

Energy Distribution of Radiation Emitted by a Black Body - Independent of Probability

Devinder Kumar Dhiman*

Damsgardsveien 165, box 1925, 5160 Laksevag, Bergen, Norway. dhimandk@rediffmail.com

Abstract

Background/Objective: To demonstrate that the formula for Energy distribution of radiation emitted from the cavity of a black-body, derived by German physicist Max Planck in the year 1900, has an alternative explanation independent of probability. **Method:** Energy released by radiations emitted from the cavity of a black-body, is compared with the energy released from an incandescent electric bulb. Electrical energy added to the bulb is envisaged to transform into energy-sphere that breaks-up into small packets of energy akin to radioactive decay of matter, and generates electromagnetic radiations causing brightness. **Finding:** Conjoining the quantization of energy and the technique of radioactive decay, a formula is derived for the brilliance of the incandescent bulb. The formula, thus derived, is found to be identical to the formula derived by Max Planck. This finding leads us to a unique outcome that the assumption of oscillators inside the cavity of black-body is not a requisite for deriving the formula as suggested by Max Planck, and believed till date. Elimination of oscillators also makes Boltzmann's statistical method, which was utilized by Max Planck in his derivation, irrelevant to the formula, thereby making it independent of probability. Use of probability in physics leads to uncertainties and weirdness which is evident in quantum physics. This technique eliminates the probability from this basic formula of quantum physics and introduces a distinctive mode of generation of electromagnetic radiations from energy-spheres. **Application/Improvements:** This technique can further be explored and applied in ascertaining the behavior of electrons and other sub-atomic particles without the usage of probability.

Keywords: Black-body, Probability, Quantization, Quantum Physics, Radiation

1. Introduction

Towards the end of nineteenth century, with the advent of incandescent electric bulbs, a need was felt to investigate the emission of light from electrically heated filament of bulb so as to give the best result with minimum input of electrical energy. This requirement led scientists to experiment with radiation emitted from the black-body as this is identical to radiation emitted by a luminous bulb.

Data was compiled for radiation emitted at various frequencies and temperatures of the black-body. From that data, radiation curves for energy emitted over a wide range of wavelength at different absolute temperatures were drawn as shown in Figure 1. Analysis of the data helped scientists to derive formula for the calculation of the energy of radiation emitted at various frequencies and

different temperatures of the black-body. An empirical formula¹ was devised by Wilhelm Wien in 1893 as

$$I(\nu, T) = \frac{2h\nu^3}{c^3} e^{-h\nu/kT}, \quad (1)$$

where, $I(\nu, T)$ is the amount of energy emitted at a frequency ν , T is absolute temperature of the black-body, h is Planck constant, c is speed of light, and k is Boltzmann's constant.

But this formula did not provide correct values at low frequencies of the radiation emitted, and lacked theoretical explanation.

In 1900, Lord Rayleigh derived formula for energy of radiation emitted from black-body, using classical electromagnetic theory, and presented his formula² as

*Author for correspondence

$$I = \frac{8\pi\nu^2}{c^3} kT. \quad (2)$$

But this formula failed at high frequencies of radiation, causing a condition known as ‘ultraviolet catastrophe’, where the energy contained in the radiation was divergent and reached infinity value for the radiation in ultraviolet region, as shown in Figure 2. Red line in the figure represents deviation of values obtained from Rayleigh-Jean formula, green line represents that of Wilhelm Wein’s approximation, whereas actual values obtained experimentally are represented by black lines.

Though Lord Rayleigh’s formula was not accurate, yet through his theoretical explanation, he provided a noteworthy concept that the modes of the standing waves formed inside the cavity of a black-body should behave analogous to the particles in an ideal monoatomic gas in thermal equilibrium. Conjoining this concept with Wilhelm Wein’s empirical formula, Stefan Boltzmann’s

statistical method and his own revolutionary idea of quantization of energy, Max Planck derived another formula³,

$$B = \frac{8\pi\nu^3}{c^3} \frac{1}{e^{\hbar\nu/kT} - 1}. \quad (3)$$

This formula gives correct values, but it is based on Boltzmann’s statistical method, which involves averaging out of a large data of oscillators and modes of standing waves. It does not tell us what is happening at the individual level of oscillators.

Therefore, in order to understand the proceedings at the smallest level, in this article, we discuss the derivation of this formula through a technique that does not require large data and is not dependent on probability, such that, it can be applied to the smallest level of source capable of emitting radiation. Technique used involves investigation of the conversion of electrical energy supplied to a luminous bulb, to light energy, and then its fragmentation and dissipation as radiation.

2. Incandescent Bulb

For more than a century, energy distribution of radiation emitted by a black-body has always been derived by the well-established method of assumption of oscillators and standing waves inside the black-body⁴, and we can add nothing more to that to make it independent of probability. But light emitted from a luminous bulb, being equivalent to black-body radiation, should be able to give us the same formula without the assumption of oscillators and standing waves. Therefore, we consider an incandescent bulb as represented in Figure 3. When the bulb filament is heated by electrical energy input, the bulb illuminates, and it has brilliance B_1 at a radial distance d_1 from the filament, at absolute temperature T_1 of the filament. Brilliance is equal in all directions, so it can be represented with a sphere around the bulb as shown around first bulb in Figure-3, and in this article, we shall call it energy-sphere.

After that, when the bulb is supplied with additional electrical energy, the brilliance spreads up to a radial distance d_2 , as shown by second bulb in Figure-3, where a second energy-sphere superimposes on the first energy-sphere. And with additional electrical energy supply, values of brilliance at distances d_1 , d_2 and d_3 increase to B_3 , B_2 and B_1 respectively, as shown by the three bulbs in Figure3.

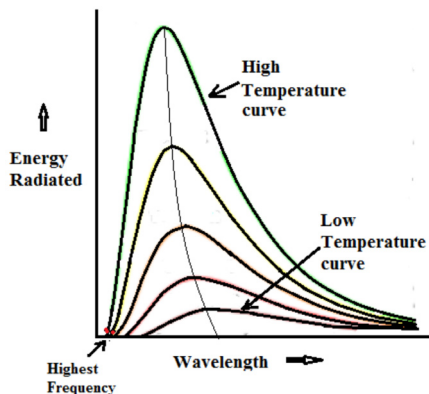


Figure 1. Curves of distribution of energy over wavelength at different absolute temperatures.

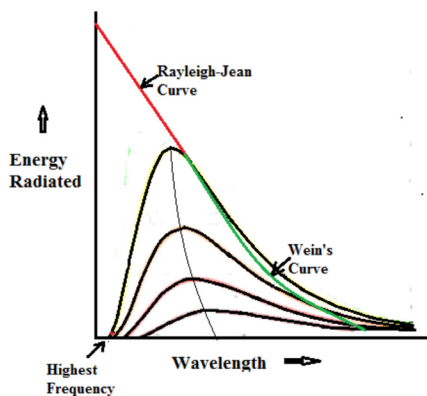


Figure 2. Deviation of Wein’s and Rayleigh-Jean’s curves from experimentally observed values.

Following the same pattern, we can envisage that new energy-spheres get added over the previous energy-spheres, thus the brilliance becomes maximum near the bulb filament, and decreases with radial distance from the bulb, in accord with Inverse Square Law of light⁵.

3. Addition of Energy

For the bulb with brilliance B_1 spread up to a distance d_1 , let the total energy supplied is E_1 , absolute temperature of the filament is T_1 , volume of energy-sphere is V_1 , and highest frequency (the frequency in the radiation curve at a point on the curve where it just touches the horizontal axis at the left most part of the curve. It is different from the frequency represented by peak of the curve) of radiation emitted is ν_1 , as shown by left-hand side sphere in Figure 4.

Then heat energy E is provided electrically to increase the total energy of the bulb to E_2 ; absolute temperature of filament will rise to T_2 , volume of energy-sphere will

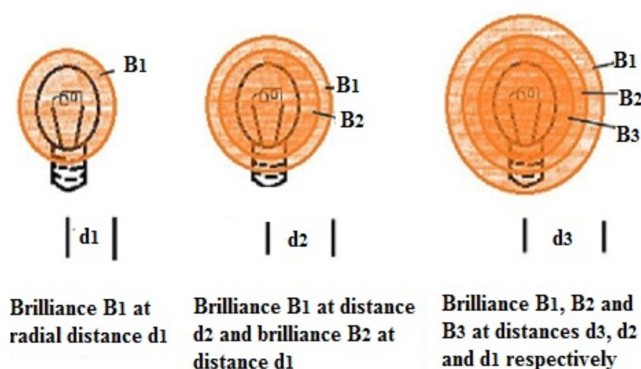


Figure 3. Brilliance of a bulb at different energy inputs.

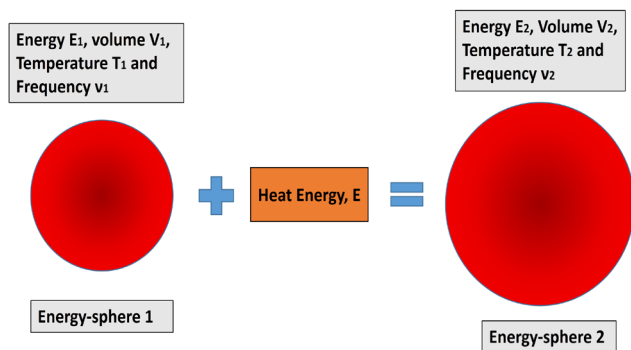


Figure 4. Conversion of Energy-sphere-1 to Energy-sphere-2 due to addition of electrical energy E .

become V_2 , and the bulb will emit radiation of highest frequency ν_2 , as shown by sphere on right-hand side in Figure 4.

4. Discarding of Additional Energy

When the bulb filament is in thermal equilibrium, the electrical energy supplied should be equal to the energy dissipated through heat and light radiation. Added energy E should, thus, get discarded completely through radiation after dissociating itself from E_2 as shown in Figure 5. The total energy of the bulb should, then, fall back to E_1 as it was before addition of energy E .

5. About Energy-spheres

Highest frequency of the radiation emitted by Energy-sphere-2 and Energy-sphere-E should be equal because supply of energy E enabled Energy-sphere-2 to emit radiation of frequency ν_2 .

And according to Rayleigh-Jean's law, the total energy of radiation over complete range of emission³ is $\frac{8\pi\nu^3}{c^3}kT$. Therefore, energy of an energy-sphere is proportional to the cube of the frequency of the radiation emitted.

Volume of the energy sphere increased from V_1 to V_2 , in same proportion as the energy increased, therefore, the volume of the energy-sphere should also be proportional to the cube of the frequency.

Above reasoning shows a similarity between energy contained and volume occupied by an energy-sphere, but there is a significant difference between these two values. Value of energy in radiation curve starts reducing after reaching a peak value, as the frequency increases, but the

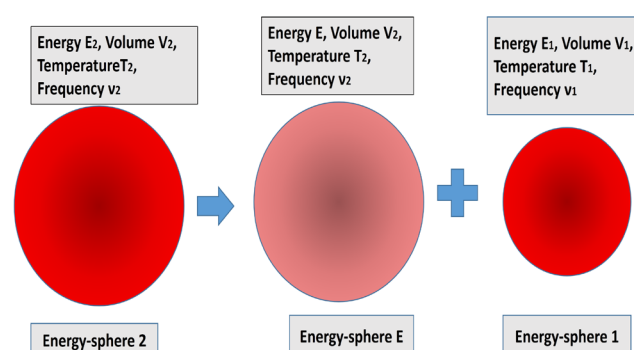


Figure 5. Division of Energy-sphere-2 into Energy-sphere-E and Energy-sphere-1.

volume occupied by energy sphere should keep increasing proportional to cube of frequency. The reason being that the energy emitted by the black-body is dependent on cube of frequency and amount of radiation emitted. But the volume occupied by the energy sphere should depend only on the cube of frequency.

6. Disintegration of Energy-sphere-E

For reduction of Energy-sphere-2 back to Energy-sphere-1, the Energy-sphere-E should vanish completely, which is envisaged to happen by breaking of Energy-sphere-E into circumferential parts of energy e_1, e_2, e_3 etc. as shown in Figure 6, and escape in the form of radiation.

Because mass and energy are equivalent as per Einstein's famous equation⁶ $E = mc^2$, the technique of mass disintegration of radioactive substances can be applied here also for calculation of the value of energy of the disintegrated parts of Energy-sphere-E.

A radioactive substance disintegrates in such a way that it reduces to half of its original mass in a certain time period, which is termed as its half-life. If the reduction factor has any other value instead of half, then the number of steps for reduction change accordingly, for arriving at same final value.

Let us assume the reduction factor for total disintegration of Energy-sphere-E as a (instead of considering 2, as in half-life), and the number of steps as n . Then the ratio of energy values of Energy-sphere-2 and Energy-sphere-1, for complete disintegration of Energy-sphere-E into small packets of energy e_1, e_2, e_3 etc. can be written as

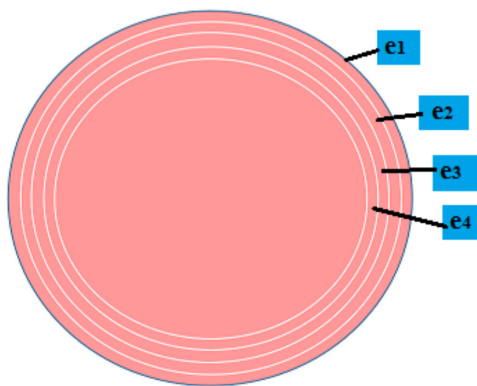


Figure 6. Circumferential partition of Energy-sphere-E.

$$\frac{E_2}{E_1} = a^n \quad (4)$$

During disintegration, the first packet of energy released will be e_1 , which shall be accompanied with a radiation of the highest frequency ν_2 . After the release of packet of energy e_1 the size of the Energy-sphere-E will decrease, therefore, second part e_2 , when released, will emit highest frequency less than ν_2 by an integral number.

Packets of energies e_1, e_2, e_3 etc. may not necessarily result in a reduction of frequency one by one, they may do so by any other integral number ε . So, the radiation of highest frequency $(\nu_2 - \varepsilon)$ should be emitted with release of second part of energy e_2 , and radiation of highest frequency $(\nu_2 - 2\varepsilon)$ with third part e_3 , and so on.

Following the above pattern, highest frequency of radiation emitted with release of energy e_N in N_{th} step can be represented by the formula

$$\{\nu_2 - (N-1)\varepsilon\} \quad (5)$$

7. Number of Steps of Disintegration

Highest frequency of the emitted radiation decreases by a fixed number ' ε ', every time a packet of energy gets dissociated from the Energy-sphere-E until it vanishes completely, therefore, the number of steps in the fragmentation of the Energy-sphere-E can be written as $n = \frac{\nu_2}{\varepsilon}$.

Bulbs at high temperature of the filament radiate more energy than at low temperature. It implies that the size of the steps in the reduction of the energy is larger at high temperature than at low temperature. In other words, we can say ε is proportional to absolute Temperature. Therefore, $\varepsilon = \mu T_2$ where μ is a constant and T_2 is absolute temperature.

From the two paragraphs, we can deduce that $n = \frac{\nu_2}{\mu T_2}$.

8. Energy Ratio

Substitute the value of n in equation (4) to get:

$$\frac{E_2}{E_1} = a^{\nu_2/\mu T_2} \quad (6)$$

After release of packets of energy e_1, e_2, e_3, e_4 etc. from the Energy-sphere-E, the ratio of energies can be written as:

$$\frac{E_2 - e_1}{E_1} = a^{\frac{\nu_2}{\mu T_2} - 1},$$

$$\frac{E_2 - (e_1 + e_2)}{E_1} = a^{\frac{\nu_2}{\mu T_2} - 2},$$

$$\frac{E_2 - (e_1 + e_2 + e_3)}{E_1} = a^{\frac{\nu_2}{\mu T_2} - 3}, \text{ and so on.}$$

Thus, after release of N packets of energy, the ratio will become

$$\frac{E_2 - \sum e_i}{E_1} = a^{\frac{\nu_2}{\mu T_2} - N}, \text{ where } \sum e_i \text{ denotes the}$$

summation of e_1, e_2, e_3, e_4 etc. for all values of i from 1 to N .

Substitute $E_2 = E_1 + E$ in the above equation to get:

$$\frac{E_1 + E - \sum e_i}{E_1} = a^{\frac{\nu_2}{\mu T_2} - N} \quad (7)$$

After rearranging the terms, the equation changes to

$$\frac{E - \sum e_i}{E_1} = a^{\frac{\nu_2}{\mu T_2} - N} - 1 \quad (8)$$

The exponential of a in equation (8) can be rewritten as

$$\frac{\nu_2 - N\mu T_2}{\mu T_2}$$

Since $\varepsilon = \mu T_2$, we can rewrite the term as $\frac{\nu_2 - N\varepsilon}{\mu T_2}$

After N times reduction of frequency in steps of ε from maximum frequency ν_2 , we get the current frequency $\nu = \nu_2 - N\varepsilon$. Therefore, the exponential term will become equal to $\frac{\nu}{\mu T_2}$.

Substituting this value of exponential of a in equation 8, we get:

$$\frac{E - \sum e_i}{E_1} = a^{\frac{\nu}{\mu T_2} - 1} \quad (9)$$

For all values of $E_1 \neq 0$, we can inverse the equation (9) completely,

$$\frac{E_1}{E - \sum e_i} = \frac{1}{a^{\frac{\nu}{\mu T_2} - 1}} \quad (10)$$

This is the energy quotient of the Energy-sphere-1 and the residual energy in the Energy-sphere-E after N time disintegration and dissipation of energy.

9. Energy Density

We have learned so far that after N times disintegration of the Energy-sphere-E, the energy packet released will be able to emit highest frequency ν , therefore volume of the Energy-sphere before the release of the packet of energy e_N should be $V = \gamma \nu^3$, where γ is a constant, and its energy should be $E - \sum e_i$, where, $i = N$.

Therefore, the ratio of the residual energy and residual volume of the Energy-sphere-E after N time disintegration can be written as

$$\rho_e = \frac{E - \sum e_i}{\gamma \nu^3}, \quad (11)$$

where, ρ_e denotes energy density.

This energy density can be envisaged to behave akin to mass density as mass and energy are equivalent. It is easier to break a part of a substance having less density than the one having more density, as the constituents of matter have less attraction towards each other than in the latter case. Therefore, if the energy density of the Energy-sphere-E increases with its disintegration, it will become more difficult for the Energy-sphere-E to disintegrate further, and as such, less radiation will escape from the bulb. Thus, the escape of energy that causes radiation, or the brilliance, should be inversely proportional to the energy density, i.e. $B \propto \frac{1}{\rho_e}$.

Disintegrated parts of energy from the Energy-sphere-E, will scatter away from the bulb as emitted radiation due to repulsion from the energy of Energy-sphere-1 present at the bulb. Therefore, the radiated energy should be proportional to the energy of that sphere, i.e. $B \propto E_1$.

10. Brilliance of the Bulb

From the combination of above two proportionalities, we have the relation $B \propto \frac{E_1}{\rho_e}$.

After introducing the proportionality constant β , equation becomes

$$B = \beta \frac{E_1}{\rho_e} \quad (12)$$

Substitute the value of ρ_e from equation (11) in equation (12) to get:

$$B = \frac{\beta E_1}{(E - \sum e_i) \gamma v^3} \quad (13)$$

After rearranging, it becomes:

$$B = \frac{\beta \gamma v^3 E_1}{E - \sum e_i} \quad (14)$$

Substituting the value of $\frac{E_1}{E - \sum e_i}$ from equation (10)

in equation (14) and changing the constant $\beta \gamma$ to K , we get:

$$B = \frac{K v^3}{a^{v/\mu T_2} - 1} \quad (15)$$

Since a is a constant, we can replace it with e^η , where η is another constant. Thus,

$$B = \frac{K v^3}{e^{\eta v/\mu T_2} - 1} \quad (16)$$

Comparing equation (16) with formula in equation (3), we find that both equations become identical at $T = T_2$, if we replace K with $8\pi h/c^3$, and η/μ with h/k . We can do this replacement as they all are constants.

Thus the formula in equation (3) can be derived without the assumption of the oscillators and standing-waves inside the black-body, through an entirely distinctive concept of formation, disintegration and dissipation of energy-spheres, when energy is supplied to bulb filament.

11. Conclusion

For derivation of the formula for energy distribution of radiation emitted by a black-body, we have pursued an approach in which additional energy supplied to the luminous bulb to maintain the temperature, results in emission

of heat and light radiation. Additional energy expands the existing energy-sphere present around the bulb, and from that an energy-sphere having an energy equal to supplied energy gets released and promptly breaks up circumferentially into parts of energy. These energy fragments are simultaneously pushed away as radiation by the existing energy-sphere around the bulb. This gives us a technique of the disintegration of the energy similar to radioactive disintegration of matter, which has been mathematically constructed step by step in order to present a derivation for the formula for the distribution of energy of a black-body- independent of probability.

Since the size of the bulb can be imagined as small as possible, the technique can be applied to the smallest level of source of radiation emission, without requirement of a large data of oscillators and standing waves. Further research and development of this technique of derivation of the formula for distribution of energy of radiation emitted- independent of probability, may be in harmony with the words of the great scientist, Albert Einstein, "I, at any rate, am convinced that He does not play dice²."

12. References

1. Marr JM, Wilkin FP. A better presentation of Plank's radiation law. American Journal of Public Health. 2012 Apr; 80(5):2. DOI:10.1119/1.3696974.
2. Kramm G, Herbert F. Heuristic derivation of blackbody radiation laws using principles of dimensional analysis [Internet]. 2008 Mar 22. p.6. Available from: <http://arxiv.org/abs/0801.2197v2>.
3. Kramm G, Herbert F. Heuristic derivation of black-body radiation laws using principles of dimensional analysis, [Internet]. 2008 Mar 22. p.11. Available from: <http://arxiv.org/abs/0801.2197v2>.
4. Kumar M. Quantum. UK: Icon Books; Chapter 1, The Reluctant Revolutionary; 2008. p.3-30.
5. The inverse square law of light. [Internet]. NASA. Available from: https://www.nasa.gov/pdf/583137main_Inverse_Square_Law_of_Light.pdf.
6. Agarwal NS. New quantum theory. Indian Journal of Science and Technology. 2012 Nov; 5(11):2. DOI: 10.17485/ijst/2012/v5i11/30649.
7. Natarajan V. What Einstein meant when he said God does not play dice. Resonance. Journal of Science Education. 2008 Jul; 13(7):651-5.