

# Performance Evaluation for Wavelet based OFDM System effected by CFO over Rayleigh Channel

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## Abstract

OFDM systems due to its orthogonality factor provides significant spectrum efficiency, robustness to fading and simple cost effective implementation. One of the factors that effects the OFDM system performance is the Carrier Frequency Offset (CFO) that results in the Inter-Carrier Interference (ICI) which causes a degradation of the OFDM system performance. In this paper, a discussion on the performance of Discrete Wavelet Transform (DWT) OFDM and Fourier Transform (FFT) OFDM is done by considering the presence of CFO in both the systems. A comparative study of both the systems is done on the basis of Bit Error Rate (BER) performance these systems show. The system is considered on the basis of improvement in the BER. Simulations conclude that the system that uses wavelet implementation performs better than the conventional OFDM system. The DWT performance even with CFO factors shows considerable improvement in the bit error performance than the FFT based traditional system. Further, the whole simulation is done under the Rayleigh channel, the performance calculated for five practical values of CFO from 0 to 0.2 with step difference of 0.05. The BER performance is simulated using MATLAB software. Further, the whole simulation is done under the Rayleigh channel, the performance calculated for five practical values of CFO from 0 to 0.2 with step difference of 0.05. The DWT based OFDM outperforms the FFT based OFDM for all the values of CFO.

**Keywords:** BER, CFO, DWT, FFT, ICI, IDWT, IFFT, ISI, OFDM

## 1. Introduction

OFDM, a multi-carrier based modulation technique is used to achieve higher data rates by transmitting some orthogonal sub-carriers that are overlapped with each other. The high data-rate sub-carriers are required to maintain an orthogonal relationship with each other so as to improve the spectral efficiency. At the receiver side, it is easy to recover data in each sub-carrier as long as carriers are orthogonal to each other<sup>1</sup>. The orthogonality present in the orthogonal carriers is considered as the primary aspect in the system. In this scheme, a large sum of carriers is transmitted in parallel that are orthogonal, overlapped sub-carriers that are narrowly spaced in the band. Each sub-carrier is then taken for modulation by any modulation scheme, i.e., Quadrature Amplitude Modulation (QAM) or Phase-Shift Keying (PSK). The FFT techniques that are used for modulation and demodulation process makes the system more efficient in computation. The guard interval

insertion mitigates the inter-symbol & inter-channel interference. Also, the code synchronization can be easily achieved by making the extension of symbol periods relating to the sum of subcarriers<sup>2</sup>. A data stream of high rate is divided into a sum of small rate streams which needs to be transmitted all together over the sum of sub-carriers employed. The increase in symbol duration for the parallel sub-carriers of lower rate due to delay in multipath further causes a reduction of dispersion in time. OFDM chooses the orthogonal sub-carriers for transmission. This leads to the reduction in cross-talk between sub-channels, so eliminating the requirement of inter-carrier bands. This greatly simplifies the design of the system as it doesn't require a separate filter for all the sub-channels. The available transmission bandwidth is divided into the carriers. At a low bit rate, the modulation of each sub-carrier is being done. The separation between the sub-carriers makes a very compact spectral utilization of the available bandwidth. OFDM has become a very trendy transmission technique that shows

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a high spectral efficiency and the robustness in opposition to the multipath fading and impulsive noise for the use of high data-rate based wireless communication systems. Sensitivity to frequency offsets like CFO and the noise in phase causes a loss of the orthogonality in subcarriers<sup>3</sup>. Fourier transform procedures help in maintaining the orthogonality factor among the carriers by defining the OFDM signal.

In this paper, an overview of the performance analysis and an improvement in the bit error is shown in comparison to the simple conventional OFDM system. A comparative study on the FFT-based and the wavelet based OFDM system is done, and their performance is analysed on the basis of bit error curve. All the results are simulated for different modulations, i.e., BPSK, QPSK, 8-PSK, 16-PSK, 32-PSK in the Rayleigh channel in the presence of CFO whose value lies in the range from 0 to 0.2.

## 2. System Description

The basic block diagram of OFDM system is defined in Figure 1. A serial stream of  $N$  number of data is converted into parallel and after suitable modulation (QPSK or BPSK or M-PSK) the respective  $N$  number of subcarriers reaches IFFT/IDWT block. In conventional OFDM, IFFT is performed and the required frequency to time domain conversion has been done. The system becomes DWT wavelet-based system by swapping IFFT and FFT modules with the respective IDWT and DWT modules<sup>4,5</sup>. Similarly, FFT OFDM system is modelled with IFFT and FFT modules. The Cyclic prefix (CP) is added to the output of IFFT in a way to keep away the signal from ICI and ISI. This is done by extending the symbol with its repetition applied at the end. The resultant output signal is then transmitted to the signal. The addition of CP increases the

system complexity. For DWT system, there is no requirement of CP, which makes this system less complicated to implement. The cyclic prefix is the redundant data which restricts the useful data rate<sup>6,7</sup>. By avoiding this, data rate and hence the efficiency can be improved.

This motivates the use of DWT based OFDM that does not use cyclic prefix<sup>8</sup>. At last, the demonstrated OFDM symbol is shown as:

$$x(n) = \frac{1}{N \sum_{m=0}^{N-1} \left( X_m e^{\frac{j2\pi mn}{N}} \right)}; 0 \leq n \leq N-1 \quad (1)$$

Here,  $X_m$  defines the baseband data available on each of the sub-carrier. The digital-to-analog conversion process creates a signal in time domain that is considered to be analog in nature and is passed from the Rayleigh channel. In Rayleigh channel, when a signal is taken for transmission, the power of the signal varies randomly corresponding to the Rayleigh distribution which is represented by a radial angle component, which is considered as the addition of two uncorrelated variables which are random and Gaussian by nature. The Rayleigh channel in wireless devices is assumed to form the propagation effect in the environment. This model is used mainly when no line of sight is present in the transmitter section and the receiver. The signal was then transmitted over the Rayleigh fading channel that causes a variation in phase, amplitude and the angle of arrival of the received signal and can be characterized by:

$$h(t, \tau) = \sum_{r=1}^R a_r(t, \tau) e^{-j\theta_r} \delta(t - \tau_r(t)) \quad (2)$$

Where, the amplitude is represented by  $a_r(t, \tau)$ , the time delay by  $\tau_r$  and at time  $t$ , for the  $n$ th multipath the phase shift is  $\theta_r = 2\pi f_c \tau_r(t)$ .  $R$  defines the maximum of possible multipath with  $\delta(\cdot)$  as the Dirac delta. So, this channel is modelled as a zero-mean complex-valued Gaussian process. The signal relating to each of the sub-carrier is further taken back for conversion in the form of a discrete  $N$ -point sequence that is denoted by  $y(n)$ . At the receiver side, the discrete signal received was further taken for demodulation process that is done by the usage of the  $N$ -point FFT operation<sup>9</sup>. The symbol stream that is demodulated expressed in the form:

$$y(m) = \sum_{n=0}^{N-1} y(n) e^{\frac{j2\pi mn}{N}}; 0 \leq m \leq N-1 \quad (3)$$

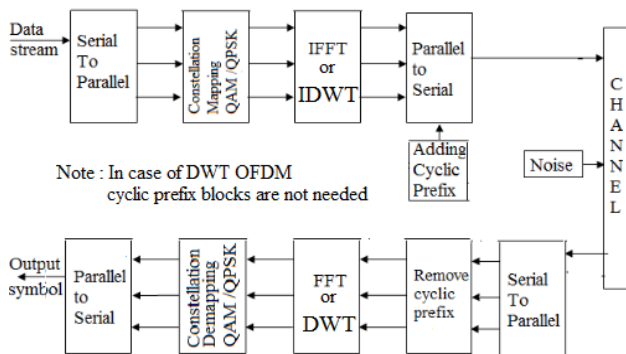


Figure 1. Block Diagram of OFDM system.

DWT-based OFDM is an efficient approach to replace FFT in conventional OFDM systems. The traditional Fourier based OFDM system, due to its excellent performance for wireless transmission of signals is preferred from a long time<sup>10</sup>. For the Fourier system, we take a signal and break it up into a set of an infinite signal that is a summation of the sines and cosines to destroy the orthogonal relationship between them. On the other hand, in the wavelet transform, firstly the signal is broken down by two filters, i.e., by a Low-Pass (LPF) and a High-Pass (HPF) filter. During this decomposition of signals by two filters, the signal is down-sampled at the filter outputs, as half of the frequency components are filtered out at the output of these filters. Wavelet transform is applied in order to mitigate the addition of cyclic prefix that helps in decreasing the wastage of bandwidth and increase the efficiency by 25% and the wavelet transform employed also reduces the transmission power. The spectral pattern of channels shows that DWT system is improved than FFT system.

In Wavelet transform, the signal that is to be transformed is decomposed into a set of basis waveform functions, named as wavelets, that helps in providing a way for the analysis of signals by making an investigation on the coefficients of transformed functions i.e. wavelets<sup>11</sup>. A wavelet is being used in various applications and has become very attractive to use among engineers, technologists and mathematicians. The basic functions of this transform make it a powerful tool by localizing both in time and frequency domains and by possessing different resolutions in respective domains<sup>12</sup>. Different resolutions correspond to analyze the behaviour of the process and the power of the transform. A low pass filter and high pass filter, two filters are implemented to work as QMF and satisfy a perfect reconstruction with orthonormal properties. In the wavelet-based system, the signals after modulation are sent for transmission by making use of zero padding and applying vector transpose on that. The wavelet technique is an efficient and flexible method used for decomposition of signals. The wavelet Low-pass filter presents approximation coefficients, where the high-pass filter shows detail coefficients as depicted in Figure 2 and Figure 3. Different wavelet families have different filters satisfying orthogonality Principle<sup>13</sup>. The IDWT transformed output can be expressed as follows:

$$x_i(n) = \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} D_k^m 2^{\frac{k}{2}} \phi(2^k n - m) \quad (4)$$

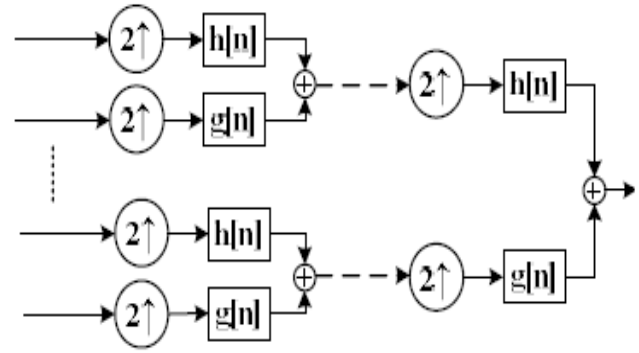


Figure 2. Block Diagram for IDWT Transform.

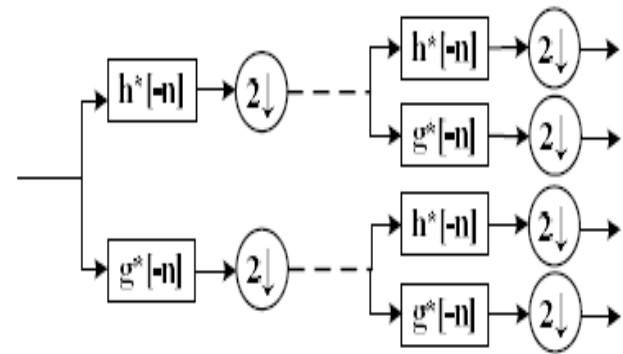


Figure 3. Block Diagram for DWT Transform.

Where  $D_k^m$  are the wavelet coefficients and  $\psi(t)$  is the wavelet basis function with compression factor  $k$  and shifted  $m$  for each subcarrier. At the receiver section, the demodulator and the DWT are used for recovering back the input data by discarding the zeros that were added at the transmitter as shown in Fig. 3. The output of DWT is given by,

$$D_k^m = \sum_{n=0}^{N-1} x_i(k) 2^{\frac{k}{2}} \phi(2^k n - m) \quad (5)$$

The DWT-OFDM has some advantages over FFT system:

- It needs less overhead as it doesn't make use of CP.
- No need of pilot tone that makes use of 8 percent of the available subbands.
- Inherently shows robustness to ICI and ISI.

### 3. Effect of CFO in OFDM

OFDM is a high data rate transmission technique used in wireless communication systems. Due to its orthogonality property between subcarriers, it provides less degradation in performance as well as gives bandwidth efficiency resulting in excellent performance parameters. But there

is one drawback in OFDM, which is its sensitivity to CFO. CFO is caused due to relative motion between transmitter and receiver resulting in loss of orthogonality between sub-carriers which in result degrade its performance or by a mismatch in the oscillator frequency placed at both ends. This system is capable of utilizing the frequency spectrum more efficiently by overlapping the sub-carriers. These carriers overlap partially with each other without making any interference with the neighbouring sub-carrier. OFDM is very sensitive to any frequency offset, so this system offers an ease of equalization of the channel in the frequency domain. CFO caused due to this offset causes a loss of orthogonal relationship between the carriers that produces ICI that degrades the system performance significantly. So, it needs to be estimated and corrected too. As mentioned above, the main causes for CFO in the communication systems are Doppler frequency shift  $f_D$  and frequency mismatch  $f_m$  between transmitter and receiver. So, if normalized CFO  $\epsilon$  is defined as a ratio of a sum  $f_D + f_m$  to spacing in subcarrier  $\Delta f$ , the received OFDM signal  $r(n)$  for transmitted signal  $s(n)$  is shown as:

$$r(n) = e^{j2\pi N(f_D + f_m) \frac{n}{B_s}} s(n) = e^{j2\pi n \epsilon} s(n) \quad (6)$$

Where  $B_s$  is the OFDM signal's bandwidth and  $B_s = N_{FFT} \Delta f$ . For an ideal case, the received signal at the front end of the OFDM receiver may be considered as given in the equation (2). But it is true and valid only if there is no CFO or Doppler-effect. That ideal condition practically doesn't exist. The presence of CFO rules out the ideal condition and hence the complex expression for the received modulated signal  $R(k)$  can be deduced:

$$R_\epsilon(k) = \text{FFT}[e^{j2\pi n \epsilon} s(n)] = s(k - \epsilon) \quad (7)$$

The ICI due to the mismatch in carrier frequencies leads to loss of orthogonality among the OFDM carriers and hence introduces high degradation in performance. In this paper, the BER performance of the OFDM is simulated in the presence of CFO for both conventional FFT-OFDM and DWT-OFDM.

## 4. Simulation Parameters

Simulation of both FFT-OFDM and DWT-OFDM is performed using MATLAB software. In DWT-OFDM, the filter banks are selected in such a way that it satisfies the condition for perfect reconstruction and renders required

orthogonality. First a simulation with the parameters defined in the table is done for the FFT model with comparison to the wavelet-based model. The study for both the system is done with different modulation techniques with five practical CFO values. A comparative study of both the systems is done, and analysis is made on the basis of their BER curve. The complete simulation is done in the Rayleigh channel.

Better BER performance is achieved by the DWT wavelet based OFDM system in comparison to Fourier based system. A signal to noise ratio gain is obtained over FFT-OFDM by the DWT-OFDM system. Also due to the addition of CP in the FFT model, this model suffers from wastage of bandwidth in the system which makes it inferior in comparison to DWT system which doesn't make use of CP in the system. Simulation parameters are as listed in the Table 1. BER versus  $E_b/N_0$  simulation in the presence of CFO is carried out for  $\epsilon$  values in the range of 0 to 0.2 in steps of 0.05.

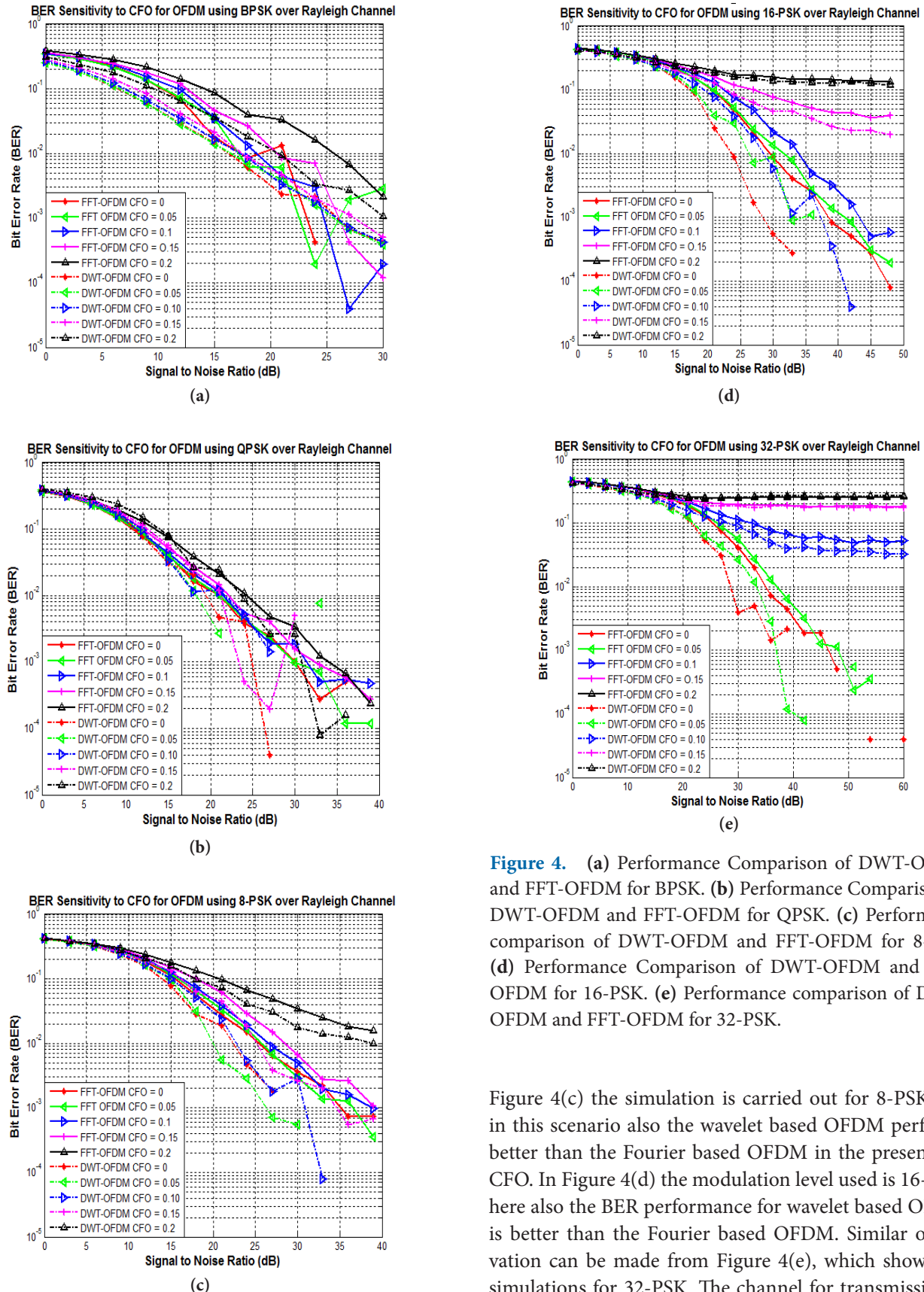
## 5. Result Discussion

In Figure 4(a-e) shows the BER performance of DWT-OFDM and FFT-OFDM in the presence of CFO for different CFO levels. In Figure 4(a) the BER performance for DWT-OFDM and FFT-OFDM is compared for BPSK modulation scheme in the presence of CFO and it clearly depicts the BER performance enhancement provided by the use of wavelet transform over Fourier transform. In Figure 4(b) the comparison is made in between the two systems employing the QPSK modulation scheme and the same observations are there in this case also. In

**Table 1.** Simulation parameters

Simulation parameters	Values for FFT OFDM	Values for DWT OFDM
Number of Data sub-carriers	256	256
Number of sub-carriers in cyclic prefix	16	16
Modulation	BPSK, QPSK, 8-PSK, 16-PSK, 32-PSK	BPSK, QPSK, 8-PSK, 16-PSK, 32-PSK
Channel	Rayleigh	Rayleigh
CFO values	0, 0.05, 0.1, 0.15, 0.2	0, 0.05, 0.1, 0.15, 0.2
Wavelet used	N/A	Haar/biorthogonal





**Figure 4.** (a) Performance Comparison of DWT-OFDM and FFT-OFDM for BPSK. (b) Performance Comparison of DWT-OFDM and FFT-OFDM for QPSK. (c) Performance comparison of DWT-OFDM and FFT-OFDM for 8-PSK. (d) Performance Comparison of DWT-OFDM and FFT-OFDM for 16-PSK. (e) Performance comparison of DWT-OFDM and FFT-OFDM for 32-PSK.

Figure 4(c) the simulation is carried out for 8-PSK and in this scenario also the wavelet based OFDM performs better than the Fourier based OFDM in the presence of CFO. In Figure 4(d) the modulation level used is 16-PSK, here also the BER performance for wavelet based OFDM is better than the Fourier based OFDM. Similar observation can be made from Figure 4(e), which shows the simulations for 32-PSK. The channel for transmission is

considered to be Rayleigh, which suffers from multipath fading as there no line of sight between both sections.

## 6. Conclusion

Simulations are done on the basis of simulation parameters defined. As the CFO rises in the system the relative error rate to that values also increases. It is seen from the graphs that the rate of error change is less between the DWT and the FFT OFDM, but the nature of the DWT it is concluded that the DWT based system gives better performance and less overhead over the FFT-based system. So after performing simulations by considering different parameters for both the systems, a conclusion on the performance of both the system is made, and it is concluded that DWT-OFDM gives better and improved performance over the conventional FFT-OFDM in the presence of CFO. Further, this work can be extended to different channels and different simulation parameters. Also with those parameters and channel conditions different CFO mitigation techniques can also be adopted.

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