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A Note on Bipolar Perfect Fuzzy Matching

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Abstract

The notion of matching in a fuzzy graph could be defined using the concept of effective edges[7] and fractional matching[3]. In this paper, we introduce the notion of bipolar fuzzy matching and bipolar perfect fuzzy matching of a bipolar fuzzy graph and prove some results.

Keywords: Bipolar Fuzzy Graph, Bipolar Fuzzy Matching, Bipolar Fuzzy Perfect Matching,

AMS Classification: 05C72, 03E72

1. Introduction

In 1965, [9]introduced the notion of fuzzy sets. It has been generalized by many researchers such as L-fuzzy sets by Gougan and Intuitionistic fuzzy sets by Atanasov. One such generalization is the concept of bipolar fuzzy sets by[8] in 1994.

In 1975, [4]introduced the notion of fuzzy graphs. In 2011, [1]introduced the notion of bipolar fuzzy graphs. Using the concept of effective edges, 7defined matching in a fuzzy graph. [3]Introduced the notion matching in a fuzzy graph using the concept of fractional matching.

In our earlier paper[5], we discussed the concept of perfect fuzzy matching on some fuzzy graphs. In[6], we defined intuitionistic perfect fuzzy matching and discussed some results. In this paper, we introduce the notion of bipolar fuzzy matching and bipolar perfect fuzzy matching of a bipolar fuzzy graph and we prove some results.

2. Preliminaries

In this section, we introduce some basic definitions that are required in the sequel.

Definition 1: [4]A fuzzy graph $G = (\sigma,\mu)$ is a pair of functions $\sigma: V \to [0,1]$ and $\mu: V X V \to [0,1]$ with $\mu(u,v) \leq \sigma(u) \square \sigma(v)$, $\forall$ u,v $\in$ V, where V is a finite nonempty set and $\square$ denote minimum.

Definition 2. [8]Let X be a non-empty set. A bipolar fuzzy set B on X is an object having the form: $B = \{ (x, m+ (x), m- (x)) / x \in X \}$, where m+ : $X \to [0,1]$ and m- : $X \to [-1,0]$ are mappings. For the sake of simplicity, we shall use the symbol $B = (m^+ , m^-)$ for the bipolar fuzzy set.
$B = \{ (x, m^+ (x), m^- (x)) / x \in X \}$.

Definition 3. [2]A bipolar fuzzy graph with underlying set (V, E) is defined to be the pair G = (A, B) where $A = (m_A^+ , m_A^-)$ is a bipolar fuzzy set on V and $B = (m_B^+ , m_B^-)$ is a bipolar fuzzy set on $E \subset V X V$ such that $m_B^+ (x, y) \leq \min\{ m_A^+(x), m_A^+(y)\}$ and $m_B^- (x, y) \geq \max\{ m_A^-(x), m_A^-(y)\}$ $\forall$ (x,y) $\in$ E.

Definition 4: [2]A bipolar fuzzy graph G = (A, B) is said to be strong if
$m_B^+ (x, y) = \min\{ m_A^+(x), m_A^+(y)\}$ and $m_B^- (x, y) = \max\{ m_A^-(x), m_A^-(y)\}$ $\forall$ (x,y) $\in$ E.
The graph G is called a complete bipolar fuzzy graph if
$m_B^+ (u, v) = \min\{ m_A^+(u), m_A^+(v)\}$ and $m_B^- (u, v) = \max\{ m_A^-(u), m_A^-(v)\}$ $\forall$ u, v $\in$ V.

Definition 5: [2]Let G = (A, B) be a bipolar fuzzy graph where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ be

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two bipolar fuzzy sets on a nonempty finite set V and E $\subset$ VXV respectively.

The positive degree of a vertex u in G is denoted by $d^+(u)$ and defined as $d^+(u) = \sum_{(u,v)\epsilon E} m_2^+(u,v)$. The negative degree of a vertex u in G is denoted by $d^-(u)$ and defined as $d^-(u) = \sum_{(u,v)\epsilon E} m_2^-(u,v)$. The degree of a vertex is $d(u) = (d^+(u), d^-(u))$.

If $d^+(u) = k_1$ and $d^-(u) = k_2$, for all u $\in$ V and k_1, k_2 are two real numbers, then the graph is called (k_1, k_2) regular bipolar fuzzy graph.

Definition 6: [3]Let G = (σ, μ) be a fuzzy graph on (V, E) where V is the vertex set and E is the set of edges with non-zero weights. A subset M of E is called a fuzzy matching if for each vertex u $\in$ V,

$$\sum \mu(u,v) \leq \sigma(u).$$

Further, a subset M of E is called a perfect fuzzy matching if for each vertex u $\in$ V,

$$\sum_{v\epsilon V} \mu(u,v) = \sigma(u).$$

3. Bipolar Fuzzy Matching

In this section, we introduce the notion of bipolar fuzzy matching and bipolar perfect fuzzy matching and prove some results.

Definition 7: Let G = (A, B) be a bipolar fuzzy graph where A = (m_1^+, m_1^-) and B = (m_2^+, m_2) be two bipolar fuzzy sets on a nonempty finite set V and E $\subset$ VXV respectively. If

$\sum_{(u,v)\epsilon M} m_2^+(u,v) \leq m_1^+(u)$ and
$\sum_{(u,v)\epsilon M} m_2^-(u,v) \geq m_1^-(u), \forall\, u\ \in V,$

then M is said to be bipolar fuzzy matching in G.

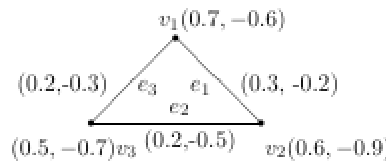
M is said to be bipolar perfect fuzzy matching if:

$\sum_{(u,v)\epsilon M} m_2^+(u,v) = m_1^+(u)$ and
$\sum_{(u,v)\epsilon M} m_2^-(u,v) = m_1^-(u), \forall\, u\ \in V.$

Definition 8: Let G be a bipolar fuzzy graph on the underlying graph (V, E). Let M be a bipolar fuzzy matching for G. Then bipolar fuzzy matching number $\Gamma(G)$ is defined as:

$$\Gamma(G) = \Big(\sum_{(u,v)\epsilon M} m_2^+(u,v), \sum_{(u,v)\epsilon M} m_2^-(u,v)\Big).$$

Example 1: Consider the following bipolar fuzzy graph G on (V, E) where V = {v_1,v_2,v_3} and E = {e_1, e_2, e3}.



E = {e_1,e_2}is a bipolar fuzzy matching for G.

For $\sum_{(v_1,v_2)\epsilon M} m_2^+(v_1,v_2) = m_2^+(v_1,v_2) + m_2^+(v_1,v_3) =$ 0.3+0.2=0.5 $\leq$ 0.7 = $m_1^+(v_1)$

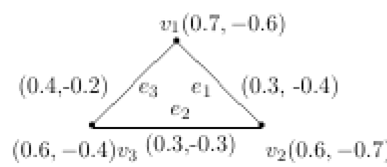
$\sum_{(v_2,v_3)\epsilon M} m_2^+(v_2,v_3) = m_2^+(v_2,v_3) + m_2^+(v_2,v_1) =$ 0.3+0.2=0.5 $\leq$ 0.6 = $m_1^+(v_2)$

$\sum_{(v_1,v_2)\epsilon M} m_2^-(v_1,v_2) = m_2^-(v_1,v_2) + m_2^-(v_1,v_3) =$ -0.2+-0.3=-0.5 $\geq$ - 0.6 = $m_1^-(v_1)$

$\sum_{(v_2,v_3)\epsilon M} m_2^-(v_2,v_3) = m_2^-(v_2,v_3) + m_2^-(v_2,v_1) =$ -0.5+-0.2=-0.7 $\geq$ - 0.9 = $m_1^-(v_2)$

Here $\Gamma(G) =$ (0.5,0.7).

Example 2: Consider the following bipolar fuzzy graph G on (V, E).



$\sum_{(v_1,v_2)\epsilon M} m_2^+(v_1,v_2) = m_2^+(v_1,v_2) + m_2^+(v_1,v_3) =$ 0.3+0.4=0.7 = $m_1^+(v_1)$

$\sum_{(v_2,v_3)\epsilon M} m_2^+(v_2,v_3) = m_2^+(v_2,v_3) + m_2^+(v_2,v_1) =$ 0.3+0.3=0.6 = $m_1^+(v_2)$

$\sum_{(v_1,v_2)\epsilon M} m_2^-(v_1,v_2) = m_2^-(v_1,v_2) + m_2^-(v_1,v_3) =$ -0.2+-0.4=-0.6= $m_1^-(v_1)$

$\sum_{(v_2,v_3)\epsilon M} m_2^-(v_2,v_3) = m_2^-(v_2,v_3) + m_2^-(v_2,v_1) =$ -0.3+-0.4=-0.7 = $m_1^-(v_2)$

Thus $\{v_1v_2, v_2v_3\}$is a bipolar perfect fuzzy matching.

Here, $\Gamma(G) =$(0.6,0.7).

Theorem 1: Let G = (A, B) be a bipolar fuzzy graph on the cycle (V, E). If $m_2^+(u,v)$ = constant = k1, (say) and $m_2^-(u,v)$ = constant = k2, (say), $\forall (u,v) \in E$ and if $m_1^+(u) = 2k_1$ *and* $m_1^-(u) = 2k_2$, then M = E is a bipolar perfect fuzzy matching.

Proof:

Since only two edges are incident with each vertex for a cycle, for any vertex $u \in V$,

$$\sum_{(u,v)\in E} m_2^+(u,v) = m_2^+(u,v) + m_2^+(u,w) \quad ,$$

where $v, w \in V$.

$= k_1 + k_1 = 2k_1 = m_1^+(u)$

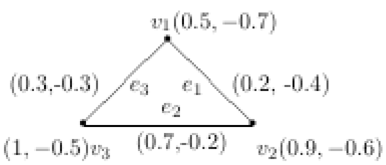
$$\sum_{(u,v)\in E} m_2^-(u,v) = m_2^-(u,v) + m_2^-(u,w)$$

$= k_2 + k_2 = 2k_2 = m_1^-(u)$

Therefore E is a bipolar perfect fuzzy matching in G.

The converse of the above theorem need not be true. This can be seen from the following example.

Example 3: Consider the following bipolar fuzzy graph G on (V, E) .



Here E is bipolar perfect fuzzy matching in G but the conditions of the above theorem are not satisfied.

Theorem 2: Let G = (A, B) be a bipolar fuzzy graph on a complete graph (V, E). If $m_1^+(u)$ = constant = k_1, (say) and $m_1^-(u)$ = constant = k_2, (say), $\forall (u,v) \in E$ and if $m_2^+(u,v) = \frac{k_1}{n} = k_3$ *and* $m_2^-(u,v) = \frac{k_2}{n} = k_4$ (say), $\forall\ (u,v)$ on the cycle C_n **and** $m_2^+(u,v) = \frac{k_1 - 2k_3}{n-3}$ *and* $m_2^-(u,v) = \frac{k_2 - 2k_4}{n-3}, \forall$ edges not on the cycle C_n then M = E is a bipolar perfect fuzzy matching.

Proof

Let G = (A, B) be a bipolar fuzzy graph on a complete graph K_n on (V, E).

For any complete fuzzy graph K_n two edges are incident with each vertex of the cycle and remaining (n-3) edges are incident with the interior vertices.

Hence,

$$\sum_{(u,v)\in E} m_2^+(u,v) = 2k_3 + (n-3)\left(\frac{k_1 - 2k_3}{n-3}\right)$$

$= 2k_3 + k_1 - 2k_3 = k_1 = m_1^+(u)$, for each vertex u.

$$\sum_{(u,v)\in E} m_2^-(u,v) = 2k_4 + (n-3)\left(\frac{k_2 - 2k_4}{n-3}\right)$$

$= 2k_4 + k_2 - 2k_4 = k_2 = m_1^-(u)$.

Therefore E is a bipolar perfect fuzzy matching in G.

The converse of the above theorem need not be true. This can be seen from the Example 3.

Theorem 3: Let G = (A, B) be a bipolar fuzzy graph on a star graph (V, E), where V = $\{v, v_1, v_2, \dots, v_{n-1}\}$. If $m_1^+(v_i)$ = constant = k_1, (say) and $m_1^-(v_i)$ = constant = k_2, (say), $\forall\ i = 1, 2, \dots, n$ and if $m_1^+(v) = (n-1)k_1$ *and* $m_1^-(v) = (n-1)k_2$, then E is a bipolar perfect fuzzy matching for G.

Proof:

For all

$v_i \in V$, $\quad m_2^+(v, v_i) = \min\{m_1^+(v), m_1^+(v_i)\}$

$= \min\{(n-1)k_1, k_1\} = k_1$

$m_2^+(v, v_i) = \max\{m_1^-(v), m_1^-(v_i)\}$

$= \min\{(n-1)k_2, k_2\} = k_2$

$\sum m_2^+(v, v_i) = \sum_{(v,v_i)\in E} k_1 = (n-1)k_1$ (Since (n-1) edges are incident with v)

$= m_1^+(v)$

$\sum m_2^-(v, v_i) = \sum_{(v,v_i)\in E} k_2 = (n-1)k_2$

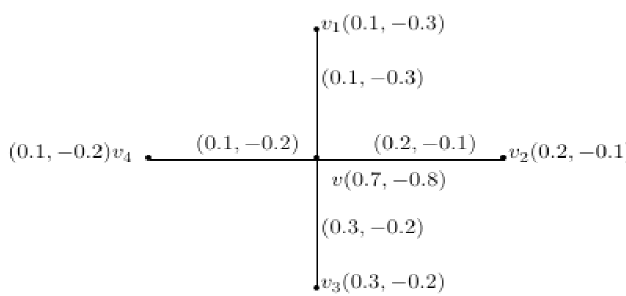
(Since (n-1) edges are incident with v)

$= m_1^-(v)$

Thus E is a bipolar perfect fuzzy matching in G.

The converse of the above theorem need not be true. This can be seen from the following:

Example 4.



Here E is a bipolar perfect fuzzy matching. But the conditions of the above theorem are not true.

The following theorem establishes that a strong regular bipolar fuzzy graph need not have a bipolar perfect fuzzy matching in G.

Theorem 4: Let G = (A, B) be a strong regular bipolar fuzzy graph on (V, E), with each vertex is of degree at least two. Then E is not a bipolar perfect fuzzy matching for G.

Proof:

Suppose E is a bipolar perfect fuzzy matching for G.

Then $\sum_{(u,v)\in E} m_2^+(u,v) = m_1^+(u), \forall\, u \in V$ and

$$\sum_{(u,v)\in E} m_2^-(u,v) = m_1^-(u), \forall\, u \in V.$$

Since G is regular, $\sum_{u\neq v} m_2^+(u,v) =$ constant = k_1 (say) and $\sum_{u\neq v} m_2^-(u,v) =$ constant = k_2 (say).

Therefore, $m_1^+(u) = k_1$ and $m_1^-(u) = k_2, \forall\, u \in V$.

Since each vertex is of degree atleast two, $m_2^+(u,v) < k_1$.

Therefore, $\min\{m_1^+(u), m_1^+(v)\} < k_1$, since G is strong.

$\Rightarrow \min\{k_1, k_1\} < k_1 \Rightarrow k_1 < k_1$, a contradiction.

Thus E is not a bipolar perfect fuzzy matching for G.

4. Conclusion

In this paper we have derived a sufficient condition for a bipolar fuzzy graph on a cycle or a complete graph or a star graph to have a bipolar perfect fuzzy matching. This could be extended for fractional fuzzy matching.

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