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# Developing a mathematical model for optimizing oil production Using Gas-Lift Technology

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## Abstract

This article is an attempt to presents a mathematical model to optimize oil production from wells which are set to produce oil in accordance with gas lift system. This process is carried out by determining the economic level of oil production of wells and also reduction of production costs by minimizing the rate of gas used for oil production at an optimum level. Therefore, the information on the characteristics of wells are collected and used as input data of the PIPESIM software application. The performance curve of each of the wells was prepared using this application. In the following section, the non-linear multi-objective programming model of oil production optimization has been developed. The validity of the optimum solution depends on the validity of the presented nonlinear mathematical programming model, therefore, the model components and the relationships between them were described. In order to test the validity of the wells' performance charts in the gas-lift system, the obtained data were compared to the real data obtained from one of the wells through a T paired comparison test and their validity was confirmed at a significance level of 5%. The point which is of more considerable importance in the results of this study is if the wells under study undergo the production at maximum level, the maximum production would be 24,326 BPD which would require 34.702 MMScf/D of gas to be injected into the wells. Now if the level of consumed gas would reduce to 14/835 MMScf/D. Decreases in oil production only 5.7%, if the saved gas is for example used in the gas lift system for a number of wells comparable to the number of wells in this study, an increase of over 100% will be achieved in oil production. This would simultaneously reduce the rate of injected gas and increase the oil production.

**Keywords:** optimization, oil production, Mathematical modeling, Gas-lift

## 1. Introduction

Due to the increasing and accelerated development of industry and population growth, and on the other hand limited oil and gas reserves, more efficient use of these resources will be felt more seriously. When thrust to bring oil from the bottom of the well to the surface of the Earth is not enough and cannot bring enough oil to the surface, Should one of the secondary recovery techniques such as artificial methods are used to strengthen the force[7]. To strengthen the force of the secondary recovery techniques such as artificial methods are used. Many of the world's oil fields are in the second half of his life, this reservoir pressure is reduced so that oil exploitation is not possible with natural methods, And the need to lift technologies including artificial lift gas technology, the use of pumps and even some of them injecting gas, water, etc[7]. One of the most important of these methods is artificial lift gas method. High investment costs to provide compressed natural gas, large differences in the characteristics of the oil fields and even large differences in the characteristics of drilling wells in an oil field and The combined economic and technical complexity of the problem, The decision on how to exploit from an oil field with a gas lift method,

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requires to modeling approaches and optimization. According to the theoretical literature and the history, most research has been done with the aim of increasing production per unit time of an oil field or choose the most economical method of oil production. And approaches to optimizing the exploitation of an oil field with a gas lift method has been less attention. This paper presents a mathematical model to optimize compressed gas rate a series of wells that use of common gas supply facility and drilled in the oil field in South West Iran, as oil production done at a favorable level.

## 2. Gas lift method of oil production

In this method natural compressed gas will be generated with some of facilities, and then through a series of pipelines to be injected into the oil well. This makes light of the fluid column within the tubing well, which will increase oil production[6].

**The advantages of the gas lift system**

- Gas lift can be used for a wide range of depth and flow rate.
- This is important when the compressed gas is available and there is no need to install compression devices.
- Its construction is less expensive than other methods.
- There are hands-on experience in many oil-producing countries.
- This method is very useful for wells that are producing sand, because they have fewer moving equipment.
- The Initial investment of this method is less than other artificial lift[6].

**Restrictions on the use of gas lift system**

- Gas should be available for injection
- If the gas used be sour, Corrosion occurs in gas lift facilities, and paralyze the operation of the oil production.
- And using gas lift technology in extraction of heavy oil to light oil is less efficient.

## 3. About oilfield

The oil field is located inside folds of the zagros at a small distance from the Persian Gulf. Compare the different layers of the reservoir fluid pressure shows, there is little distance between the current pressure and the pressure of the bubble. In addition, Because of the high gas-oil ratio, the reservoir so is ideal for using gas lift technology. Reservoir fluid profile is presented in In Table 1.

**Table 1.** Reservoir fluid profile

| Static Pressure Bottom-Well | Solution Gas-Oil Ratio (GOR) | Flow Pressure Bottom-Well | Base Depth | API Specific Gravity | Average Water Saturation | Formation Volume Factor | Relative Permeability | Saturation Pressure | Base Depth Temperature (°F) |
|---|---|---|---|---|---|---|---|---|---|
| 420 psig | 450 scf/stb | 3220 psig | 7600 ft | 29 °API | 25% | 1.25 bbl/stb | 4 md | 3100 Psi | 185 °F |

**Table 2.** Gas lift system design parameters

| temperature | Specify gravity | Pressure |
|---|---|---|
| 80°F | 0.72 | 1600 PSIG |

**Table 3.** selected wells profile

| flow Pressure Bottom-Well (Psi) | Gas injection depth (m) | Wellhead flow pressure (Psi) | Perforation depth (m) | Casing diameter (Inch) | Static Pressure(Psi) | Productivity index (Stb/Psi. Day) | Well |
|---|---|---|---|---|---|---|---|
| 2875 | 2047 | 737 | 2589 | 3 1/2 | 3045 | 15.62 | W12 |
| 2875 | 2810 | 730 | 2253 | 3 1/2 | 3040 | 15 | W13 |
| 2467 | 2594 | 821 | 2824 | 4 1/2 | 3147 | 5.25 | W15 |
| 2930 | 2480 | 799 | 2496 | 3 1/2 | 3040 | 2.8 | W16 |
| 2122 | 2694 | 752 | 2813 | 2 7/8 | 3161 | 0.53 | W22 |
| 2636 | 2335 | 770 | 2640 | 4 1/2 | 3024 | 4.55 | W23 |
| 2622 | 2419 | 714 | 2771 | 3 1/2 | 3101 | 7 | W29 |
| 2650 | 2383 | 811 | 2755 | 3 1/2 | 2935 | 7.94 | W30 |
| 3041 | 2086 | 687 | 2805 | 3 1/2 | 3080 | 5.75 | W52 |
| 2452 | 2467 | 786 | 2775 | 4 1/2 | 3072 | 7.57 | W69 |
| 1928 | 2159 | 758 | 2367 | 2 7/8 | 3075 | 0.52 | W91 |
| 2817 | 2501 | 715 | 2512 | 3 1/2 | 3060 | 18 | W98 |

Wells choice mainly due to a reduction in wellhead flowing pressure, unable to flow to the first stage separation of unit operation. So in addition to reducing production, the imbalance in the gas collection system, burning part of the associated gas produced, wasting national wealth and also causing environmental pollution[7]. Gas lift system design parameters and selected wells profile are presented in In Tables 2 and 3.

## 4. Gas lift system costs

Wells repair costs for the installation of gas lift equipment (Valves lift,) Included the cost of killing the well, Installation equipment lift and reclamation of wells. The average repair cost is estimated at 250 thousand dollars. The cost of purchasing and installing the equipment is estimated 2,500 $ per horsepower. Cost lift valves and other equipment needed is almost a thousand dollars. Considering the average age of 25 years for the compressor, the annual maintenance cost of each well is considered the equivalent of 4 percent. Gas injection costs and operational costs are calculated according to the scheduled program. Costs related to artificial lift system with a total cost of gas wells to be considered. The calculations with respect to the input and output pressure compressors and gas injection rate, the amount of power required by the compressor is calculated through the equation (1).

HP=0/193 * Qg[(P2/P1)* 0. 230-1] (1)

P2: Discharge pressure of the compressor (Psi)

P1: Suction pressure of the compressor (Psi)

Qg: gas volumetric rate at standard condition, Mscf/d

According to the above described, In order to supply 22 million cubic feet of gas by a compressor with a capacity of 2300 Hp, the cost of Gas lift system implementation and the cost of a cubic foot of gas is given in table4. It should be noted that the estimated costs for the year 2011. Obviously, with the inflation rate can be changed[1].

## 5. Parameters and decision variables:

The first set of decision variables of this study is related to the amount of gas injected into each of the wells. Another set is the amount of oil produced by each of the wells. As following:

$X_{i:}$ rate of gas injected in well No. i

Qi: The rate of oil production from well No. i

i = 12, 13, 15, 16, 22, 23, 29, 30, 52, 69, 91, 98

More parameters and technical coefficients are the coefficients of the equation as well performance curves after determining that the equation will be determined. The parameters of this study are presented below.

- Economic border of oil production from each of the wells with gas lift method.
- The flow capacity of the pipeline to transport oil from the well to the unit operation. The pipe diameter is 8 inches and have the capacity of 60000 barrels of crude oil per day.
- The share of wells from oil Daily demand, the value of this parameter is variable in the distance minimum capacity and the economic border of oil production from all of wells.

To access the details of the objective functions and constraints, we need to performance curve of each of the wells, this primarily required having the cure performance of each of the wells. These charts are needed to achieve the specifications of each of the wells is under study, as well as the use of specialized software. In order to obtain the diagrams of each of the wells, the software is intended to help PIPESIM. For example, the performance curve of well No.12 is shown in the graph (Figure 1), which is obtained by PIPESIM software.

**Table 4.** Gas lift method costs (US $)

| Compressor cost per HP | Compressor power(hp) | Compressor Cost | Overhaul compressors cost |
|---|---|---|---|
| 2500 | 2300 | 5750000 | 230000 |
| Repairing cost a well | lift valves Cost for a well | Number of wells | purchasing valves and repair wells cost |
| 250000 | 50000 | 10 | 3000000 |
| Total costs per thousand cubic feet of gas injection wells for a | Total costs of gas daily injection per thousand cubic feet | total annual costs of gas injection | Total cost of gas injection |
| 6.5 | 22000 | 8030000 | 52195000 |

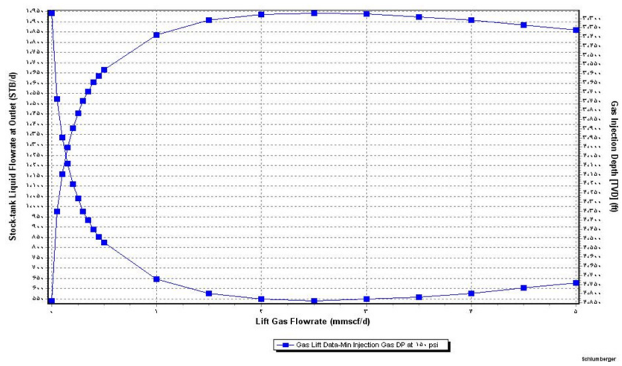


**Figure 1.** The performance graph of well No. 12

In order to fit the function on performance curve wells, several models have been provided by various scholars. The most important of them have been provided by Alarcon G.[12] and Khamehchi E.[11]. In This research have been used the exponential function with the following form and parameters:

$f(x) = a * exp(-b*x^{d})+c \;\; a<0 \;\; b>0 \;\; c>0 \;\; 0<d<1$

The result of fitting function on Well No.12 performance curve, with MATLAB software is as follows and figure 3.

| | |
|---|---|
| General model: | Goodness of fit: |
| f(x) = a*exp(-b*x^d)+c | SSE: 605.1 |
| Coefficients (with 95% confidence bounds): | R-square: 0.9998 |
| a = -1421(-1408, -1434) | Adjusted R-square: 0.9998 |
| b = 2.461(2.41, 2511) | RMSE: 5.244 |
| c = 1960(1954, 1966) | |
| d = 0.6308 (0. 619, 0.6427) | |

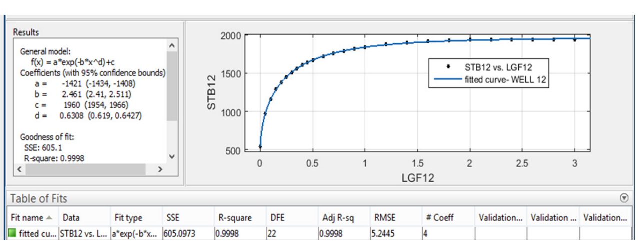


**Figure 2.** Fitted function on well No. 12 performance curve by MATLAB software

Therefore fitted function on well No. 12 performance curve is as follows:

$Q_{12}=-1421e^{-2.461G_{12}^{0.630s}}+196_0$

Math functions fitted on performance curve of each of the wells (In terms of oil production per gas injection) are as follows:

$Q_{13}=-1731e^{-2.339G_{13}^{0.5754}}+1374$ $Q_{15}=-5019e^{-1.018G_{15}^{0.853s}}+482_0$

$Q_{16}=-175\, e^{-2.513G_{16}^{0.5556}}+1736$ $Q_{22}=-1937e^{-2.461G_{22}^{0.53}}+149_2$

$Q_{23}=-1482\, e^{-2.496G_{23}^{0.6215}}+1914$ $Q_{29}=-1890\, e^{-2.516G_{29}^{0.5337}}+165_4$

$Q_{30}=-1571\, e^{-2.496G_{30}^{0.599}}+1853$ $Q_{52}=-1050\, e^{-2.411G_{52}^{0.7474}}+225_9$

$Q_{69}=-1747\, e^{-2.518G_{69}^{0.5623}}+1751$ $Q_{91}=-1149\, e^{-2.415G_{91}^{0.7175}}+215_3$

$Q_{98}=-1828\, e^{-2.515G_{98}^{0.544}}+1712$

## 6. Objective functions

This research includes an objective function "to minimizing gas consumption" in the production of oil from twelve wells by using gas lift technology, in the "favorable maximum oil production". These functions are formulated as follows.

Gas LIFT:$Z_1$

$$\text{Min } Z_1 = \sum_i X_{12}$$

$i = 12,13,15,16,22,29,30,52,69,91,98$

Oil Production:$Z_2$

$$\text{Max } Z_2 = \sum_i Q_i$$

$i = 12,13,15,16,22,23,29,30,52,69,91,98$

According to the obtained functions in before, second objective function with following form, is a function of the amount of gas injected into each wells.

*Max* $Z_{\downarrow}2$=–1432e↑(-2.461 [$G_{\downarrow}1_{2]}$↑0.6308) -1731e↑(-2.339 [$G_{\downarrow}1_{3]}$↑0.5754) -5019e↑(+1

## 7. Model constraints

Gas lift process is expensive. As if the case is uneconomically it should be ignored. Therefore, one of the main constraints the model is Determine the economic level of gas lift process for each of the wells (Figure 3).

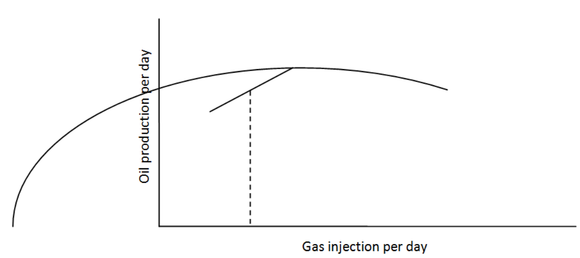


**Figure 3.** Economical level of gas lift process

Economic frontier is maximum gas injected into oil production so oil production be cost-effective. The average production cost of the company›s operation is approximately US $ 1.8 that one-third of that is the cost of extraction And two-thirds Is the cost of processing and transmission. In this study, injection of compressed gas to produce one barrel of crude oil supply costs less than US $ 6.0 (cost of production) is as the limit in oil production with gas lift method. According to the data related to the cost of oil production with gas lift method calculate in Table 4, Total cost of gas used in the gas lift process in 2011, equivalent to $ 22,000 for injection 22 Million Standard Cubic Feet oil industry (MMSCF) of gas per day[1]. Due to changes in the value of the dollar in 2015 compared to 2011 (from 10000 Rials to 32,000 Rials in other words at a rate of 3.2),

And 83% of providing gas costs are domestic (energy, labor, etc.) and 17% are foreign (equipment and repairs). Therefore the cost of providing each million cubic feet of gas in 2015 in Iran is equivalent $ 2656 estimated at. According to what was said, Limit point of oil production with gas lift method, is at least 379.43 barrels per million cubic feet of gas is injected per day. To find this crossing point, Use of math derivative techniques, or the slope of the tangent at the point. So it is derived from mathematical functions fit to the curves of each of the wells to variables injected gas ($x_i$) which results in the simplified as follows.

$X_{12} = 0.6687 \qquad X_{13} = 0.7327 \qquad X_{15} = 2.5995$

$X_{16} = 0.6885 \qquad X_{22} = 0.7119 \qquad X_{23} = 0.6731$

$X_{29} = 0.6944 \qquad X_{30} = 0.6787 \qquad X_{52} = 0.6371$

$X_{69} = 0.6872 \qquad X_{91} = 0.6529 \qquad X_{98} = 0.6901$

Base on amount of gas injection in economic levels, calculated before for each of the wells, and the mathematical function of the performance of each of the wells, for all of them oil production at the economic level be able to calculate (Table 5). The minimum oil produced (without the use of gas lift system) and a maximum oil production (using a gas lift system) is set as the output application (Table5).

One of the important parameters is change in market demand, therefore, if the allocation of market demand to the wells be D barrels per day, the following linear constraint will be added to the model.

$$Q_{12}+Q_{13}+Q_{15}+Q_{16}+Q_{22}+Q_{23}+Q_{29}+Q_{30}+Q_{52}+Q_{69}+Q_{91}+Q_{98} = D$$

According to the obtained functions before, this constraint with following form, is a function of the amount of gas injected into each wells.

$$-1421e^{-2.461G_{12}^{0.6308}}-1731e^{-2.339G_{13}^{0.5754}}-5019e^{-1.018G_{15}^{0.8538}}-1775e^{-2.513G_{16}^{0.5556}}-$$

The result of solving the model are provided in the tables 6 and 7.

**Table 5.** The level of economic production and the minimum and maximum oil production wells under study

| oil production in economic level (using a gas lift system) | | maximum oil production (using a gas lift system) | | minimum oil produced (without the use of gas lift system) | Well |
|---|---|---|---|---|---|
| Oil production (Stb/day) | Gas injection (MMSCF) | Oil production (Stb/day) | Gas injection (MMSCF) | | |
| 1931 | 1.4742 | 1942 | 2.1367 | 540 | W12 |
| 1306 | 1.5497 | 1346 | 2.209361 | 233 | W13 |
| 3307 | 3.274 | 4697 | 4.5462 | 0 | W15 |
| 1667 | 1.4325 | 1713 | 2.36255 | 628 | W16 |
| 1423 | 1.5345 | 1460 | 2.26865 | 317 | W22 |
| 1862 | 1.4488 | 1900 | 2.2941 | 906 | W23 |
| 1578 | 1.3823 | 1625 | 2.44099 | 514 | W29 |
| 1798 | 1.4651 | 1835 | 2.29889 | 812 | W30 |
| 2222 | 1.4475 | 2252 | 2.4847339 | 1215 | W52 |
| 1695 | 1.5002 | 1727 | 2.294715 | 652 | W69 |
| 2122 | 1.3931 | 2143 | 2.13832 | 1009 | W91 |
| 1654 | 1.5102 | 1686 | 2.289559 | 591 | W98 |
| 22/22456 | 19.4121 | 24326 | 29.764777 | 7417 | عومجم |

**Table 6.** optimum rate of gas used for the production of oil at different level (By using gas-lift technology)

| rate of gas lift (million cubic feet per day) | | | | | | | | | WELL |
|---|---|---|---|---|---|---|---|---|---|
| 0/233 | 0/284 | 0/350 | 0/435 | 0/551 | 0/722 | 1/005 | 652/1 | 2/701 | W12 |
| 0/258 | 0/313 | 0/382 | 0/473 | 0/600 | 0/788 | 1/108 | 1/869 | 2/808 | W13 |
| 1/107 | 1/320 | 1/570 | 1/872 | 2/250 | 2/753 | 3/503 | 4/982 | 4/748 | W15 |
| 0/253 | 0/304 | 0/368 | 0/452 | 0/569 | 0/743 | 1/041 | 1/500 | 2/803 | W16 |
| 0/262 | 0/313 | 0/379 | 0/466 | 0/588 | 0/770 | 1/084 | 1/460 | 2/742 | W22 |
| 0/239 | 0/291 | 0/356 | 0/441 | 0/557 | 0/726 | 1/008 | 1/447 | 2/785 | W23 |
| 0/257 | 0/307 | 0/371 | 0/456 | 0/574 | 0/750 | 1/053 | 1/387 | 2/440 | W29 |
| 0/244 | 0/295 | 0/360 | 0/445 | 0/561 | 0/732 | 1/020 | 1/463 | 2/792 | W30 |
| 0/193 | 0/247 | 0/316 | 0/404 | 0/521 | 0/688 | 1/955 | 1/445 | 2/664 | W52 |
| 0/252 | 0/303 | 0/367 | 0/452 | 0/568 | 0/742 | 1/037 | 1/499 | 2/803 | W69 |
| 0/208 | 0/262 | 0/330 | 0/418 | 0/536 | 0/705 | 0/217 | 1/977 | 2/617 | W91 |
| 0/254 | 0/305 | 0/369 | 0/453 | 0/570 | 0/745 | 1/279 | 1/509 | 2/800 | W98 |
| 3/761 | 4/544 | 5/518 | 6/768 | 8/446 | 10/864 | 14/83 | 23/835 | 34/702 | Gas Lift |
| 1700 | 18000 | 19000 | 20000 | 21000 | 22000 | 23000 | 24000 | MAX: 24326 | Oil Prod. (BPD) |

**Table 7.** Optimum rate of gas used (million cubic feet per day) at different levels of production of all wells by using gas-lift system

| Oil production (BPD) | Rate of Gas Used | Oil production (BPD) | Rate of Gas Used |
|---|---|---|---|
| 19200 | 5.743 | 24326 | 34.702 |
| 19000 | 5.518 | 24300 | 30.502 |
| 18800 | 5.305 | 24200 | 27.742 |
| 18600 | 5.102 | 24000 | 23.835 |
| 18400 | 4.907 | 23800 | 21-98 |
| 18200 | 4.723 | 23600 | 19.021 |
| 18000 | 4.544 | 23400 | 17.363 |
| 17800 | 4.375 | 23200 | 15.994 |
| 17600 | 4.212 | 23000 | 14.835 |
| 17400 | 4.055 | 22800 | 13.835 |
| 17200 | 3.906 | 22600 | 12.560 |
| 17000 | 3.761 | 22400 | 12.185 |
| 16800 | 3.623 | 22200 | 11.490 |
| 16600 | 3.490 | 22000 | 10.864 |
| 16400 | 3.360 | 21800 | 10.295 |
| 16200 | 3.237 | 21600 | 9.775 |
| 16000 | 3.118 | 21400 | 9.267 |
| 15800 | 3.003 | 21200 | 8.855 |
| 15600 | 2.892 | 21000 | 8.446 |
| 15400 | 2.784 | 20800 | 8.065 |
| 19200 | 5.743 | 24326 | 34.702 |
| 19000 | 5.518 | 24300 | 30.502 |
| 18800 | 5.305 | 24200 | 27.745 |
| 18600 | 5.102 | 24000 | 23.835 |
| 18400 | 4.907 | 23800 | 21.098 |
| 18200 | 4.723 | 23600 | 19.021 |
| 18000 | 4.544 | 23400 | 17.363 |

## 8. Conclusion

In this study, 12 of the wells digging in an oil fields located in the South West of Iran chosen for optimize oil production with gas-lift technology. The reason for these wells are as follows: First, due to the natural production decline in reservoir pressure is stopped, first, because of the drop in reservoir pressure natural oil production is stopped, Therefore it is necessary by one of the operation methods such as gas lift technique must be done. And required gas selected wells is supplying from Common facilities.

The results indicate that the economic limit of oil production in all the selected wells is negligibly lower than their maximum production capacity by the gas-lift system. In other words, the maximum level of oil production is not significantly different from the amount of oil produced at the economic level (less than 7%). On the other hand, the amount of gas consumed at the maximum level of oil production is significantly different from the amount of gas consumed at the economic level of oil production (44.6%). Therefore, the application of the research results in all gas-lift systems would lead to dramatic reduction of production costs in the gas-lift methods.

Secondly, in cases of need for production of oil below the economic level, the non-linear programming model provided in this study and the results of its resolution, presented in tables 6 and 7, including the least amount of gas used for the production of oil at an optimum level and also the gas rate which should be injected to each of the wells can be used as a reference. It can be concluded that this can also lead to considerable saving on the cost of gas supply, overhead costs, control of equipment wearing rate by reducing their function time and also provision of opportunities for the overhaul of the equipment.

The point which is of more considerable importance in the results of this study is the comprehensive view of the production of all wells in the oil field or in a specific area where gas-lift artificial system is used for oil production. Such that if the wells under study undergo the production at maximum level, according to the results provided in Table6, the maximum production would be 23,326 barrels per day which would require 34.702 million cubic feet of gas to be injected into the wells on a daily basis. Based on the results presented in Table 7 that show the optimal rate (minimum) of injected gas (million cubic feet per day) at different levels of production in the wells under study using the gas-lift system, if for example a total of 23,000 barrels of oil are produced from these wells on a daily basis, the level of consumed gas would reduce to 15/286 cubic feet per day. In other words, a cut of about 6/6% in oil production will lead to saving of 57% in gas consumption equivalent to 19/986 million cubic feet per day. If the saved gas is for example used in the gas lift system for a number of wells comparable to the number of wells in this study, based on the results in Table 8, about 23800 barrels of oil can be produced per day, which including 23,000 barrels of oil produced from the wells under study, an increase of over 100% will be achieved in oil production. This would simultaneously reduce the rate of injected gas and increase the oil production.

## 9. Acknowledgment

Here's the place to appreciate NIOC that provided data related to selected oil field and wells to modeling, solve and analysis.

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