

Solving Planar Graph Coloring Problem using Enhanced Cuckoo Search

Sudhanshuprakashwari*, M. VijayaRaju, Gurbakashphonsa and Deepak kumar Deepu

Department of Computer Science and Engineering, LPU, Phagwara - 144411, Punjab, India;
Sudhanshu.iitg@gmail.com, Vijaya.14844@lpu.co.in, Gurbakash.15483@lpu.co.in, deepaku371@gmail.com

Abstract

Objectives: This paper aims towards giving colors optimally to the vertices of the graph so that graph coloring constraints can be satisfied. **Methods:** A hybridized algorithm that consists of Cuckoo Search along with LDO algorithm is proposed to show a comparative and more optimal algorithm for graph coloring problem. **Findings:** Graph coloring relates graph regions coloring in a way so that sequence of coloring will meet with all the constraints of coloring. **Novelty:** Hybridization of nature-inspired algorithm with Mantegna algorithm finds out the new nest positions when the nest having worst survival rate are destroyed. The Largest degree Ordering is utilized in assigning the color coding to the nodes with the nodes having the largest degree first. The experimental results prove that hybridized solution works well using moderate size and provides another approach for the same.

Keywords: Degree based Ordering, Graph Coloring

1. Introduction

Graph coloring¹ shows a way for coloring adjacent vertices of the graph and whole vertices must be chosen for coloring process. A chromatic number represents a minimum number of required colors graph coloring. So a graph represents set of objects interconnected with edges.

Many nature inspired techniques are there and their comparative study² is also available. The CS comes under the category of metaheuristic approach which is a straightforward technique and capable of solving general N-dimensional, linear and nonlinear optimization problems³. Cuckoo Search (CS)⁴ was developed by Yang and Deb as search swarm intelligence algorithm.

Thus several hybrid optimization solutions are available for this particular problem. Several different hybrid algorithms are proposed on the similar set of problems.

Optimization and degree based algorithms⁵ will be discussed further. This paper aims towards combining all of these algorithms upon the same set of the problem to optimize the results.

Other parts of the paper are as organized: Part 2 and 3 is for planar graph coloring, Cuckoo Search and a hybrid of that algorithm. Final solutions are being displayed in Part 4 and explained another search algorithm to the similar set of problem. Part 5 makes a conclusion.

2. Planar Graph Coloring Problem

By using four various colors all the nodes and any adjacent two nodes are colored. Various papers^{1,6,7} have applied 0 to 3 numbers for various four colors, and based on that specific coloring algorithm is prepared.

Let sequence of colors for proposed coloring program is represented as $S = q_1, \dots, q_n$. Here $q_i \in \{0, 1, 2, 3\}$. For the

*Author for correspondence

node $v_i \in V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and assigned color to this node is q_i . Edges have the representation as:

$$r_{ij} = \begin{cases} 1, & \text{two nodes are connected and } i \neq j \\ 0, & \text{two nodes are disconnected, or } i = j \end{cases}$$

So for any other node $V_i \in V$ (where $i \neq j$ and $r_{ij} = 1$) as well as same color q_i , will be termed as a conflict. For this conflict, conflict representing matrix conflict_{xy} will be created so that a conflict representation can be shown for those vertices

$$\text{Conflict}_{xy} = \begin{cases} a_{xy} & \text{if } r_x = r_y \text{ and } x \neq y, \\ 0 & \text{otherwise} \end{cases}$$

So fitness value will be $n \cdot n$

$$f(R) = \sum_{x=1}^n \sum_{y=1}^n \text{Conflict}_{xy}$$

3. Standard Cuckoo Search algorithm

This meta-heuristic algorithm^{8,7} simulating behavior of Cuckoo, used for getting solutions of optimizing problems from various domains.

Interesting part is its lifestyle and egg generation process. Firstly, it shows an initial population. Survival rate for cuckoos regions is the main emphasis of this algorithm. Some of cuckoos or their eggs end in between the survival competition. The immigrant survived cuckoos in new environment start producing eggs again. With the same profit values for all cuckoos' survival rate can converge. It contains radius for laying eggs:

Egg Laying Radius (ELR) which is defined as:

$$\text{ELR} = \alpha \times \text{current cuckoo's eggs count} \times (\text{Var}_{hi} - \text{Var}_{low}) \quad (1)$$

Total eggs count

α represents integer for handling largest value of ELR,

Var_{hi} represents upper value for variables,

Var_{low} represents lower value for variables.

3.1 Mantegna Algorithm

This algorithm^{3,9} is required ingredient for cuckoo search implementation. It is used to produce random values as per a symmetric levy stable distribution. Input function uses distribution's parameters, c , and n number of iteration as well as random points to be generated. Error message in terms of array of NaNs will be shown if input value is beyond valid range.

The algorithm contains three processes to be executed:

$$V = \frac{x}{|y|^{1/\alpha}} \quad (2)$$

X, y represents normally distribution variables along with standard deviations.

Distributed x, y values calculated as following:

$$\sigma_x = \frac{\Gamma\left(\frac{1+\alpha}{2}\right) \sin\left(\frac{\pi\alpha}{2}\right)^{1/\alpha}}{\Gamma\left(\frac{1+\alpha}{2}\right) / 2\alpha 2^{(\alpha-1)/2}} \quad (3)$$

$$\sigma_y = 1; \quad (4)$$

x, y calculated from normal distribution from above values.

3.2 Largest Degree Ordering

Algorithm belongs to degree based ordering technique. It is used to select the next node for coloring process. Algorithm¹⁰ selects a node whose neighbor has highest number. Suppose node possesses 5 neighbors and other node possesses 6 neighbors, 6 neighbor vertex will be chosen to color first.

3.3 Enhanced Cuckoo Search (ECS)

The complete procedure of Enhanced Cuckoo Search is described in this algorithm.

Algorithm 1

1. Initialize all nest's position (i.e. coloring nodes) with i (i.e. initial color 0).
2. Find out the vertex to be colored using LDO.
3. Assign nest with cuckoo's egg.
4. while the stop criteria not met do for nest do
5. Compute nest's current position.
6. Compute the survival rate of nest using ELR (eq. 1).
7. Evaluate its Profit function/fitness F_i .
8. Compute step size of nest i according to Mantegna algorithm (eq. 2,3,4). for all nest
9. Check for conflict.
10. Retain the best solutions.
11. end for
12. end for
13. end while

4. Experimental Results

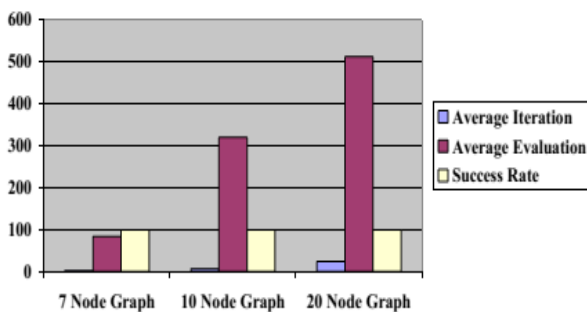
Performance of ECS algorithm is estimated through experiments. Related experiments are performed on turbo C++, and run on Intel® Core(TM)2 Duo CPU T4400 @2.20 GHz with 3 GB memory capacity. ECS is simulated for 30 runs. For maximum number of evaluations 20000 maximum cuckoos considered are 4 as only four colors available to color the graph.

Validation of results is shown through performance comparison of ECS and PSO. Algorithm is applied to graph of seven nodes and results are shown in Table 1. Graphs are also shown for various results.

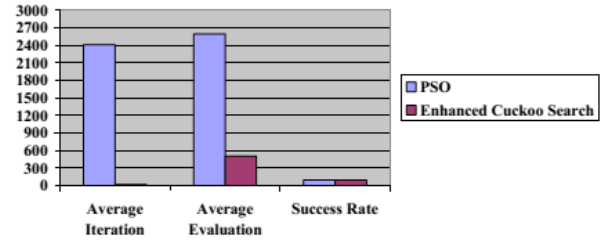
Color coding for best individual is 0 1 1 0 2 2 1.

Table 1. Experimental Results

Node	Algorithm	Average Iterations	Average Evaluations	Success Rate
7	PSO	6	129	100%
	Enhanced Cuckoo Search	3	84	100%
10	PSO	32	650	100%
	Enhanced Cuckoo Search	7	320	100%
20	PSO	2418	2600	100%
	Enhanced Cuckoo Search	24	512	100%
30	PSO	6432	70000	0%
	Enhanced Cuckoo Search	76.5	576	100%



Graph 1 Bar chart of results obtained.



Graph 2. Results of twenty node graph.

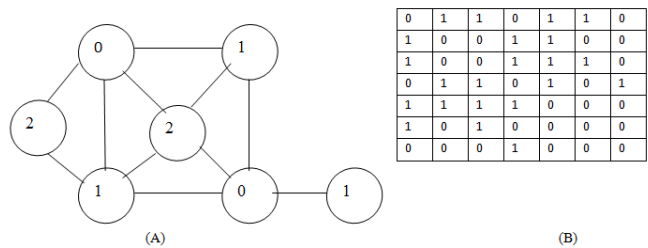


Figure 1. Graph coloring of seven nodes of graph and adjacency matrix.

5. Conclusion

ECS will help to optimize certain results. We have applied this algorithm to the problem to optimize the average iterations performed and the average evaluations. The use of nature-inspired algorithm helps us to get the best possible results. Mantegna algorithm is used to find out the new nest positions when the nest having worst survival rate are destroyed. The Largest degree Ordering is utilized in assigning the color coding to the nodes with the nodes having the largest degree first.

6. References

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