

Reconstruction of hv-Convex Binary Images with Diagonal and Anti-Diagonal Projections using Genetic Algorithm

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Abstract

This paper is related to reconstruct the binary images from small number of projections. **Objectives:** To enhance the unambiguosness of image reconstructed. Other goal isto reduce the time of reconstruction of image. **Method:** In this paper, we put forward a new approach for reconstruction. Firstly, image is reconstructed using Chang's Algorithm and we utilize two projections first diagonal projections and second anti-diagonal projections for reconstruction. Then Genetic Algorithm is applied on image reconstructed after Chang's algorithm. **Findings:** The images reconstructed after Chang's algorithm and Genetic Algorithm are compared with original images and they are contrasted with each other also. We also calculate the percentage change of images reconstructed after Chang's Algorithm and Genetic Algorithm with original images. It is concluded that images reconstructed after Genetic Algorithm have less percentage difference than Chang's Algorithm. **Improvements:** The accuracy of images reconstructed after Genetic Algorithm is much better than reconstructed by Chang's Algorithm alone.

Keywords: Diagonal and Anti-diagonal Projections, Discrete Tomography, Genetic Algorithm, Hand Binary Images, hv-Convex

1. Introduction

Computer Tomography also known as Co-axial tomography is concerned with the complication of discovering the dimensional information and structure of an object from a set of projections. It necessitates tremendous number projection set to evoke dimensional information of internal part of an object and shape of an object¹. But it is not the case in the discrete tomography as it requires minute number of projections. This is the only reason that why it is observed as a special case of computerized tomography, in which the projection data is obtainable in very few lines (mostly two to four directions only). This method is used when

new projections for reconstructing an image are accessible. The primary issue in discrete tomography is recreation of discrete sets from insignificant number of projections.

In this paper we had taken into account the obstacle of recreation of a binary image with the help diagonal and anti-diagonal projections. The crucial benefit is that, we are able to grab more information about the image using diagonal and anti-diagonal projections only and furthermore, it can also lessen the number of possible solution.

The issue is that recreation of a binary image from little number of projections produce enormous number of solutions. So it became mandatory to reduce the solution space. Now to cope with this solution space, some

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priori information is used. This priori information is called constraint¹. Connectivity, convexity and periodicity are the some examples of these constraints.

In order to decrease the number of results we use convexity constraint in our work. The problem of reconstruction is converted to optimization with projections and a-priori information. The methods which are used to find solution of reconstruction problem are known as reconstruction algorithm which is based on the work of Chang. These algorithms are used to get initial solution, as they provide a large number of solutions for reconstruction problem. Finally, genetic algorithm is applied to optimize the solution.

2. Preliminary

2.1 Binary Matrix

Let $H = (h_1, \dots, h_m)$ and $V = (v_1, \dots, v_n)$ be the nonnegative integer vectors. Binary matrix can be represented by $BM(H, V)$ and class of all binary matrices $A = (a_{ij})$ satisfying: $\sum_{j=1}^n a_{ij} = h_i$ for $i = 1, \dots, m$ and $\sum_{i=1}^m a_{ij} = v_j$ for $j = 1, \dots, n$. Here, H and V are known as horizontal and vertical projections of any matrix in $BM(H, V)$ ². Figure 1 shows the binary matrix.

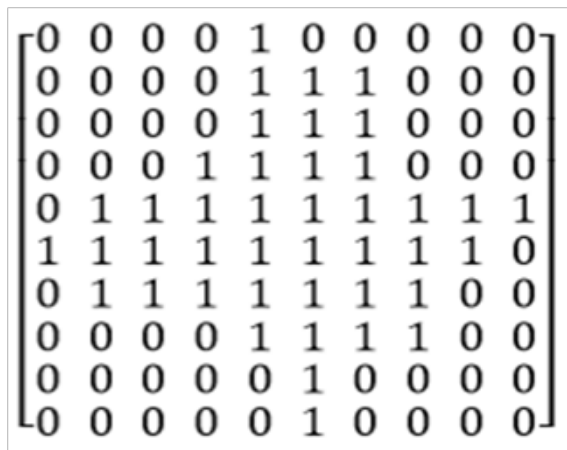


Figure 1. Binary matrix

2.2 Binary Image

Taken into account the two dimensional integer lattice $\mathbb{Z}^2$, and a non null finite subset X of $\mathbb{Z}^2$, called the discrete set, the elements of $\mathbb{Z}^2$ are ordered pairs $(i, j) \quad \forall i, j \in \mathbb{Z}$.

Thus, $X = \{(i, j) : 1 \leq i \leq m, 1 \leq j \leq n, m, n \in \mathbb{Z}\}$, is a discrete set. The function

$F : X \rightarrow \{0, 1\}$ is the binary image to be reconstructed, we denote by $\mathcal{F}$ the class of binary images satisfying the properties of application area. Some of these properties are known a priori and will be called constraints, for example convexity, connectivity, periodicity etc. Since the function consist only two values 0 or 1, thus the binary image is represented by binary matrix

$$F = (f_{ij})_{m \times n}, \text{ with } f_{ij} = F(X(i, j))^3$$

The Figure 2 represents the discrete set with the black dots denoting the points of discrete set on which the image function takes value 1, and the corresponding binary image in which black pixels represent 0 and 1 is shown by white pixels. Figure 2 represents the discrete set and binary image.

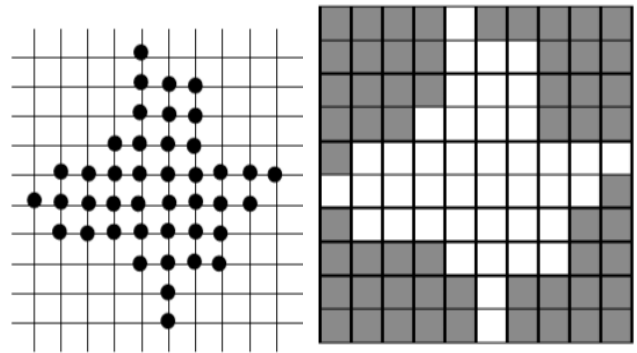


Figure 2. Discrete set and binary image

2.3 Projections

The projections are the sum of image points (1's in binary matrix) along the parallel lines in a few directions. We will consider the projection line through the pixel (i, j) of image $F = (f_{ij})_{m \times n}$ in the direction $v = (s, t)$ as a subset of X with all points common with straight line in the direction $v = (s, t)$ passing through the point (i, j) . This line is denoted by $L_k(v)$, so

$$L_k(v) = \{(i_1, j_1) : i_1 = i + ls, j_1 = j + lt, l \in \mathbb{Z}, \text{ for } k = 1, 2, \dots, k_v\}$$

Where k_v is number of lines in the directions $v = (s, t)$ thus the set of lines in direction v , is

$$\mathcal{L}(v) = \{L_k(v) : k = 1, 2, \dots, k_v\}^3$$

In this paper we considered two directions $v_1 = (1, -1)$ and $v_2 = (1, 1)$. These directions are known as diagonal and anti-diagonal directions.

A. Diagonal Projections

The projections along diagonal direction $v_1=(1,-1)$ is also referred as diagonal sum and defined as

$$P(\mathcal{L}(1, -1)) = D = \{d_1, d_2, \dots, d_{m+n-1}\} \quad \text{with } d_k = \sum_{(i,j) \in \mathcal{L}(1,-1)} f_{ij}$$

Figure 3 displays the binary image with the diagonal projections.

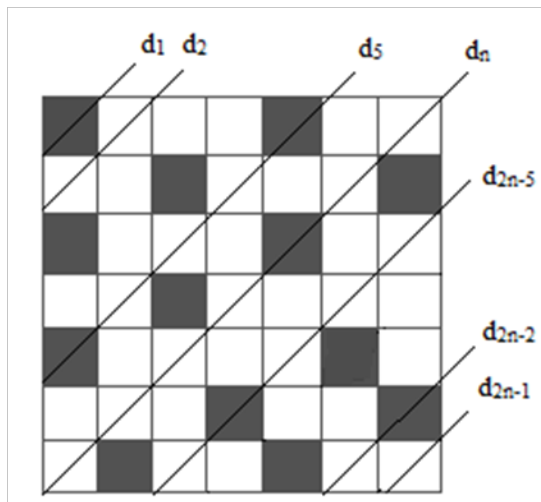


Figure 3. Binary Image With Diagonal Projections

B. Anti-Diagonal Projections

The projections along anti diagonal direction $(1,1)$ is also referred as anti diagonal sum and

$$P(\mathcal{L}(1, 1)) = A = \{a_1, a_2, \dots, a_{m+n-1}\} \quad \text{with } a_k = \sum_{(i,j) \in \mathcal{L}(1,1)} f_{ij}$$

Figure 4 displays the binary image with the anti-diagonal projections.

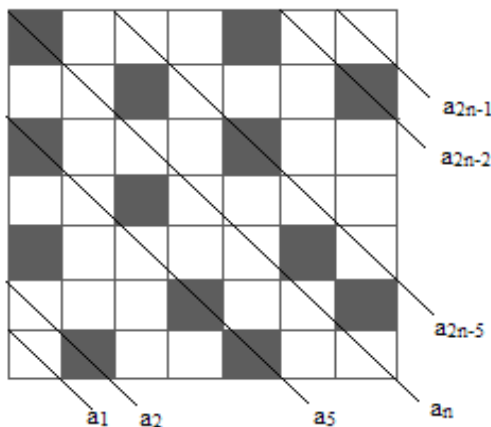


Figure 4. Binary image with anti-diagonal projections

The set of all projections available in given directions is called a projection set, for example the projection set with horizontal and vertical direction projections will be denoted by $P(R,C)$ with $R=\{r_1, r_2, \dots, r_m\}$, $C=\{c_1, c_2, \dots, c_n\}$ and with $P(R,D)$ we denote the projections set with $(1, 0)$ and $(1, -1)$ directions or projection set having row sum and diagonal sum, in general the projection set with ℓ directions is denoted as $P=\{P(\mathcal{L}(v_1)), P(\mathcal{L}(v_2)), \dots, P(\mathcal{L}(v_\ell))\}$.

2.4 Convexity

A binary matrix is said to be h-convex (horizontal convex) if the ones in each row form a continuous interval. Similarly a binary matrix is called v-convex (vertical convex) if the ones occurs consecutively in each column². The hv-convex (horizontal and vertical convex) binary matrix is matrix which is h-convex and v-convex. Figure 5 demonstrate the h-convex binary image, v-convex binary image and hv-convex binary image.

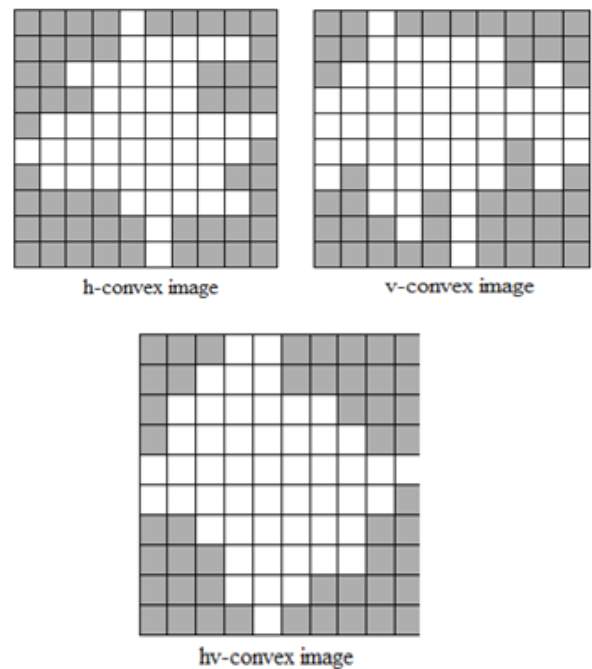


Figure 5. hv-Convex image

In¹ revealed again about problem of reconstruction of binary images using scarcely any number of projections. He mentioned that there are large numbers of solutions due to rare number of projections in reconstruction. To reduce the number of solutions some foregoing information about the object is required. This information is called

constraints. They use periodicity constraint to lessen the number of end result. They utilized Genetic algorithm for optimization. Genetic algorithm is used for very first time in the reconstruction with periodicity constraints.

In² described the issue of using projections horizontal and vertical in reconstruction of images which are hv-convex. To get the optimal solution, Simulated Annealing (SA) technique is used. Increasing the number of adjoining ones is considered as an objective function while reconstructing hv-convex image using simulated annealing algorithm.

In³ has discussed the reconstruction of binary images using only two projections. The new algorithm to regenerate binary images using diagonal and anti-diagonal projections is proposed and it is concluded that end results that are attained from proposed algorithm are good enough to accept in correspondence to any other algorithm.

In⁴ discussed about three reconstruction algorithms i.e. algorithm, Simulated Annealing and the algorithm that rely on the Location of Components. According to Algorithm, pace at which can be reconstructed and superiority of reconstruction pivot on the size of the image, and the number, the position, and the size of the components whereas this is not the case of Simulated Annealing it actually rely on the site where switching components are located in image. The switching component effect the time taken for reconstruction of image as well as efficiency of reconstruction algorithm. They finally concluded that fast reconstruction algorithm is combination of the switching operators, the localization of the components and the core-shell operators.

In⁵ performed reconstruction of image from its projection using genetic algorithm. Firstly binary image is converted into string and after that reproduction crossover and mutation operators are performed on image. Binary image is encoded string by putting pixels values of image in row order. Reproduction operator find the probability of survival of each new string in next generation and best string is selected according to likelihood. The operators such as crossover and mutation are used to create new strings. These three operations are repeated until stopping criteria meets that is if the number of iterations exceeds the maximum number. When this condition is satisfied then string at that point becomes the final solution. The image reconstructed using this method agrees well with the original image.

In⁶ illustrated that prominent problem in discrete tomography is to reconstruct discrete sets with the help of small number of projections and it lead to huge number of solutions. To decrease the number of solution, geometrical properties of set can be used. They applied connectivity restraint to attain polynomial-time recreation of hv convex discrete sets. Hv-convex discrete sets which have connectedness property can be reconstructed using various recursive formulas. Some statistics have been collected using these recursive formulas. These statistics can be used to analyze the performance of reconstruction algorithms.

In⁷ has proposed a new algorithm which costs utmost $O(\min(m,n)^2, mn \log mn)$ in reconstructing convex polyominoes which have size of $m \times n$ cells and their algorithm imposed on horizontal and vertical projections. They brought to an end that there is significant prominence of reconstruction of discrete set in innumerable issues such as pattern recognition, image processing and data compression.

In⁸ made comparison among three reconstruction algorithms that are used to regenerate hv-convex sets using row and column sums. The algorithms are compared from two perspectives first is the time that the algorithm takes to reconstruct the binary image and second is memory space and it has been concluded that the algorithm which was more effective from the perspective of worst time complexity was imperfect from the perspective of average time complexity and memory requirements. They combine two algorithms to implement a new method of reconstruction which inherits the best properties of previously implemented methods.

In⁹ had taken into account several versions of genetic algorithms. They construct large number of convergence statements and acquired theoretical estimates for their convergence. They performed a series of numerical experiments to discover empirically how the mean convergence rate of an algorithm depends on the parameters of the algorithm and compared this result with the theoretical conclusions.

In¹⁰ presented in their work the reconstruction of cranio-facial image with the help of GC¹ rational cubic Ball curves and they utilize three free parameters. They used genetic algorithm for their work and it is used for optimization of free parameters. And, finally they evaluated the errors and they illustrated that their suggested work is suitable for solving reconstruction problem in cranio-facial image.

3. Genetic Algorithm

Genetic Algorithm (GA), is inspired from Darwin's theory. The fundamental principle of genetic algorithm was first proposed by from Michigan University¹¹. It depends on natural selection. It is a stochastic search method. It is an evolutionary computing technique to find out the optimum solution. In genetic algorithm, the population of chromosomes or the genotype of genome, which encode the solution (called creatures, individuals etc.) and optimization problems evolves towards better solutions. The process of generation initiates from a population of arbitrary produced solutions and requires numerous steps to complete. In every single step the fitness of result is estimated where the multiple solutions are randomly selected, usually with higher probability of selecting more fit (having high fitness value) solutions. Then these solutions are arbitrary mutated generate a new population as next generation by modification or recombination of the results. Finally, the procedure of progress terminates either when an adequate fitness level has been achieved in the population or when adequate number of generations has been reached.

3.1 A Genetic Algorithm Necessitates

1. The genetic depiction of solution space, and
2. A fitness function which will make prediction about the improvements in the solution¹²⁻¹⁴.

The fitness function is forever rely totally on the nature of problem. Once we decide the genetic depiction as well as fitness functions after that the genetic algorithm gain to initialize a population of solutions arbitrary, then tend to refine through numerous steps of mutation, crossover, inversion and selection operator.

3.2 Genetic Algorithm in Image Reconstruction

GA is an evolutionary algorithm which is based on principles of evolution theory and same concept can be used in discrete tomography for image reconstruction. For using GA in image reconstruction, binary image is used as chromosome and each point in image is used as gene. We define the genetic representation of solution, fitness function, guided crossover, selection criteria, mutation, guided and mutation for developing genetic algorithm. For genetic representation of solutions in genetic

algorithm, each genome is an array that represents a binary image i.e. $F = (f_{ij})_{m \times n}$ is the genome and gene is a cell (i, j) .

In GA, just half part of the population is being utilized in reproduction of new population, which substitutes the other population partially. Then the new population generated is evaluated using Fitness Function. We use the fitness function which is the variation between projections of original image and projections of reconstructed image¹².

Then crossover and mutation method is applied to generate next generation of population. In crossover, each pair of parent yields one child only which replaces the weakest element of population. The crossover is defined as AND operator, which is applied on chromosomes (solution). The mutation operator interchanges the values one and zero of same chromosome. Then again fitness is evaluated using Fitness Function. This process repeated until we get satisfactory solution.

3.3 Algorithmic Steps

1. First of all, decide the primary population of solutions rely on Chang's algorithm
2. Enact mutation and crossover operators.
3. Then evaluate the fitness of solution with the help of the fitness function
4. Iterate till termination conditions meet such as time, sufficient fitness achieved, etc.:
 - i. Select the best-fit solutions for reproduction
 - ii. Breed new solutions through crossover operation as defined above and use mutation to give birth to new offspring
 - iii. Evaluate the individual fitness using fitness function
 - iv. Replace the least-fit population with new solutions¹².

4. Proposed Methodology

Firstly image is extracted from the data set. Then it is reconstructed by utilizing diagonal and anti-diagonal projections by Chang's Algorithm. In last, Genetic Algorithm is applied on images that are reconstructed after Chang's Algorithm.

The subsequent steps demonstrate the variety of phases that need to be accomplished:

Step-1: Select an image from data set.

Step-2: Computation of diagonal and anti-diagonal projections.

Step-3: By utilizing Chang's Algorithm regenerate the image with the help of diagonal and anti-diagonal projections.

Step-4: Display reconstructed image.

Step-5: Apply Genetic Algorithm on image reconstructed by Chang's Algorithm.

Step-6: Display final reconstructed image.

Figure 6 represents the block diagram of proposed work and Figure 7 shows the working of Genetic Algorithm.

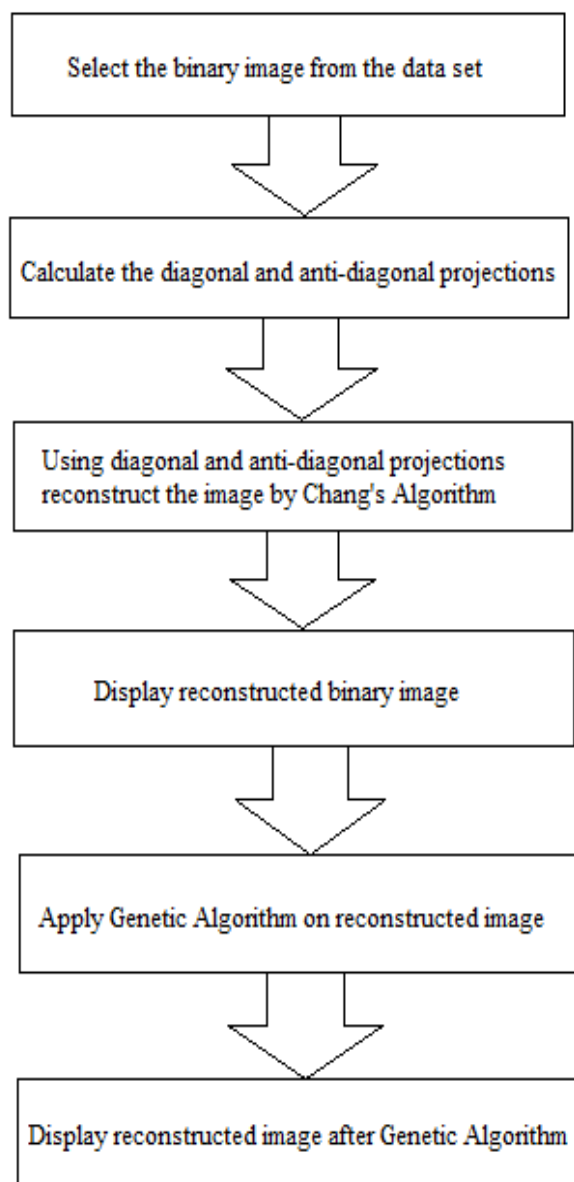


Figure 6. Block diagram of proposed work

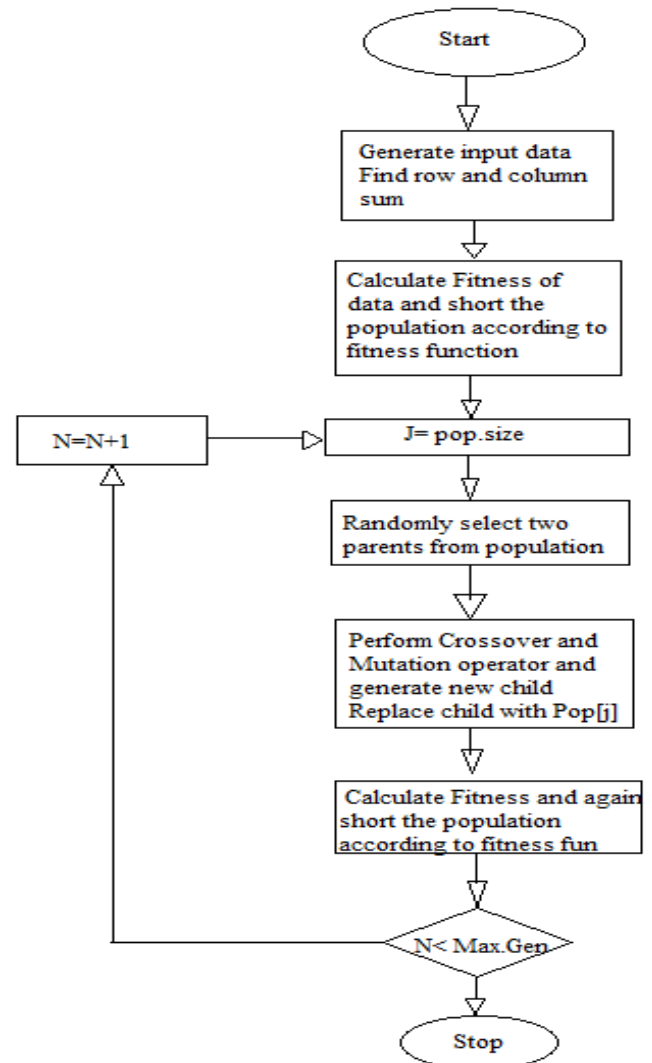


Figure 7. Flowchart of genetic algorithm for reconstruction of image

5. Experimental Result

We have taken a data set of binary images of different sizes varying from 10x10 to 150x150 pixels. The whole implementation is being done in MATLAB. Table 1 represents the images reconstructed by implementing Chang's and Genetic algorithm by employing diagonal and anti-diagonal projections.

Table 2 represents the percentage of difference between Original images and images reconstructed by Chang's and Genetic algorithm.

Table 1. Comparison of images reconstructed by chang's and genetic algorithm

Image Size (in pixels)	Original image	Reconstructed image with Chang's Algorithm using Diagonal projection	Reconstructed image with Genetic Algorithm using Diagonal projections	Reconstructed image with Chang's Algorithm using Anti-diagonal projection	Reconstructed image with Genetic Algorithm using Anti-diagonal projections
50x50					
60x60					
70x70					
80x80					
90x90					
100x100					
150x150					

Table 2. Misclassification table

Image Size (in pixels)	Percentage difference with Chang's Algorithm using Diagonal Projections	Percentage difference with Genetic Algorithm using Diagonal Projections	Percentage difference with Chang's Algorithm using Anti-diagonal Projections	Percentage difference with Genetic Algorithm using Anti-diagonal Projections
50x50	7.76	4.28	8.16	4.28
60x60	5.17	3.39	4.5	3.39
70x70	4.24	3.2	4.45	3.2
80x80	3.19	2.64	3.5	2.64
90x90	4.47	2.05	4.62	2.05
100x100	4.28	2.15	4.36	2.15
150x150	2.85	1.27	3.25	1.27

6. Conclusion

Reconstruction of an image is taken as an inverse problem in discrete tomography. Because enormous number of solution are generated while reconstructing an image using only little number of projections. To acquire the solution one required some constraints that are described which help in diminishing the number

of solutions. In order to overcome the number of solutions we brought into action hv-convex images in our work. We proposed a method of binary image reconstruction that satisfies diagonal and anti-diagonal projections using Genetic Algorithm. Also it is very clear from the results that the image reconstructed by Genetic Algorithm is much better than images obtained by Chang's algorithms.

7. References

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