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Use of Apriori Information in Estimating Mean of a Normal Population– Second Order Relative Efficiencies

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Abstract

Background/Objective: Modified estimator for population mean using known coefficient of variation was proposed by[1]. Such an estimator has minimum mean squared error but it is unbiased. In[2] has proposed an estimator for population mean using an estimator for coefficient of variation. Motivated by the distribution of the sample root mean square s given in[3] an estimator was constructed for mean of Normal population by[4] and the relative efficiency of the proposed estimator over the conventional unbiased estimator was derived up to first order approximation. The present paper is focused on constructing two estimators for the mean of Normal population using known coefficient of variation. The relative efficiencies are derived upto second order and the numerical results are tabulated. **Methods:** Basing on Searle method two estimators are constructed for mean μ of Normal population. Former is a linear combination whereas the latter being a convex combination of $\overline{X}$ and s. **Results:** The Relative efficiencies of the proposed estimators over the conventional unbiased estimator $\overline{X}$ are obtained. **Conclusions/Applications:** It is established that the first estimator is more efficient than that of $\overline{X}$ where as the second estimator is equally efficient to that of $\overline{X}$ under first order approximation. Under the second order approximation it is observed that both first and second estimators are more efficient than $\overline{X}$ and the first estimator is more efficient than that of second estimator.

Keywords: Normal Population, Squared Minimum Mean Squared Error, Unbiased Estimator
Subject Classification: 62F10

1. Introduction

Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from the normal population with mean μ and known coefficient of variation $\sqrt{v}$

Define Sample mean= $\overline{X} = \dfrac{1}{n}\sum_{i=1}^{n} x_i$ (1)

Sample mean square= $s^2 = \dfrac{1}{n-1}\sum_{i=1}^{n}(y_i - \overline{Y})^2$ (2)

Following the fact that $v\mu^2$ involves μ, two estimators for μ are constructed using $\overline{X}$ and s. The relative efficiencies of the estimators over $\overline{X}$ are computed upto second order approximation and the numerical illustrations are provided.

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2. A Linear Estimator for Mean of N $(\mu, v\mu^2)$

Using standard results $\overline{X} \sim N(\mu, \dfrac{v\mu^2}{n})$ (3)

and for large n, n-1 $\cong$ n and $\dfrac{\sqrt{2n}}{\mu\sqrt{v}} s \sim N(\sqrt{2n}, 1)$.

Consider $T = b(\overline{X} + s)$ (4)

as an estimator for μ, where b is a scalar to be determined to minimize the MSE(T). Expressions for bias in T, B(T) and minimum MSE(T) are derived below to 0(n[-1]).

B (T) = E (T) - μ

$= \mu\ [\ b(\sqrt{v} + 1) - 1\ ].$ (5)

$MSE\ (\ T) = V\ (\ T\ ) + [\ B\ (\ T)\ ]^2.$ (6)

But

$$V(T) = b^2[\frac{v\mu^2}{n} + \frac{v\mu^2}{2n}] = \frac{3b^2\mu^2 v}{2n}. \tag{7}$$

From Equation (5) and Equation (6),
M(T)=

$$MSE(T) = \frac{3b^2\mu^2 v}{2n} + \mu^2[b(\sqrt{v}+1)-1]^2 \tag{8}$$

Differentiating Equation (8) with respect to k and equating to zero, we get

$$b = \frac{1}{\sqrt{v}+1}\left[1 - \frac{3v}{2n(\sqrt{v}+1)^2}\right] \tag{9}$$

It is verified that the second order derivative is positive for this choice of b.

Using Equation (9) in Equation (4)

$$T = \frac{1}{\sqrt{v}+1}\left[1 - \frac{3v}{2n(\sqrt{v}+1)^2}\right](\overline{X} + s) \tag{10}$$

$$B(T) = \frac{-3v}{2n(\sqrt{v}+1)^2}\mu, \tag{11}$$

and

$$M(T) = \frac{3v\mu^2}{2n(\sqrt{v}+1)^2}. \tag{12}$$

Above equations are respectively the expressions for the proposed estimator T, B(T) and M(T).

2.1 Comparison of T with $\overline{X}$

From Equation (7) and Equation (12), relative efficiency of T over $\overline{X}$ is ,

$$REF(T,\overline{X}) = \frac{V(\overline{X})}{M(T)} = \frac{2(\sqrt{v}+1)^2}{3}. \tag{13}$$

It can be noticed that T is more efficient than $\overline{X}$ if $REF(T,\overline{X}) > 1 \Rightarrow \sqrt{v} > 0.2247.$ The values of $REF(T,\overline{X})$ for specified values of $\sqrt{v}$ are presented in Table 1.

Table 1. Percentage $REF(T,\overline{X})$

$\sqrt{v}$	0.5	1.5	2.5	3.5	4.0
$REF(T,\overline{Y})$	150.00	416.66	816.66	1350.00	2016.66

It can be observed from Table 1 that efficiency of T increases rapidly with increase in co-efficient of variation.

3. Convex Combination Estimator for Mean of N ($\mu, v\mu^2$)

Consider

$$T' = \mathrm{p}\overline{X} + (1-\mathrm{p})s \tag{14}$$

as an estimator for μ, where p is to be determined such that T' has minimum MSE. Expressions for bias, $B(T')$ and the minimum $MSE(T')$ are derived below upto $0(n^{-1})$

$$B(T') = E(T') - \mu = (1-\mathrm{p})\mu(\sqrt{v}-1) \tag{15}$$

$$MSE(T') = V(T') + [B(T')]^2. \tag{16}$$

But $$V(T') = \frac{\mathrm{p}^2 v\mu^2}{n} + (1-\mathrm{p})^2\frac{v\mu^2}{2n}. \tag{17}$$

From Equations (15), (16) and (17)

$$M(T') = \frac{\mathrm{p}^2 v\mu^2}{n} + (1-\mathrm{p})^2\frac{v\mu^2}{2n} + (1-\mathrm{p})^2\mu^2(\sqrt{v}-1)^2 \tag{18}$$

Differentiating $MSE(T')$ with respect to **p** and equating to zero, $p = 1 - \frac{5v}{4mn}$, (19)

where $(\sqrt{v}-1)^2 = m$; $\sqrt{v} \neq 1$ (20)

It is verified that the second derivative is positive for this choice of p

Using Equation (19) in Equation (14)

$$T' = (1 - \frac{5v}{4m})\overline{X} + \frac{5v}{4m}s \tag{21}$$

$$B(T')=\frac{5v\mu}{49(\sqrt{v}-1)n}. \quad (22)$$

and $$M(T')=\frac{v\mu^2}{n} \quad (23)$$

are respectively the expressions for the proposed estimator T', $B(T')$ and $MSE(T')$ upto o(n[-1])

3.1 Comparison of T' with $\bar{X}$

From Equations (3) and (12)

Relative efficiency of T' over $\bar{X}$ is given by

$$REF(T',\bar{X})=\frac{V(\bar{X})}{M(T')}=1. \quad (24)$$

Thus T' and $\bar{X}$ are equally efficient and hence upto $0(n^{-1})$ convex combination does not improve the efficiency.

Remark: It can be seen that

$$REF(T,T')=\frac{M(T')}{M(T)}=REF(T,\bar{X}). \quad (25)$$

3.2 Second Order Approximation

By considering the Minimum mean squared error in t upto $0(n^{-2})$, we obtain

$$M(T')=\frac{v}{n}\left[1-\frac{15v}{16mn}\right], \quad (26)$$

$$REF(T,T')=\frac{(\sqrt{v}+1)^2\left(16-\frac{15v}{mn}\right)}{24} \quad (27)$$

and

$$REF(T',\bar{X})=\frac{16mn}{16mn-15v}. \quad (28)$$

Since $REF(T',\bar{X})>1$, T' is more efficient than $\bar{X}$. The values of relative efficiency of T over T' for different sample sizes and the co-efficient of variation are presented in Table 2.

From Table 2 it can be observed that T is more efficient than T'. Also it is observed that for fixed values of co-efficient of variation, the relative efficiency of T and T' increases with increasing sample size. It is interesting

Table 2. Percentage $REF(T,T')$

n √c	0.5	1.5	2	2.5	3	3.5	4
20	143	240.9	487.5	710.3	954.2	1226	1527.8
25	144.4	276	510	731.6	976.7	1250.8	1555.6
30	145.3	299.5	525	745.8	991.7	1267.3	1574.1
35	146	316.2	535.7	755.9	1002.4	1279.1	1587.3
40	146.5	328.8	543.7	763.5	1010.4	1288	1597.2
45	146.9	338.5	550	769.4	1016.7	1294.9	1604.9
50	147.2	346.4	555	774.1	1021.7	1300.4	1611.1

Table 3. Percentage $REF(T',\bar{X})$

n √c	0.5	1.5	2	2.5	3	3.5	4
20	104.918	172.973	123.0769	114.9701	111.7904	110.117	109.0909
25	103.8961	150.9434	117.6471	111.6279	109.215	107.9331	107.1429
30	103.2258	139.1304	114.2857	109.5057	107.563	106.5246	105.8824
35	102.7523	131.7647	112	108.0386	106.4133	105.5409	105
40	102.4	126.7327	110.3448	106.9638	105.567	104.8149	104.3478
45	102.1277	123.0769	109.0909	106.1425	104.918	104.2572	103.8462
50	101.9108	120.3008	108.1081	105.4945	104.4046	103.8152	103.4483

to note that for a fixed sample size, the relative efficiency of T over T' increasing value of co-efficient of variation.

It is noticed from Table 3 that T' is more efficient than $\overline{X}$. Also, it is observed that for a fixed value of co-efficient of variation, the relative efficiency of T' over $\overline{X}$ decreases with increase in the values of sample size. It is interesting to note that for fixed sample size, the relative efficiency of T' over $\overline{X}$ increases in the beginning but later on decreases with increase in the values of co-efficient of variation. Hence, it is established that a convex combination improves the efficiency under second order approximation.

4. References

1. Searles DT. The utilisation of a known co-efficient of variation in the estimation procedure. Journal of American Stat Asso. 1964; 30:1225–6.
2. Srivastava VK. On the use of co-efficient of variation in estimating mean. Journal of Indian Society for Agri Stats. 1974; 26:33–6.
3. Kendall MG, Stuart A. The advanced theory of statistics. London: Charles Griffin and Co.
4. Sarma BBR. Estimating mean of a normal population using known co-efficient of variation. International-e-Journal of Mathematics and Engineering. 2010; 61:616–9.
5. Sarma BBR. On estimation of variance of a population using known co-efficient of kurtosis. International Journal of Statistics and Systems. 09732675, 20090601.
6. Sukhatme PV. Sampling theory of Surveys with Applications. Asia Publishing House; 1970.
7. Jeelani MI, Mir SA, Nazir N, Jeelani F. Modified ratio estimators using linear combination of co-efficient of skewness median of auxiliary variable under rank set sampling and simple random sampling. Indian Journal of science and Technology. 2014 Jan; 7(5). DOI: 10.17485/ijst/2014/v7i5/50159.
8. Jafarzadeh H, Torkashv RR, Asgari C, Amiry A. Provide a new approach for mining fuzzy association rules using apriori algorithm. Indian Journal of Science and Technology. 2015 Apr; 8(S7). DOI: 10.17485/ijst/2015/v8iS7/71227.
9. Saleena Y, Dharun VS. A new improved multistage eight directional median filter. Indian Journal of Science and Technology. 2015 Oct; 8(26). DOI: 10.17485/ ijst/ 2015/ v8i26/ 81053.