:

$$\nabla^2 z=\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=-\frac{q}{s} \tag{10}$$

By comparing two Equations (6) and (10) and considering the border condition for film deformation (Z=0), we can find the similarity between these two relations. It is enough that in Equation (6), we use q/s instead of and by determining Prandtl stress function, we express peak of film as function of x and y. For more description, we considered a rectangular membrane sample with dimensions 2a*2b using membrane analogy analysis and determined displacement, torsion anchor, shear tensions and maximum shear tension functions.

Regarding that Z deformation function has symmetry in relation with axis of membrane, border conditions x = ± a and necessitates that Z = 0; therefore, deformation function is expressed as unlimited series:

$$Z=\sum_n^\infty=1,3,5\,b_n Y_n \cos\frac{n\pi x}{2a} \tag{11}$$

In this Equation, b_n is fixed coefficient and Y_n are functions of y. Now, we will express the right side of Equation (10) as Fourier series in interval –a<x<a, and we have:

$$-\frac{q}{s}=-\sum_n^\infty=1,3,5\frac{q}{s}\frac{4}{n\pi}(-1)\frac{n-1}{2}\cos\frac{n\pi x}{2a} \tag{12}$$

By substituting Equations (11) and (12) in differential Equation (10) and solving it and by imposing border conditions and substituting 2 G θ for q/s, Prandtl stress function is:

$$\frac{32G\theta}{\pi^3}\sum_n^\infty\frac{1}{n^3}(-1)\frac{n-1}{2}[1-\frac{\cosh\frac{n\pi y}{2a}}{\cosh\frac{n\pi b}{2a}}\cos\frac{n\pi x}{2a} \tag{13}$$

By assuming b>a, we can conclude that the maximum shear stress occurs in the middle of larger side; therefore, maximum shear stress in points is equal with:

$$\tau_{\max}=2G\theta a-\frac{16G\theta a}{\pi^2}\sum_n^\infty=1,35\frac{1}{n^2\cosh\frac{n\pi b}{2a}} \tag{14}$$

Equation (4) gives maximum shear stress value for different b/a ratios. Table 1 demonstrates the maximum shear stress for different $\frac{b}{a}$ ratios. Based on this, maximum shear stress is considered as a coefficient of maximum shear stress for $\frac{b}{a}\rightarrow\infty$ and we have:

$$\tau_{\max}=K(2G\theta a) \tag{15}$$

Now, by having Prandtl stress function of Equation (13) and placing it in Equation (7), torsional anchor is:

$$\frac{16G\theta ba^3}{b}(1-\frac{192a^2}{b}\sum_n^\infty 1,3\frac{1}{n^5\tanh}\frac{n\pi b}{2a}$$

Table 1. Maximum shear stress for different b/a ratios

b/a	*K*
1	0.675
2	0.930
5	0.999
10	1
∞	1

Equation (16) indicates the value of torsional anchor to different b/a ratios. Table 2 demonstrates torsional anchor for different $\frac{b}{a}$ ratios. Based on this, we will consider torsional anchor value as a coefficient of torsional anchor for $\frac{b}{a} \to \infty$ and we have:

$$M_t = K_1(16G\theta ba^3) \tag{16}$$

Table 2. Torsional anchor for different b/a ratios

b/a	K_1
1	0.140
2	0.229
5	0.291
10	0.312
∞	0.333

2.3 Plastic Theory and Sand Pile Analogy Method

By assuming an elastic-plastic curve for stress-strain curve, we can obtain the stress values in those parts of cross-section which have reached to yield limit by help of plasticity theory. If in the torsion of prismatic members, shear stress reaches to yield point, shear stress components i.e., τ_{yz}, τ_{xz} should held in following formula:

$$\tau_{zx}^2 + \tau_{zy}^2 = \tau_{yeild}^2 \tag{17}$$

On the other hand, regarding stress balance equations and plastic function F(x,y), we have:

$$\begin{cases} \dfrac{\partial \tau_{zx}}{\partial x} + \dfrac{\partial \tau_{zy}}{\partial y} = 0 \\ \tau_{zx} = \dfrac{\partial F}{\partial y} \\ \tau_{zy} = \dfrac{\partial F}{\partial x} \end{cases} \tag{18}$$

By substituting Eequation (19) in yield surface equation, we have:

$$\left|\nabla F\right| = \tau_{yeild} \tag{19}$$

Since none of the stress components are vertical on the surface, we have: τ_{yz}, τ_{xz}

By using above Equation and Equation (19), we can show that dif (F) = 0;

Therefore, plastic stress function has fixed value along with the mass border. This value if usually considered zero because its value has no effect on the values of stresses. Since the slope of assumed stress function is fixed, the volume surrounded by F curve is similar to sand pile which is accumulated on the vertical plate with similar surface to the cross-section of membrane under torsion and for this reason, it is called sand pile analogy method. If all cross-section reaches to plastic state, we can reach following conclusions using sand pile analogy method:

- Shear stress is a fixed value throughout the cross-section.
- Total torsional anchor imposed to the membrane is corresponding with sand pile volume.
- Stress function value is independent of torsion angle.
- A tall and thin member has lower resistance compared to a member with square cross-section with same cross-section area.
- Plastic torsional strength of all tall cross-section with C, L, T, U shape is approximately same with plastic torsional strength of rectangular cross-section with same thickness and a length equal with total length of previous cross-section.

By using sand pile analogy method, we can obtain the plastic torsion value for members with rectangular cross-section as below:

$$T_u = \frac{1}{2}b^2\left((d - \frac{1}{3}b)\tau_{\max}\right) \tag{20}$$

Where,

T_u: Plastic torsional anchor.

τ_{max}: Maximum shear stress of aggregates.

d: Depth of cross-section.

b: Width of cross-section.

If only some parts of a cross-section reaches to plastic limit and other parts remain in elastic state, stress function and torsional anchor value can be obtained such that first we draw the sand pile surface, then elastic membrane extended inside it. Membrane contact point with sand pile cross-section shows the border between elastic and plastic states.

3. Torsion Theories of Concrete Members

Torsion theories of concrete members can be divided into three general classes based on the type of loading:

- Torsion.
- Combination of torsion and bending.
- Combination of torsion and bending and shear.

In Table 3, different theories have been classified[2]. Approximately, all theories less considered the stiffness factor which can be used in analyzing and calculating the

Table 3. Torsion theories until 1964[3].

Type of loading / Type of aggregate	Torsion	Torsion & bending	Torsion & bending & shear
Simple concrete	1-Elastic theory 2-plastic theory 3-Miyamoto (1927) 4-Turner and Davies (1934) 5-Marshal and Tembe (1941)	1-Fisher (1950) 2-Cowan (1953)	
Reinforced concrete	1-Rausch (1929) 2-Turner and Davies (1934) 3-Marshal and Tembe (1941) 4-Cowan (1950) 5-Ernst (1957)	1-Nylander (1945) 2-Cowan (1953) 3-Lessing (1959) 4-Yudin (1962) 5-Gesund (1964)	1-Nylander (1945) 2-Lessing (1959) 3-Yudin (1962)
Prestressed concrete	1-Nylander (1945) 2-Cowan (1953)	1-Cowan (1953) 2-Gardner (1960) 3-Swamy (1962) 4-Reeves (1962)	

membrane strength in initial cracking and break out. The only exception in this case is classic elastic theory that can calculate the stiffness of concrete membrane before cracking with reasonable confidence. None of these methods used for estimating and calculating the stiffness of a concrete membrane after cracking.

4. Torsional Stiffness of Reinforced Concrete after Cracking

In stable undetermined reinforced concrete structures, torsional anchor can be measured when torsional stiffness of reinforced concrete members is known. In this section, by using new concept shear module post-cracking, torsional stiffness post-cracking theory is expressed. Using this new concept leads to a general theory which is useable for each desired cross-section and simplifies the calculation of torsional stiffness after cracking. In ACI-1971 regulation, new criteria have been presented for reinforced concrete members in torsion. If torsional anchors were known, we can easily use these criteria. In a stable determined structure like balcony cantilever slab, torsional anchor can be obtained from static balance equations and regulations' criteria can be used without any problem. But calculation of torsional anchor in a stable undetermined structure like beam, needs compatibility conditions, in addition to balance equations. In other words, beam torsional stiffness should be considered.

Torsional stiffness of reinforced concrete cross-section before cracking has been calculated with reasonable accuracy by Saint-Venant using elastic theory[1]. But instead of research like[3,4] about torsion or a combination of torsion, bending and shear, our knowledge of torsional stiffness after cracking is not high and need to this knowledge highlights with this reality that torsional members often crack under service loads and design regulations for concrete structures remain silent about torsional stiffness of cracked members. Besides, torsional stiffness after cracking is only a small fraction of its value before cracking (%10). In a stable undetermined structure, this effective reduction in torsional stiffness after cracking will significantly influence the distribution of anchors.

By conducting experiments in Portland Cement Association (PCA), has presented empirical equations for calculating the torsional stiffness after cracking. Results of these experiments indicates that torsional stiffness after cracking is a function of torsional reinforcements' percent[5,6].

Scientific calculation of torsional stiffness after cracking was first done by Robinson. His suggested equations were reasonable, although these equations were only applicable for circular cross-sections and torsional stiffness should be calculated with trial and error method. For rectangular cross-sections which have higher use in practice, torsional stiffness after cracking was obtained. He did it by obtaining compatibility equation for a thin-wall rectangular cross-section. His report explicitly shows that torsional stiffness is a function of percent of steels used in the cross-section.

In this section, by using new concept shear module after cracking, post-cracking torsional stiffness theory is presented. Using this new concept leads to a general theory which can be used for each desired cross-section including rectangular and circular cross-sections. Using this can significantly simplify the calculation of torsional stiffness after cracking.

4.1 Torsional Stiffness after Cracking

Experiments show that concrete middle zone has no effect on torsional behavior o cross-section after cracking[3]. In other words, resistance and torsional stiffness of a filled cross-section is similar to resistance and torsional stiffness of a hollow cross-section. Based on these observations, we can obtain stiffness equation after cracking for hollow cross-section and then, we can use it for filled cross-sections. Consider prismatic thin-wall member with desired cross-section under torsional anchor T (Figure 4).

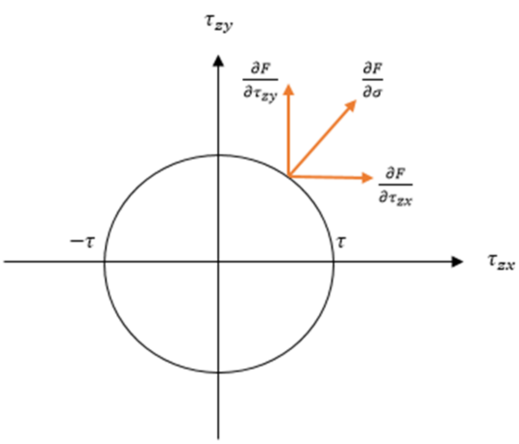

Figure 4. Mass yield surface under torsion.

Cross-section wall is shown by dashed line. By imposing this torsional anchor in the wall of member, shear stress τ develops that its value according to thin-wall cross-section theory is:

$$\tau = \frac{T}{2Ah} \tag{21}$$

Where,

A: Area of enclosed part by center line of member wall.

h: Thickness of wall.

Because of this shear stress, other stresses develop in the steel and concrete. According to Rausch theory[4], in order to calculate these stresses, given cross-section is considered as a spatial truss (Figure 5). In this method, which is known as truss method, three following assumptions are considered:

- Reinforced concrete acts as spatial truss after cracking that its diagonal members have angle 45^0 and made from concrete; longitudinal and medial members form reinforcements which connect to each other by joint connection in nodes.
- Metal concrete members tolerate only pressure (there is no shear strength for them).
- Longitudinal and lateral reinforcements tolerate only axial tension force.

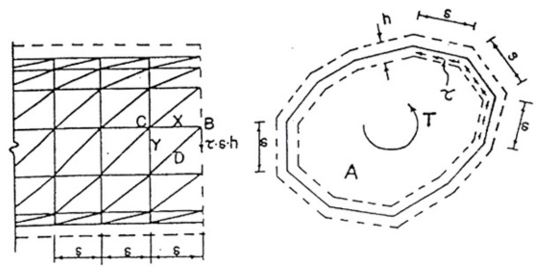

Figure 5. Thin-wall cross-section and spatial truss model[5].

Regarding above assumptions and using balance equations in nodes, Rausch showed that in all diagonal members, forces are equal with D. In a similar way, all exiting forces in all tights are Y and in all longitudinal reinforcements are X. Balance equation for node B along with tight gives:

$$D = \sqrt{2}\tau sh$$

where s is the distance between tight and longitudinal reinforcement. Therefore, stress in the concrete member (diagonal) is:

$$f_c = \frac{D}{\frac{sh}{\sqrt{2}}} = 2\tau \tag{22}$$

Balance equation for node B in length direction gives:

$$X = \frac{D}{\sqrt{2}} = \tau sh \tag{23}$$

Balance equation node C along with radial direction necessitates that inward and outward forces along with diameters leading to node were equal; therefore:

$$Y = \frac{D}{\sqrt{2}} = \tau sh \tag{24}$$

By assuming linear stress-strain relation between steel and concrete, we can obtain values of strain in the concrete members and reinforcements from Equations (24), (25) and (26). These strains create shear deformation in member's wall. This shear deformation can be obtained from compatibility of deformations. Figure 6 shows shear deformation caused by 1. Concrete members, 2. Longitudinal reinforcements and 3. Annular reinforcements.

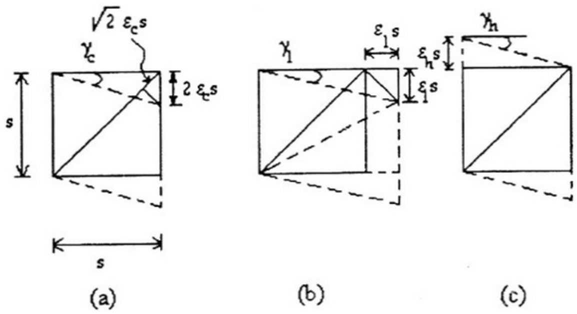

Figure 6. TShear deformation of reinforced concrete member[5].

Therefore, total shear deformation is:

$$\gamma = 2\gamma_c + \gamma_h + \gamma_l = 2\varepsilon_h + \varepsilon_l \tag{25}$$

By substituting available strains, the relationship between stress and shear strain or shear module is:

$$G_{cr} = \frac{\tau}{\gamma} = \frac{E_s}{(4n + \frac{sh}{A_l} + \frac{sh}{A_h})} \tag{26}$$

where A_l is cross-section area of a longitudinal reinforcement and A_h is cross-section in steel tight and n is steel to concrete module ratio. This new concept of shear module after cracking is a very suitable concept and describes the general property of each torsional member after cracking. If torsional coefficient of a cross-section shows torsional coefficient of a thin wall cross-section with same thickness, C_{cr}, we can calculate it according to thin-wall cross-section theory as following:

$$C_{cr} = \frac{4A^2 h}{p} \tag{27}$$

In Equation (29), p is the perimeter of a ring which passes the tight central line. We know that torsional stiffness is equal with shear module times to torsional coefficient. Critical torsional stiffness after cracking the concrete and before full rupture is:

$$K_{cr} = \frac{4A^2 E_s}{(\frac{4nh}{p} + \frac{sp}{A_l} + \frac{sp}{A_h})} \tag{28}$$

4.2 Effect of Wall Thickness on the Torsional Stiffness after Cracking

Applying Equation (30) for filled cross-sections raises this question that what is the effect of wall thickness on this equation? In fact, for a hollow cross-section, we cannot use real thickness of wall because only some part of it is effective. Achieving a theoretical method in order to determine effective thickness of wall is difficult. Robinson tried to achieve this but his attempts made the theory more sophisticated and difficult. His suggested solution led to trial and error method. Besides, validity of his assumptions are doubtful. On the other hand, by ignoring the first expression in the denominator of Equation (30) which includes thickness of wall, we can simply avoid the determining of effective thickness of wall. Because effective thickness of wall influences only on one of the expressions in the denominator of equation related to the torsional stiffness, it seems that the study of effects of wall thickness is not desired. On the other hand, avoidance of determining the effective wall thickness by ignoring related expression overestimates the torsional stiffness. As a result, in following, effective wall thickness is calculated empirically by comparing theoretical equations and test results. Because test results are available for rectangular cross-sections, these cross-section are used to calculate effective wall thickness. Laboratory observations indicate that effective wall thickness is a function of smaller dimension of rectangular cross-section (x) and volumetric ratio of steels ($\rho_l + \rho_h$).

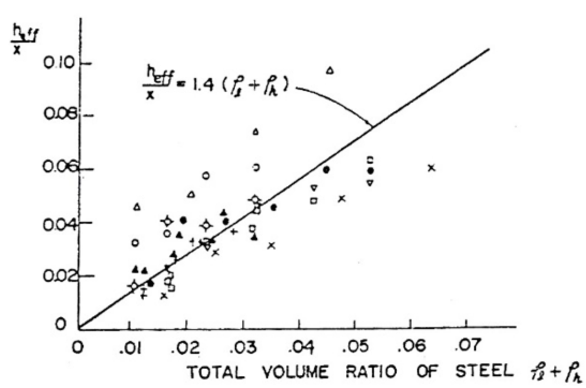

Figure 7. Effective wall thickness based on total steel[5].

By drawing $\frac{h_{eff}}{x}$ diagram based on the $(\rho_l + \rho_h)$ in Figure 7, we can express effective wall thickness by following equation:

$$h_{eff} = 1.4(p_l + p_h)x \tag{29}$$

4.3 Changes of Torsional Anchor based on Torsion Angle after Cracking

Torsional anchor-angle of twist curve after cracking is a straight line in the beginning and then, by downward concavity, it approaches the break-out point. Torsional stiffness after cracking ($G_{cr}C_{cr}$) is equal with the slope of linear area of curve after cracking. In Figure 8, an instance of torsional anchor-angle of twist curve is shown.

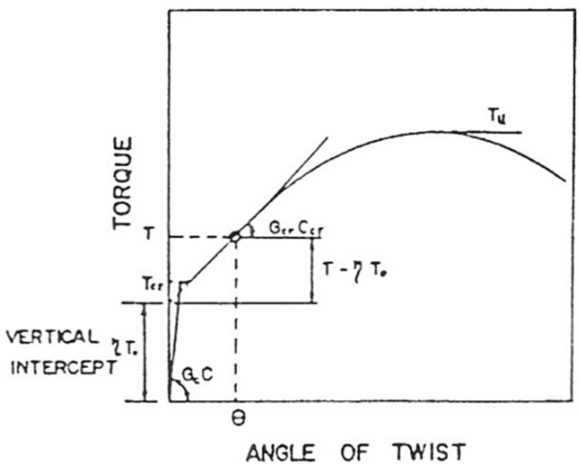

Figure 8. Anchor changes based on the angle of twist.

Along with linear zone after cracking, we can observe that this line has not passed the origin of coordinates but it cuts the vertical axis in a point with ηT_0 width that T_0 is the share of concrete from torsional strength that after cracking, it reduces from the torsional strength of cross-section.

$$T_0 = \frac{2.4}{\sqrt{4}} x^2 y \sqrt{f_c^1} \tag{30}$$

and is an empirical coefficient. Therefore, the relationship between torsional anchor and angle of twist for linear zone after cracking is:

$$T = \eta T_0 + G_{cr} C_{cr} \theta \tag{31}$$

Experiments done by Hsu in Portland Cement Association showed that ηT_0 is a function of real thickness of wall which is shown in Figure 9 and Figure 10.

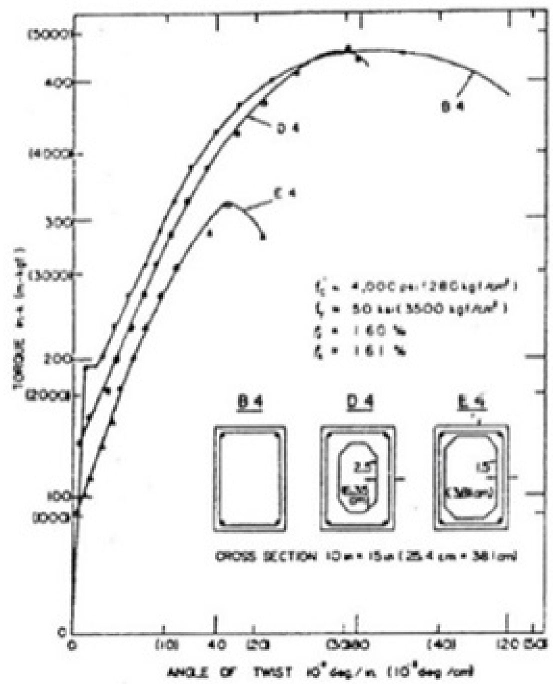

Figure 9. Anchor changes based on the angle of twist for three cross-sections with different wall thickness[15].

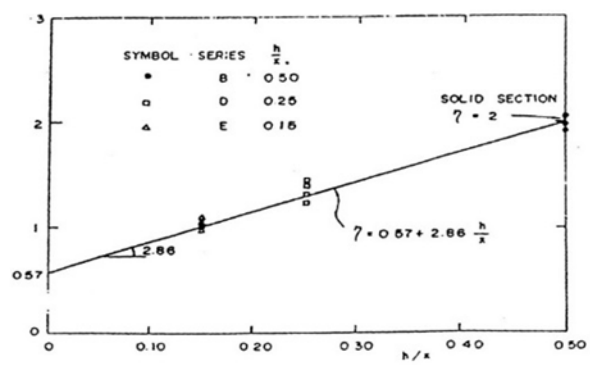

Figure 10. Changes of τ_1 based on the wall thickness[5].

By increasing the thickness of wall, η increases. By drawing η as a function of $\frac{h}{x}$, we can obtain their relationship between these two as following:

$$\eta = 0.57 + 2.86\frac{h}{x} \tag{32}$$

5. Nonlinear Analysis of Torsion in Reinforced Concrete Members

Nonlinear analysis of reinforced concrete members under torsion is conducted by using truss model theory[7–9].

Based on this theory, for non-stresses concrete members, 16 basic equations were obtained based on the balance equations, compatibility conditions and stress-strain dependencies of aggregates (steel and concrete). In this section, by referring to these 16 basic equations, we present a method for drawing T - θ diagram or τ - γ for reinforced concrete members under torsion after cracking. Consider a reinforced concrete member with rectangular cross-section under torsional anchor T (Figure11(a)).

Shear flow with q size flows on the external ring with thickness t_d which is called shear flow zone. In Figure 11(b), an element is shown along with d-r coordinate axis. Compressive stress σ_d along with axis d and tensile stress σ_r is imposed along with r axis. The angle between d axis and r axis is shown by α. In Figure 11(c) a square element is shown along with l and t axis that shear stress τ_{lt} equal with $\frac{q}{t_d}$ influences this element. Finally, an element of member based on the truss model concept located in the shear flow zone, is shown in Figure 11(d). In reference[10,11], balance equations, compatibility conditions and stress-strain equations are discussed for this member. By referring truss model balance condition, we can write balance equations as following:

$$\begin{cases} \sigma_l = \sigma_d \cos^2 \alpha + \sigma_r \sin^2 \alpha + p_l f_l \\ \sigma_t = \sigma_d \sin^2 \alpha + \sigma_r \cos^2 \alpha + p_t f_t \\ \dfrac{T}{2A_0 t_d} = (-\sigma_d + \sigma_r) \sin\alpha \cos\alpha \end{cases} \tag{33}$$

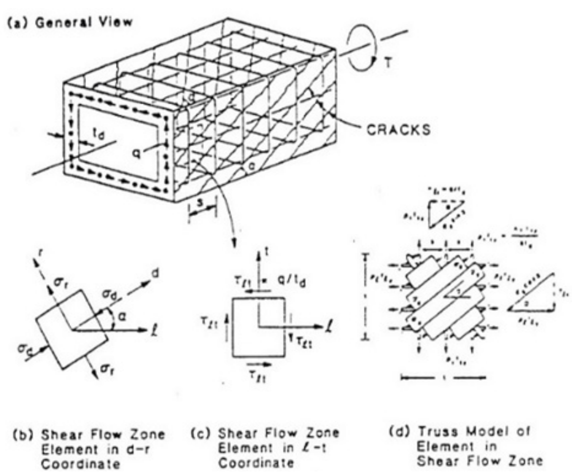

Figure 11. (**a**) Reinforced concrete torsion member.(**b**) Main element of member under major stresses.(**c**) Member element in coordinate system. (**d**) Truss model for element in shear flow zone[7].

In these Equations, and, is steel ratios along with axis l and t which are and, respectively. A_0 is enclosed surface by central line of shear flow zone which is a function of thickness of this zone. is central line perimeter of shear flow zone.

Using compatibility condition, seven compatibility equations are obtained as following:

$$\begin{cases} \varepsilon_l = \varepsilon_d \cos^2 \alpha + \varepsilon_r \sin^2 a \\ \varepsilon_t = \varepsilon_d \sin^2 \alpha + \varepsilon_r \cos^2 a \\ \dfrac{A_0\theta}{p_o} = (-\varepsilon_d + \varepsilon_r) \sin^2 a \cos\alpha \end{cases} \tag{34}$$

$$\varepsilon_{ds} = -t_d \theta \sin 2a \tag{35}$$

where is compressive strain of concrete in concrete diagonal levels.

5.1 Stress-Strain Relation of Concrete in Tension and Compression Modes

- In tension mode: Regarding small value of , we can ignore concrete tensile strength in torsion. In other words, σ_r is zero.
- In compression mode: Stress-strain equation of concrete along with axis d can be written as following:

$$\sigma_d = k_l \eta f_c^1 \tag{38}$$

In which,

f_c': Compressive strength of cylindrical concrete sample.

k_l: Ratio of average stress to maximum stress in compressive tension concrete block.

η: Softness coefficient equal with.

$$\eta = \frac{1}{\sqrt{0.7 - \frac{\varepsilon_r}{\varepsilon_d}}} \tag{39}$$

5.2 Simplifying Equations for Achieving Solutions

By looking obtained equations, we find that we have a system with 16 equations and 19 unknowns. By knowing three unknowns, we can solve 16 equations. For a concrete member under torsion, vertical tensions σ_t , σ_l which are imposed to the element' faces in shear flow zone, are zero. Because compressive strain changes from zero to maximum value, by choosing different ε_d values in the allowed range of system, we can solve equation system for 16 remained unknowns. Therefore, by assuming that ε_d is known, we first calculate t_d. Finally, by using cumbersome calculations for chosen ε_d and t_d, we calculate torsional anchor, shear stress and torsion angle values on length unit. By selecting a new value for ε_d, all steps were implemented and or curve can be drawn.

6. Results of Nonlinear Analysis of Concrete Torsion Members using nlarctm Program

In this section, 23 concrete rectangular cross-sections with different characteristics are analyzed with nlarctm program. This program is written based on the method discussed in previous sections and is introduced in the appendix. In order to prevent the volume, we ignored printing all output files and only drawn related diagrams and analysis of outputs.

In Table 4, characteristics of all cross-sections are shown. Quantities in this table are:

b : Width of cross-section (cm).
h : Height of cross-section.
f'_c : Compressive strength of cylindrical concrete instance (kg/cm^2).
f_{ly}: Stress for longitudinal reinforcements (kg/cm^2).
f_{ty}: Stress of closed collars (kg/cm^2).
s: Distance between closed collars (cm).
n: Number of longitudinal reinforcements.
:Φ_l : Diameter of longitudinal reinforcements (cm).
:Φ_t : Diameter of closed reinforcement.

6.1 Changes of Torsional Anchor based on Angle of Twist on Length Unit ()

Using "nlarctm" program, all above cross-sections are analyzed for torsion loading state. It is necessary to mention that analysis trend presented by this program is nonlinear analysis of cross-section after cracking. Only related diagrams are presented and the study of torsional anchor changes based on the angle of twist on length unit will be discussed later.

Table 4. Characteristics of all cross-sections.

Number of cross-section	*b*	*h*	f'_c	f_{ly}	f_{ty}	*S*	*n*	Φ_l	Φ_t
1	30	45	200	2400	2400	10	4	2	6/0
2	30	45	200	2400	2400	10	4	2	8/0
3	30	45	200	2400	2400	10	4	2	1
4	30	45	200	2400	2400	10	4	2	2/1
5	30	45	200	2400	2400	10	4	2	4/1
6	30	45	200	2400	2400	10	4	6/1	1
7	30	45	200	2400	2400	10	4	2	1
8	30	45	200	2400	2400	10	4	4/2	1
9	30	45	200	2400	2400	10	4	8/2	1
10	30	45	200	2400	2400	10	4	2/3	1
11	30	45	150	2400	2400	10	4	2	2/1
12	30	45	200	2400	2400	10	4	2	2/1
13	30	45	250	2400	2400	10	4	2	2/1
14	30	45	300	2400	2400	10	4	2	2/1
15	30	45	350	2400	2400	10	4	2	2/1
16	30	45	300	2400	2400	10	4	2	2/1
17	30	45	300	3600	3600	10	4	2	2/1
18	30	45	300	4500	4500	10	4	2	2/1
19	30	45	300	2400	2400	6	4	2	2/1
20	30	45	300	2400	2400	8	4	2	2/1
21	30	45	300	2400	2400	10	4	2	2/1
22	30	45	300	2400	2400	12	4	2	2/1
23	30	45	300	2400	2400	14	4	2	2/1

6.2 Evaluating the Result of Nonlinear Analysis of Reinforced Concrete under Torsion

In this section, first we will discuss about the shear flow zone and its changes based on the total percent of longitudinal and transverse reinforcements[12,13]. Then, we will study the angle of cracks and finally, changes of torsional strength and torsional stiffness of cross-sections will be evaluated based on the changes in the different quantities. All figures were not drawn in Section (6.1) due to their large volume and only 3 first cross-sections and three last cross-section were drawn (Figure 12 to 19).

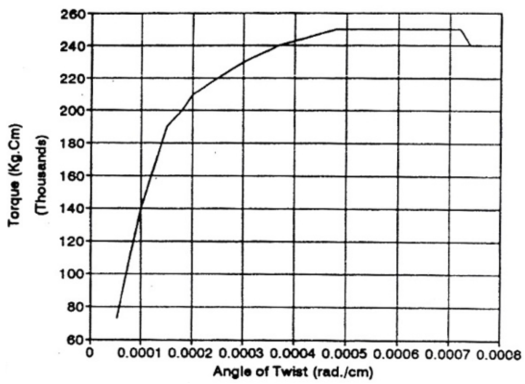

Figure 12. Cross-section 1.

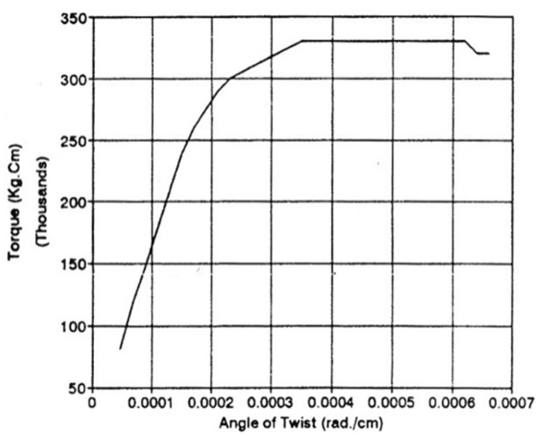

Figure 13. Cross-section 2.

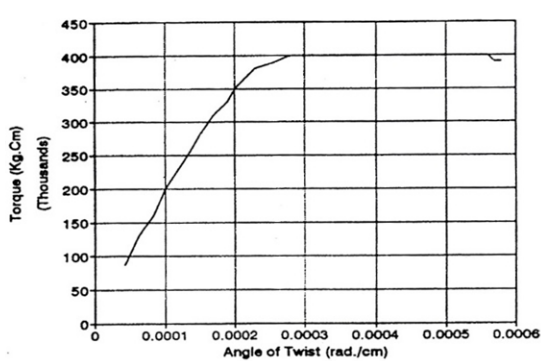

Figure 14. Cross-section 3

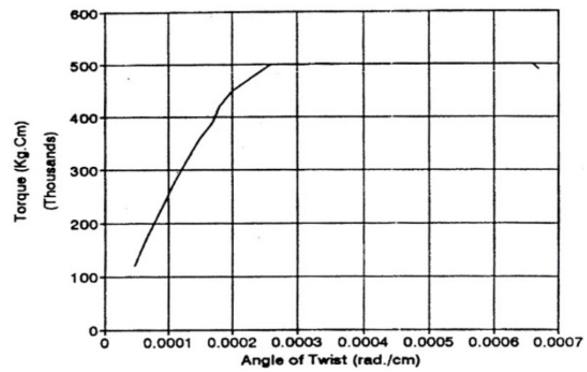

Figure 15. Cross-section 21.

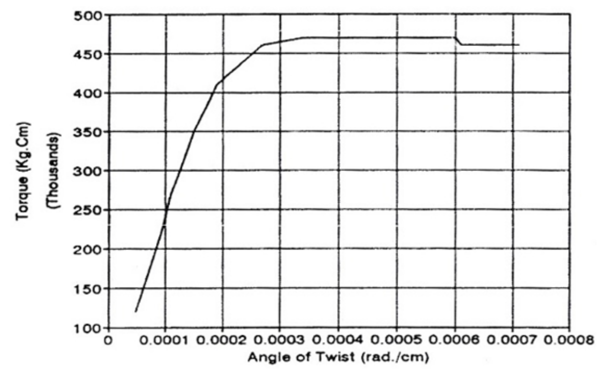

Figure 16. Cross-section 22.

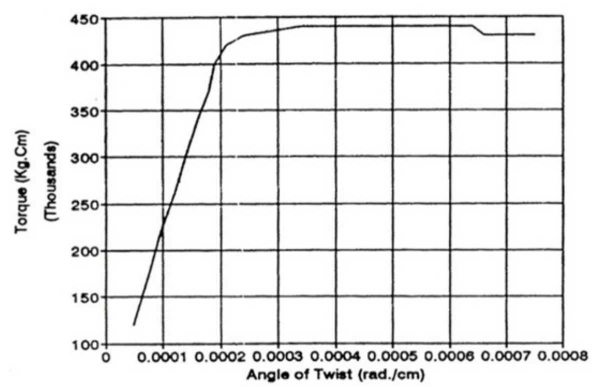

Figure 17. Cross-section 23.

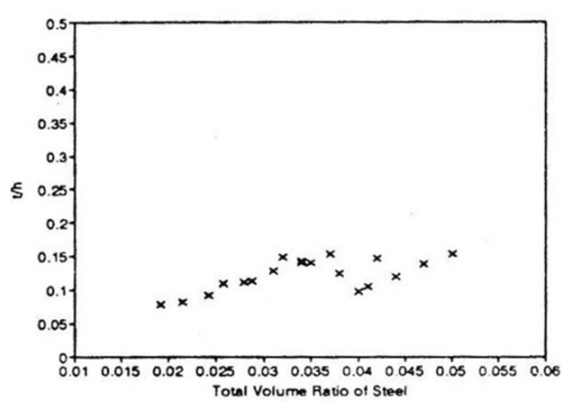

Figure 18. Diagram for changes based on $\rho 1 + \rho_t$.

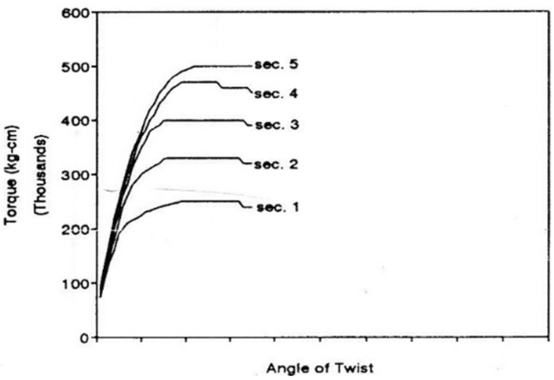

Figure 19. Diagram for changes based on $\rho_1 + \rho_t$.

6.2.1 Thickness of Shear Flow Zone

- The study of thickness changes of shear flow zone based on the total percent of steels:
  If changes of $\frac{t_d}{b}, \frac{t_d}{h}$ based on $(\rho_1 + \rho_t) = \rho$ were linear,

by using least square method, the equation of best lines is as follows:

$$\frac{t_d}{h} = \begin{cases} 11.27\rho - 0.19 & \rho < \frac{3}{100} \\ 4.07\rho & \frac{3}{100} \leq \rho \frac{4}{100} \\ 5.04\rho - 0.1 & \rho \geq \frac{4}{100} \end{cases} \quad (40)$$

$$\frac{t_d}{b} = \begin{cases} 2.09\rho + 0.11 & \rho < \frac{3}{100} \\ 6.12\rho & \frac{3}{100} \leq \rho \frac{4}{100} \\ 8.44\rho - 0.19 & \rho \geq \frac{4}{100} \end{cases} \quad (41)$$

In Table 5, comparison of Equations (40) and (41) with t_d values in[14,15] has been done. As seen, ratio of t_d in[14,15] to value obtained by Equations (40) and (41) for

Table 5. Comparing the thickness of shear flow thickness obtained by "nlarctm", equation (40) and Equation (41)

Number of cross-section	nlarctm $t_d^{(nla)}$	Equation (40) $t_d^{(1)}$	Equation (41) $t_d^{(2)}$	$\frac{t_d^{(nla)}}{t_d^{(1)}}$	$\frac{t_d^{(nla)}}{t_d^{(2)}}$
1	9/4	5/4	9/4	08/1	00/1
2	1/5	0/6	1/5	85/0	00/1
3	8/5	7/5	7/5	01/1	01/1
4	4/6	2/6	3/6	03/1	01/1
5	9/6	8/6	8/6	01/1	01/1
6	0/5	1/5	1/5	98/0	98/0
7	8/5	7/5	7/5	01/1	01/1
8	3/6	4/6	4/6	98/0	98/0
9	6/6	1/5	9/4	29/1	34/1
10	9/6	8/6	9/6	01/1	00/1
11	7/7	2/6	1/5	24/1	51/1
12	4/6	2/6	3/6	01/1	01/1
13	6/5	9/6	9/6	81/0	81/0
14	1/5	8/4	7/4	06/1	08/1
15	4/4	1/6	2/6	72/0	71/0
16	1/5	8/4	7/4	06/1	08/1
17	3/6	2/6	3/6	01/1	00/1
18	7/6	9/5	9/5	13/1	13/1
19	2/6	1/6	2/6	01/1	00/1
20	4/5	5/5	4/5	98/0	00/1
21	1/5	8/4	7/4	06/1	08/1
22	7/4	8/4	7/4	98/0	00/1
23	4/4	5/4	4/4	98/0	00/1
Total mean & standard deviation of sample					
(Arithmetical mean)				1/008	1/003
(Sample standard deviation)				0/131	0/167

23 cross-sections, has standard deviation 0.131 and 0.167, respectively. Since in a rectangular cross-section, the highest shear stress caused by torsion occurs in the middle of larger side of cross-section $\left(\frac{h}{2}\right)$ not in the middle of smaller side $\left(\frac{b}{2}\right)$, it is suggested that Equations (40) were used in order to guess the initial value of shear flow zone thickness for certain cross-section.

- Comparing the thickness of shear flow zone with the height of stress block

Whitney equivalence in bending

About the bending of reinforced concrete members of bending, the height of Whiney equivalent stress block is:

$$a=\frac{A_s f_y}{0.85 f_c^1 b} \quad (42)$$

In this Equation, A_s is the area of longitudinal cross-section located in the tensile zone. Regarding above equation, it is clearly seen that quantity of Whitney stress block height is corresponding with diameter and yield stress of longitudinal reinforcement and has inverse relationship with concrete compressive strength and width of cross-section. By referring to Equations (40) and (41), we can find that changes in the thickness of shear flow zone is similar to the changes of Whitney stress block height; in other words, thickness of shear flow zone corresponds with diameter and yield stress of the longitudinal reinforcements and has inverse relationship with compressive strength of concrete. An important point is the effects caused by changes in the diameter and the distance between closed collars on these two quantities. Equation (42) shows that change in the diameter and the distance between closed collars has no effect on the height of Whitney stress block; while values mentioned in the Table 4 and equations (40) and (41) indicates that the thickness of shear flow zone has direct relationship with the closed collars and inverse relationship with their distance. Therefore, in order to find a relationship between the height of Whitney stress block and thickness of shear flow zone, we should consider the effect of changes in the diameter and the distance between closed collars. Proposed method indicates the relationship between thicknesses of shear flow zone with the Whitney stress block height, and in order to guess the initial value of shear flow zone thickness, we can simply use it.

$$t \ = a + \quad (43)$$

$$t_d = a + \frac{5.7\phi_t}{s}\left(\frac{b}{\phi_l} - 10\right)$$

In Table 6, values obtained for the thickness of shear flow zone from Equation (43) with corresponding values of "nlarctm" program.

As seen, t_d ratio to value obtained from Equation (43) for 23 cross-sections in[14,15] has standard deviation 0.082 and mean 0.950 which shows higher accuracy of Equation (43) in order to calculate the thickness of shear flow zone.

6.2.2 Torsional Cracks' Angle

In most previous theories, like[16,17], truss analogy method has been used in order to analyze reinforced concrete cross-sections under torsion. One of the essential assumptions in this method is that reinforced concrete cross-section after cracking acts like spatial truss that its diagonal members are concrete and have angle 45^0. In fact, we can claim that by imposing this assumption, the angle of torsional cracks are 45^0. Results of analyzing 23 given cross-section show that the angle of torsional cracks fluctuates between 34 to 50^0 but if their related values evaluated more accurately, following results are obtained:

- By increasing the percent of collars, angle of cracks will increase.
- By increasing the percent of longitudinal reinforcements, angle of cracks will decrease.
- Changes in the compressive strength of concrete (f'_c0 and yield stresses of closed collars and longitudinal reinforcements (f_{ly},f_{ty}) has no effect on the angle of cracks.
- By increasing the distance between collars, angle of cracks will decrease.

6.2.3 Final Torsional Strength and Torsional Stiffness after Cracking

Figures 20 to 24 show the diagram of torsional anchor variations based on the torsional angle on length unit for different values of S, f_y, f_c', Φ_l, Φ_t. It should be mentioned that during changing above quantities, all other quantities are fixed. Yield stress of longitudinal and transverse reinforcements are same f_y.

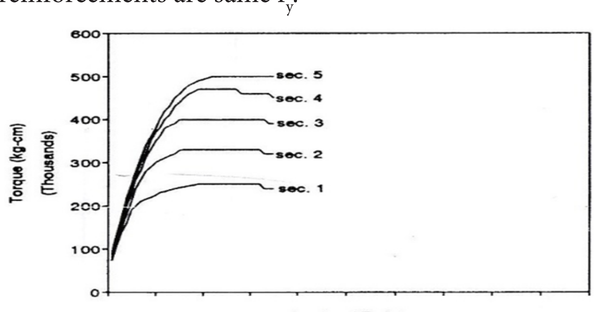

Figure 20. Cross-section number 1 to 5.

Table 6. Comparing the thickness of shear flow zone obtained from "nlarctm" program and Equation (43)

Number of cross-sections	$t_d^{(nla)}$ (cm)	α (cm)	ξ	Equation (43) $t_d^{(4)} = \alpha + \xi$	$\frac{t_d^{(nla)}}{t_d^{(4)}}$
1	9/4	0/3	7/1	7/4	04/1
2	1/5	0/3	2/2	2/5	98/0
3	8/5	0/3	8/2	8/5	00/1
4	4/6	0/3	4/3	4/6	00/1
5	9/6	0/3	0/4	0/7	99/0
6	0/5	9/1	0/5	9/6	72/0
7	8/5	0/3	8/2	8/5	00/1
8	3/6	3/4	4/1	7/7	82/0
9	6/6	8/5	5/0	3/6	05/1
10	9/6	5/7	4/0-	1/7	97/0
11	7/7	0/4	4/3	4/7	04/1
12	4/6	0/3	4/3	4/6	00/1
13	6/5	3/2	4/3	7/5	98/0
14	1/5	0/2	4/3	4/5	94/0
15	4/4	7/1	4/3	1/5	86/0
16	1/5	0/2	4/3	4/5	94/0
17	3/6	0/3	4/3	4/6	98/0
18	7/6	7/3	4/3	1/7	94/0
19	2/6	0/2	7/5	7/7	81/0
20	4/5	0/2	2/4	2/6	87/0
21	1/5	0/2	4/3	4/5	94/0
22	7/4	0/2	8/2	8/4	98/0
23	4/4	0/2	4/2	4/4	00/1
		Arithmetical mean & sample standard deviation			
		(Arithmetical mean)			95/0
		(Sample standard deviation)			082/0

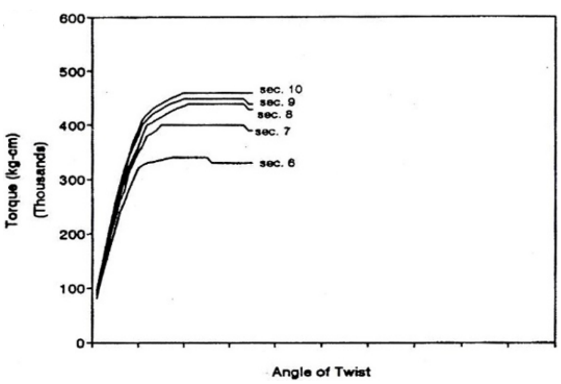

Figure 21. Cross-sections number 6 to 10.

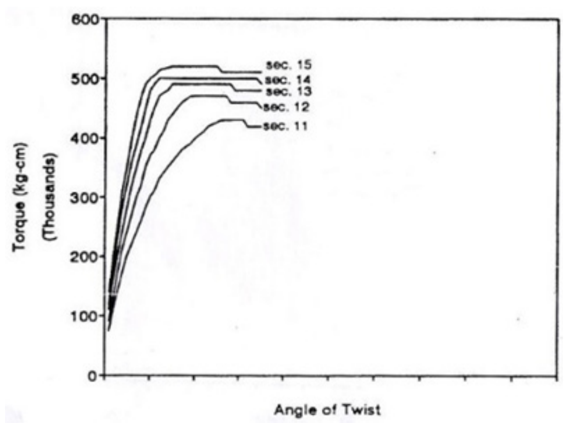

Figure 22. Cross-sections number 11 to 15.

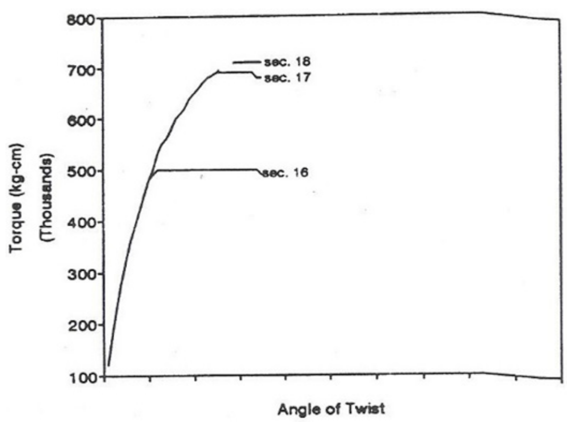

Figure 23. Cross-sections number 16 to 18.

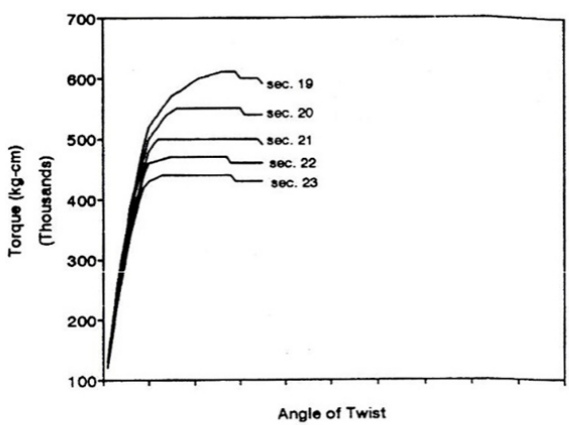

Figure 24. Cross-sections number 19 to 23.

If the tangent on T - θ curve in each point is considered as torsional stiffness of that point, regarding Figures 20 and 21, we can find that in general case, by increase in the percent of longitudinal reinforcements or collars, torsional stiffness of cross-section increases after cracking and final torsional strength of the cross-section will increase[18]. Figure 22 shows that the increase in compressive strength of concrete (f'_c) increases the final torsional strength and without tangible change in the cracking angle, it increases the torsional stiffness of cross-section after cracking. It seems that in truss theory, the effect of increase in the compressive strength of diagonal members is ignored, while it is clear that by increase in concrete compressive strength (f'_c), we expect increase in the strength of members. Regarding Figure 23, we can find that by increasing the yield strength of longitudinal and transverse steels, torsional stiffness of cross-section after cracking will not change but the torsional strength will increase. Figure 24 shows that increasing the distance between closed collars will reduce the torsional stiffness of cross-section after cracking and final torsional strength. Therefore, we can conclude that in torsional stiffness of a cross-section after cracking, torsional reinforcement's percent has the highest influence, while change in other physical quantities of cross-section including concrete compressive strength (f'_c) is also effective.

7. Guide for Computer Program Nonlinear Analysis of Reinforced Torsion Members

7.1 Programming Language and Solving Equations in the Program

This program is nonlinear analysis of reinforced concrete torsion members

This program is written by MATLAB language and its main steps are considered in Section 5 and Section (2.5) the method to solve equations.

7.2 Introducing Input File

Input file is torsion.xlsx in which given quantities in excel file. In order to run a program for a certain cross-section, it is enough that by using program written in common window environment, we select "Import data" option and designed excel file, then we go to Editor and run the program. No particular unit is considered for given quantities but coordination between units is necessary. In general, width and height of cross-section and diameter of longitudinal reinforcements and collars and the distances between collars is in length unit; yield stresses of longitudinal reinforcements and collars and concrete compressive strength and steel elastic module is in length unit and n which shows the number of longitudinal reinforcements has no unit. Concrete compressive strength should be considered with negative sign.

8. Conclusion

25 percent increase in longitudinal reinforcement strength and closed collars increases torsional strength as 2 percent after cracking which is minor and regarding this, cross-section number (17) is more optimal than cross-section (18).

133 percent increase in concrete compressive strength results in 25 percent increase in torsional strength after cracking which is significant.

133 percent increase in the distance between closed collars results in 33 percent reduction in torsional strength after cracking which is significant.

By comparing three effects the resistance of reinforcements, compressive strength and the distance of collars, we can conclude that all of them are very effective in increase or decrease.

Diagrams for Figures 20, 21 show that increase in the cross-section of longitudinal and transverse reinforcements increases the torsional strength after cracking.

Increase in the cross-section of transverse reinforcements will increase COD of cracks.

Increase or decrease in resistance of reinforced concrete members has minor effects on opening of initial crack.

Increasing the distance between transverse reinforcements closes the cracks.

9. References

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