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Least Square based Signal Denoising and Deconvolution using Wavelet Filters

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Abstract

Noise, the unwanted information in a signal reduces the quality of signal. Hence to improve the signal quality, denoising is done. The main aim of the proposed method in this paper is to deconvolve and denoise a noisy signal by least square approach using wavelet filters. In this paper, least square approach given by Selesnick is modified by using different wavelet filters in place of second order sparse matrix applied for deconvolution and smoothing. The wavelet filters used in the proposed approach for denoising are Haar, Daubechies, Symlet, Coiflet, Biorthogonal and Reverse biorthogonal. The result of the proposed experiment is validated in terms of Peak Signal to Noise Ratio (PSNR). Analysis of the experiment results notify that proposed denoising based on least square using wavelet filters are comparable to the performances given by deconvolution and smoothing using the existing second order filter.

Keywords: Least Square, Peak Signal to Noise Ratio (PSNR), Signal Denoising, Wavelet Filters

1. Introduction

Any unwanted signal is defined as noise. In real world, signals are often affected by device-specific noise. Therefore, signal denoising is a challenging task for researchers. The noise is caused due to several reasons such as electrical fluctuations in devices and electromagnetic interference. Noise adds unwanted information to the signal which leads to distorted signal. To overcome this, we use different denoising techniques in signal processing. The main objective in signal denoising is to remove maximum noise to get a clean signal. Denoising is the kernel of signal processing. Removing the noise and retaining details of a signal is the key goal of signal denoising techniques.

Signal denoising is a vital task in research areas and for the same, various techniques have been proposed. Signals in medical applications such as, ECG signals essentially depend on denoising techniques[1]. Signal denoising techniques which use different notch filters and Signal-Noise residue algorithm is introduced in [2]. This approach is basically used to remove disturbances such as, power line interferences.

The current trend in signal processing includes wavelets. In this paper, we discuss the methods of denoising based on least square approach using wavelet filters. The function of filters is to remove noise from the original signal. Every wavelet filter used for denoising satisfies invertible property. One of the wavelet filters that hold much of the recent applications is Haar filter. It has only very few computing requirements justifying its wide usage.

Generally, wavelets are designed for signal processing[3–6]. Wavelet decomposition is also mathematically reversible[7]. If random noise is present in the signal, the removal of small variations within the signal can help to denoise the signal[8]. Recently, significant applications of wavelet analysis are applied to a wide variety of problems. Diverse fields including mathematics, physics, computer science and engineering use wavelets[9].

In this paper, we are interested to use different wavelet filters in the place of second order filter along with the least square weighted regularization framework applied

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for signal deconvolution and denoising approach. The wavelet filters used are Daubechies, Symlets, Haar, Coiflets, Biorthogonal and Reverse biorthogonal. Among them, Daubechies 2, Symlet 3, Coiflet 1, Biorthogonal 2.2 and Reverse biorthogonal 3.1 are chosen. The proposed technique is validated and compared in terms of standard quality metric called Peak Signal to Noise Ratio (PSNR) against the least square based approach using second order filter proposed by Selesnick[10].

The mathematical background of the proposed work is discussed in section II. The proposed system and the experimental results and analysis are given in section III and IV respectively. The paper is concluded in section V.

2. Mathematical Background

The mathematics behind the concept of least square based approach for signal deconvolution and denoising is discussed in this section.

2.1 Deconvolution

The problem of determining the input to a Linear Time Invariant (LTI) system when the output signal is known, is termed as Deconvolution. Let $y(n)$ be the output signal which is given by,

$$y(n) = h(0)\,x(n) + h(1)\,x(n-1) + \ldots\ldots + h(N)\,x(n-N)$$

where, $x(n)$ is the input signal and $h(n)$ is the impulse response. $y(n)$ can be written in terms of $y = Hx$, where, H is given by the matrix,

$$\mathbf{H} = \begin{bmatrix} h(0) & & & \\ h(1) & h(0) & & \\ h(2) & h(1) & h(0) & \\ \vdots & & & \ddots \end{bmatrix}$$

The H matrices are constant valued along the diagonal and are called Toeplitz matrices. The input signal x should atleast approximately satisfy $y = Hx$. The problem formulation for the same is given by

$$\min_x \|y - \mathbf{H}x\|_2^2 + \lambda \|\mathbf{D}x\|_2^2 \qquad (1)$$

where, D is represented by,

$$D = \begin{bmatrix} 1 & -2 & 1 & & & \\ & 1 & -2 & 1 & & \\ & & \ddots & & \ddots & \\ & & & 1 & -2 & 1 \end{bmatrix}$$

The solution for signal denoising by minimizing Equation (1) is given by,

$$x = (H^T H + \lambda\, D^T D)^{-1} H^T y \qquad (2)$$

2.2 Smoothing

In this method, the idea is to obtain a denoised signal closer to noisy input by least squares weighted regularization approach[10]. As smoothness of the signal increases, the energy of its derivative becomes smaller. In other terms, it is interpreted as if x is smooth, $\|Dx\|_2^2$ becomes small.

Let $y(n)$ be the noisy input signal and $x(n)$ be the output that approximates $y(n)$, then the problem formulation is given by,

$$\min_x \|y - x\|_2^2 + \lambda \|\mathbf{D}x\|_2^2 \qquad (3)$$

where, Dx is the second order differentiation of $x(n)$. λ is a parameter on which x depends, for smoothening of a noisy signal. The signal x get smoother as the value of λ increases. The mathematical equation for signal denoising using least squares weighted regularization in the method of smoothing is given by,

$$x = (I + \lambda\, D^T D)^{-1} y \qquad (4)$$

where, I is the identity matrix of same size as that of D.

3. Proposed System

The existing method of denoising is implemented by using second order differential matrix [11]. In our proposed system, this matrix is replaced with the high pass decomposition coefficients of different wavelet filters in the least squares weighted regularization method of denoising applied for deconvolution and smoothing.

3.1 Deconvolution

The block diagram for deconvolution by the least squares weighted regularization method is depicted in Figure1. An input signal of size n is convolved with a filter and a random noise is added to the convolved signal to get a noisy signal. Different wavelet filters are chosen for D in Equation (2). Further, this noisy signal of size n, convolution matrix $H_{n\times n}$ and the chosen $D_{n\times n}$ are passed to the least square based solution for deconvolution which is given in Equation (2). The deconvolved signal obtained in this step is validated with a PSNR quality metric against existing second order.

3.2 Smoothing

The block diagram for smoothing by the least squares weighted regularization method is also depicted in Figure 1. A noisy ECG signal of size n is loaded as input to get it denoised. The D matrix given in Equation (4) is replaced with high pass decomposition coefficients of different wavelet filters. The noisy input of size n and matrix $D_{n \times n}$ are passed to the Equation (4), which is the solution given for smoothing by least squares weighted regularization method. Thus obtained denoised signal is validated with a PSNR quality metric with input signal as reference.

4. Experimental Results and Analysis

In this paper, we proposed the use of different wavelets for signal denoising by least squares weighted regularization approach. The proposed method is applied on deconvolution and smoothing techniques. The wavelets included in the experiment are Daubechies, Symlets, Haar, Coiflets, Biorthogonal and Reverse biorthogonal[11]. Along with second order sparse matrix, Daubechies 2, Symlet 3, Coiflet 1, Biorthogonal 2.2 and Reverse biorthogonal 3.1 are used. An input signal is convolved and a random noise is added. For deconvolution, the D matrix is replaced with the high pass coefficients of different wavelets filters. In case of smoothing, a noisy ECG signal is passed as input and is denoised using high pass coefficients of different wavelet filters.

The results obtained by the chosen filters for deconvolution and smoothing are illustrated in Figures 2 and 3 respectively. In Figure 2, X-axis represents the length of the signal and Y-axis represents the function obtained for each x. In Figure 2, (a) shows the input signal of length 300 and (b) shows noisy signal obtained by adding random noise to the convolved signal. The deconvolution obtained for Second order, Haar, Daubechies 2, Symlet 3, Coiflet 1, Biorthogonal 2.2 and Reverse biorthogonal 3.1 are depicted in Figure 2 (c), (d), (e), (f), (g), (h) and (i) respectively. For validation of the results obtained, the measurement of PSNR quality metric is used. Similarly, Figure 3 (a) shows the noisy input signal for smoothing. The denoising obtained by smoothing technique for Second order, Haar, Daubechies 2, Symlet 3, Coiflet 2, Biorthogonal 2.2 and Reverse biorthogonal 3.1 are depicted in Figure 3 (c), (d), (e), (f), (g) and (h) respectively.

Table 1 exhibit the PSNR values obtained for deconvolution and smoothing with input signal as reference. The PSNR values are compared with the conventional second order filter to validate the improvement on using wavelet filters. The signal which has high PSNR value is supposed to have less noise. From Table I, it can be inferred that the PSNR value is high when Reverse Biorthogonal wavelet filter is used. Haar, Daubechies 2,

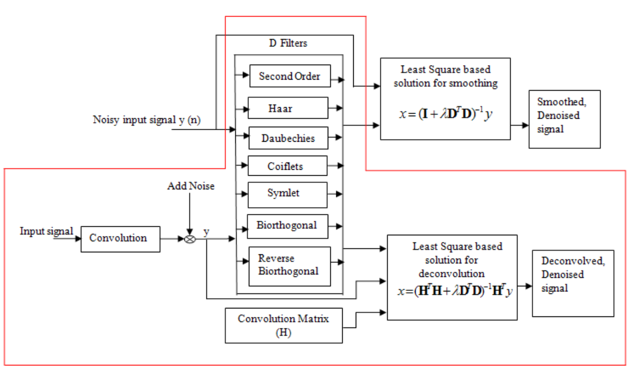

Figure 1. Block diagram for smoothing and deconvolution.

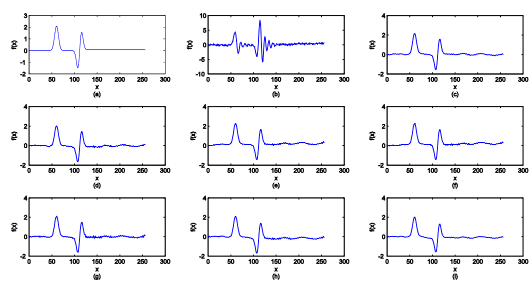

Figure 2. Deconvolution: (a) Input signal (b) Output signal (noisy) (c) Second order (d) Haar (e) db2 (f) sym3 (g) bior2.2 (h) coif1 (i) rbio3.1.

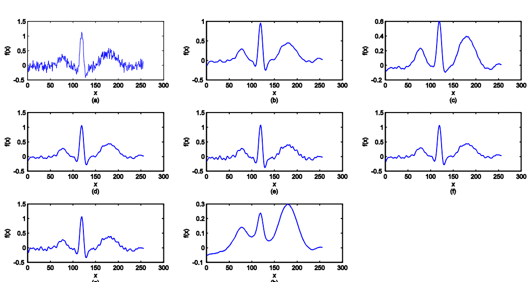

Figure 3. Smoothing: (a) Data (b) Second order (c) Haar (d) db2 (e) sym3 (f) coif1 (g) bior2.2 (h) rbio3.1.

Table 1. Comparison of PSNR (db) values obtained for proposed method based on least squares (wavelets) against existing second order filter

Method of Denoising	Second order			Wavelets (Proposed)			
		Haar	Daubechies	Symlets	Coiflets	Biorthogonal	Reverse Biorthogonal
Deconvolution	32.5913	32.0972	31.8481	31.7353	31.8159	30.8854	32.6520
Smoothing	22.0606	19.3663	22.8835	23.4117	22.9432	23.4946	15.9736

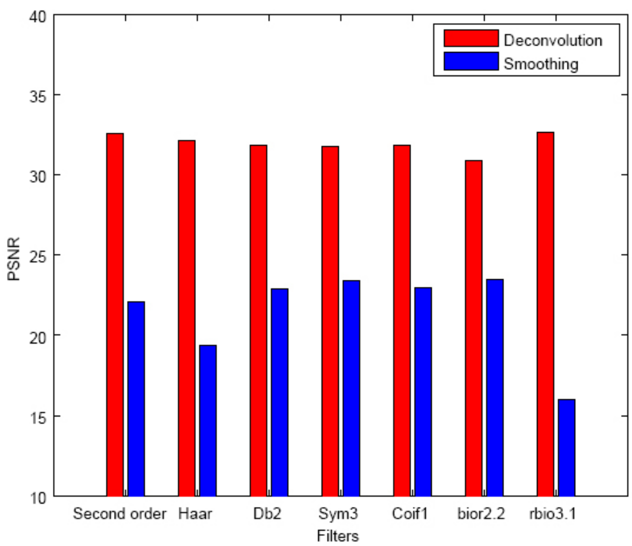

Figure 4. Comparison of PSNR values obtained for six wavelet filters in deconvolution and smoothing.

Symlet 3, Coiflet 1 and Biorthogonal 2.2 are with PSNR values 32.0972 dB, 31.8481 dB, 31.7353 dB, 31.8159 dB and 30.8854 dB respectively. Since the PSNR value of the Second order wavelet filter is 32.5913 dB, other filters are found to be comparable with second order filter. Similarly, in smoothing, Biorthogonal wavelet filter results in better denoising with PSNR value 23.4946 dB. Haar, Daubechies 2, Symlet 3, Coiflet 1 and Reverse Biorthogonal 3.1 wavelet filters have their PSNR values as 19.3663 dB, 22.8835 dB, 23.4117 dB, 22.9432 dB and 15.9736 dB respectively. The PSNR value obtained by using second order wavelet filter is 22.0606 dB. Thus, it is observed that wavelet filters are comparable with second order filter in denoising. The analysis of PSNR values on using different wavelet filters for deconvolution and smoothing is graphically depicted in Figure 4. The horizontal axis represents different wavelet filters while vertical axis represents the PSNR values obtained for deconvolution and smoothing. Figure 4 and Table I illustrates that analysis of proposed method based on least square approach using wavelet filters is much comparable with the existing second order filter.

5. Conclusion

To do any signal analysis, it is necessary to get the signal cleaned first. The process of denoising is employed in this task. In the proposed paper, denoising is achieved by least square weighted regularization method using wavelets on deconvolution and smoothing techniques. The insight of using different wavelets helps in understanding the effectiveness of each wavelet that is being used. From the experimental results, it can be inferred that most of the other wavelet filters used is also giving comparable results when compared with the conventional second order filter. In the proposed experiment, Reverse Biorthogonal wavelet filter performs better in signal deconvolution and Biorthogonal wavelet filter gives about 1 dB PSNR improvement in signal denoising when compared with second order wavelet filter. The deconvolution and smoothing programs given in are reprogrammed by replacing different wavelets filters for denoising and the efficiency of the same has been determined using the quality metric, PSNR.

6. References

1. Thomas S, Velayudhan A, Baby N, Peter S. Study of ECG signal denoising and peak detection techniques. Journal of Network Communications and Emerging Technologies. 2015 Aug; 3(2):1–3.
2. Joshi SL, Vatti RA, Tornekar RV. A survey on ECG signal denoising techniques.Communication Systems and Network Technologies, 2013 International Conference;2013 Apr. p. 60–4,
3. Bruni V, Vitulano D. Wavelet-based signal de-noising via simple singularities approximation. Signal Processing. 2006 Apr; 86(4):859–76.
4. Rioul O, Vetterli M. Wavelets and signal processing. IEEE Signal Processing Magazine. 1991Oct; 8(4):14–38.

5. Chen G, Bui T. Multi wavelets denoising using neighboring coefficients. Signal Processing Letters. 2003 Jul; 10(7):211–14.
6. Crouse MS, Nowak RD, Baraniuk RG. Wavelet-based statistical signal processing using hidden Markov models. IEEE Transactions onSignal Processing. 1998 Apr; 46(4):886–902.
7. Soman K. Insight into wavelets: from theory to practice. Prentice Hall India Learning Private Limited, 3rd edition; 2010.
8. Jensen A, La Cour-Harbo A. Ripples in mathematics: the discrete wavelet transform. Springer Science and Business Media; 2001. p. 51–60.
9. Debnath L. Brief historical introduction to wavelet transforms. International Journal of Mathematical Education in Science and Technology. 1998; 29(5):677–88.
10. Selesnick I. Least squares with examples in signal processing. Open Stax-CNX. Module:m46131; 2013 Apr. p. 1–25.
11. Aarthy G, Amitha P, Krishnan T, Pillai S, Sowmya V, Soman K. A comparative study of spike and smooth separation from a signal using Different over complete dictionary. 2013 International Multi-Conference on Automation, Computing, Communication, Control and Compressed Sensing (iMac4s); 2013 Mar. p. 590–5.