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Bulk Arrival, Fixed Batch Service Queue with Unreliable Server and with Compulsory Vacation

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Abstract

Objectives: To derive a steady state solution of bulk arrival queue with fixed batch service, in addition the server may break down and the server takes compulsory vacation at each service completion points. **Methods/Statistical Analysis**: Using supplementary variable technique and Laplace Stieltjes transform, the probability generating functions has been obtained for finding the steady state solution. **Findings:** Using the properties of probability generating function, some performance measures of the queuing model have been obtained at various server state. The model has been compared with existing once by assuming particular distributions to respective random variables. **Application/Improvements:** The results obtained has been comprehended by illustrating numerical examples.

Keywords: Batch Arrival, Batch Service, Compulsory Vacation, Probability Generating Function and Operating Characteristics, Unreliable Server

1. Introduction

The congestion situatins encountered in computer, communication, manufacturing, production system, etc, can be modelled as queueing system with vacation. Several researchers have contributed significantly on vacation models and the corresponding works are in[1,2,10,12,14] and paper[5,14] are the two excellent survey works on vacation queues. In practical phenomina it is usual that the server may break downs. So that a more suitable queueing model is that which consider the assumption of unreliable server. Many researchers have contributed on queue with unreliable servers and the papers are in[13,15,16]. Some notable works on queuing with break down are discused in[15-17] and also in[9]. Many researchers have studied the queueing model with unreliable server under several assumptions and given idea to model the related works. In[6] incorporate the server breakdown on vacation queueing model. The Poisson arrival of batch size X and general service given by single service queuing system of unreliable server and with single vacation studied in[7].

In[3] the Poisson arrival with a single unreliable server queue with Bernoulli vacation with two phases of gerneral service. In[4], the random arrival size X and Bernoulli vacation queue with two phases of general unreliable service also repair may starts with some delay time has been considered. In[11] the Poisson arrival of random size X with general service by a single unreliable server queuing system with an repair time, also server may take multiple vacation and follow randomized vacation policy has been discussed. The Poisson arrival of random size X and server give service of fixed size K with Bernoulli vacation has been studied in[8]. In this article we consider an $M^{[X]}/G^{K}/1$ queue with unreliable server and with compulsory vacation. This type of queuing system exits in manufacturing industries, transportation system etc. In manufacturing industries, after products are approved for transportation to customer shops, they are transported to the shops in bulks by truck. After transporting the products, the truck is sent for maintenance (vacation period). During the service period (transportation period), the trucks may break down. This situation can be modeled as an $M^{[X]}/G$

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^{K} /1 queue with unreliable server and compulsory vacation. The remainder of this article is organized as follows: Section 2 provides the model description and mathematical analysis. In section 3, we obtain some queuing characteristics of the model discussed in this paper. In section 4, we present some particular models. In section 5, we illustrate the model by some numerical examples. Finally, In section 6, we present a conclusion.

2. The Model and Analysis

Let us assume an $M^{[X]}/G^K/1$ queue, arrival follows compound Poisson process with parameter λ and the size of the successive arriving batches is a random variable with probability P{X=j}=C_j ,whose probability generating function is defined as $C(z)=\sum_{j=1}^{\infty} C_j z^j$. The services are given by single server in batches of fixed size 'K' and the service time distribution of each batch is generally distributed with distribution function $G(x)$. After completion of each service, the server takes a compulsory vacation of random duration. The vacation period is also generally distributed with distribution function $B(x)$. In addition, the server may breakdown during a service and the breakdowns follows a Poisson process with the parameter 'α'. The interrupted customer will stay in the queue as a first person to get service again and the repair to server starts immediately when the server breakdown. The duration of the repair period is generally distributed with distribution function $H(x)$. Immediatly after the broken server is repaired, the server is ready to start its service. Further, we assume that the input process, server life time, server repair time, sevice time and vacation times are independent of each other. The analysis of this model is based on supplementary variable technique and the supplementary variable is elapsed service time/ elapsed vacation time/elapsed repair time. We define the following conditional probabilities and probabilities:

$\mu(x)=\dfrac{g(x)}{1-G(x)}$ is the conditional probability that the completion of service during the interval $(x, x+dx)$, here 'x' is the elapsed service time.

$\beta(x)=\dfrac{b(x)}{1-B(x)}$ is the conditional probability that the completion of vacation during the interval $(x, x+dx)$, here 'x' is the elapsed vacation time.

$\gamma(x)=\dfrac{h(x)}{1-H(x)}$ is the conditional probability that the completion of repair during the interval $(x, x+dx)$, here 'x' is the elapsed repair time.

The Markov process related to this model is $\{N(t), S(t) : t \geq 0\}$ where $N(t)$ be the number of customer in the queue and $S(t)$ be the supplementary variable at time t. and

$S(t)=S_1(t)$, the elapsed service time

$=S_2(t)$, the elapsed vacation time

$=S_3(t)$, the elapsed repair time

$P_n(t,x)$ =Probability that, at time 't', there are 'n' customers in the queue (excluding the customer in service) and the elapsed service time is 'x'

$V_n(t,x)$ =Probability that, at time 't', there are 'n' customers in the queue and the elapsed vacation time is 'x'

$R_n(t,x)$ =Probability that, at time 't', there are 'n' customers in the queue and the elapsed repair time is 'x'

$Q_n(t)$ =Probability that, at time 't', there are n customers in the queue and the server is idle

The differential-difference equations for this model are

$$\frac{dP_0(x)}{dx}=-(\lambda+\mu(x)+\alpha)P_0(x) \tag{1}$$

$$\frac{dP_n(x)}{dx}=-(\lambda+\mu(x)+\alpha)P_n(x)+\lambda\sum_{j=1}^{n}C_jP_{n-j}(x),\ for\ n\geq 1 \tag{2}$$

$$\frac{dV_0(x)}{dx}=-(\lambda+\beta(x))V_0(x) \tag{3}$$

$$\frac{dV_n(x)}{dx}=-(\lambda+\beta(x))V_n(x)+\lambda\sum_{j=1}^{n}C_jV_{n-j}(x),\ for\ n\geq 1 \tag{4}$$

$$\frac{dR_0(x)}{dx}=-(\lambda+\gamma(x))R_0(x) \tag{5}$$

$$\frac{dR_n(x)}{dx}=-(\lambda+\gamma(x))R_n(x)+\lambda\sum_{j=1}^{n}C_jR_{n-j}(x),\ for\ n\geq 1 \tag{6}$$

$$0=-\lambda Q_n+\lambda(1-\delta_{n,K})\sum_{j=1}^{n}c_jQ_{n-j}+\int_0^\infty R_n(x)\gamma(x)dx+\int_0^\infty V_n(x)\beta(x)dx \tag{7}$$

The boundary conditions are

$$P_n(0)=\int_0^\infty V_{n+K}(x)\beta(x)dx+\int_0^\infty R_{n+K}(x)\gamma(x)dx+\lambda\sum_{j=0}^{K-1}C_{n+K-j}Q_j,\ for\ n\geq 0 \tag{8}$$

$$V_n(0)=\int_0^\infty P_n(x)\mu(x)dx,\ for\ n\geq 0 \tag{9}$$

$$R_n(0)=\alpha\int_0^\infty P_{n-K}(x)dx,\ for\ n\geq K \tag{10}$$

$$R_n(0)=0,\ for\ n<K \tag{11}$$

and the normalization condition is

$$\sum_{n=0}^{K-1}Q_n+\int_0^\infty\sum_{n=0}^{\infty}[P_n(x)+V_n(x)+R_n(x)]dx=1 \tag{12}$$

For the analysis, we define the following probability generating functions

$$P(x,z)=\sum_{n=0}^{\infty}P_n(x)z^n,\ V(x,z)=\sum_{n=0}^{\infty}V_n(x)z^n,\ R(x,z)=\sum_{n=0}^{\infty}R_n(x)z^n,\ Q(z)=\sum_{n=0}^{K-1}Q_nz^n$$

Multiplying equation (2) by z^n and applying $\sum_{n=1}^{\infty}$, we have

$$\frac{\partial\sum_{n=1}^{\infty}P_n(x)z^n}{\partial x}=-(\lambda+\mu(x)+\alpha)\sum_{n=1}^{\infty}P_n(x)z^n+\lambda\sum_{n=1}^{\infty}\sum_{j=1}^{n}C_jP_{n-j}(x)z^n$$

Adding the above equation with equation (1), we have

$$\frac{\partial P(x,z)}{\partial x}+(\lambda-\lambda C(z)+\mu(x)+\alpha)P(x,z)=0 \tag{13}$$

Multiplying equation (4) by z^n and applying $\sum_{n=1}^{\infty}$, we have

$$\frac{\partial\sum_{n=1}^{\infty}V_n(x)z^n}{\partial x}=-(\lambda+\beta(x))\sum_{n=1}^{\infty}V_n(x)z^n+\lambda\sum_{n=1}^{\infty}\sum_{j=1}^{n}C_jV_{n-j}(x)z^n$$

Adding the above equation with equation (3), we have

$$\frac{\partial V(x,z)}{\partial x}+(\lambda-\lambda C(z)+\beta(x))V(x,z)=0 \tag{14}$$

Multiplying equation (6) by z^n and applying $\sum_{n=1}^{\infty}$, we have

$$\frac{\partial\sum_{n=1}^{\infty}R_n(x)z^n}{\partial x}=-(\lambda+\gamma(x))\sum_{n=1}^{\infty}R_n(x)z^n+\lambda\sum_{n=1}^{\infty}\sum_{j=1}^{n}C_jR_{n-j}(x)z^n$$

Adding the above equation with equation(5) , we have

$$\frac{\partial R(x,z)}{\partial x}+(\lambda-\lambda C(z)+\gamma(x))R(x,z)=0 \tag{15}$$

Multiplying equation (8) by z^{n+K} and applying $\sum_{n=0}^{\infty}$, we have

$$\sum_{n=0}^{\infty}P_n(0)z^{n+K}=\lambda\sum_{n=0}^{\infty}\sum_{j=0}^{K-1}Q_jC_{n+K-j}z^{n+K}+\int_0^\infty\gamma(x)\sum_{n=0}^{\infty}R_{n+K}(x)z^{n+K}dx+\int_0^\infty\beta(x)\sum_{n=0}^{\infty}V_{n+K}(x)z^{n+K}dx$$

$$=>z^KP(0,z)=\int_0^\infty\gamma(x)\sum_{n=K}^{\infty}R_n(x)z^ndx++K(z)+\int_0^\infty\beta(x)\sum_{n=K}^{\infty}V_n(x)z^ndx \tag{16}$$

where $K(z)=\lambda\sum_{n=0}^{\infty}\sum_{j=0}^{K-1}Q_jC_{n+K-j}z^{n+K}$

We multiplying equation (7) by z^n and applying $\sum_{n=0}^{K-1}$, we have

$$0=-\lambda\sum_{n=0}^{K-1}Q_nz^n+\int_0^\infty\gamma(x)\sum_{n=0}^{K-1}R_n(x)z^ndx+\int_0^\infty\beta(x)\sum_{n=0}^{K-1}V_n(x)z^ndx+\lambda\sum_{n=0}^{K-1}(1-\delta_{n,K})\sum_{j=1}^{n}C_jQ_{n-j}z^n$$

$$0=-\lambda Q(z)+\int_0^\infty\gamma(x)\sum_{n=0}^{K-1}R_n(x)z^ndx+\int_0^\infty\beta(x)\sum_{n=0}^{K-1}V_n(x)z^ndx+\lambda L(z)$$

where

$$L(z)=\sum_{n=0}^{K-1}(1-\delta_{n,K})\sum_{j=1}^{n}C_jQ_{n-j}z^n$$

We add the equations (17) and (16), we have

$$z^KP(0,z)=+\int_0^\infty\beta(x)V(x,z)dx+\int_0^\infty\gamma(x)R(x,z)dx-\lambda Q(z)+K(z)+\lambda L(z)$$

where $K(z)=\lambda[C(z)Q(z)-L(z)]$

$$z^KP(0,z)=+\int_0^\infty\beta(x)V(x,z)dx+\int_0^\infty\gamma(x)R(x,z)dx-\lambda Q(z)+[\lambda C(z)Q(z)-\lambda L(z)]+\lambda L(z)$$

$$z^KP(0,z)=\int_0^\infty\beta(x)V(x,z)dx+\int_0^\infty\gamma(x)R(x,z)dx+\lambda[C(z)-1]Q(z) \tag{18}$$

Multiplying equation (9) by z^n and applying $\sum_{n=0}^{\infty}$, we have

$$V(0,z)=\int_0^\infty\mu(x)P(x,z)dx \tag{19}$$

Multiplying equation (10) by z^n and applying $\sum_{n=1}^{\infty}$, we have

$$\sum_{n=1}^{\infty}R_n(0)z^n=\alpha\int_0^\infty\sum_{n=1}^{\infty}P_{n-K}(x)z^ndx$$

Adding the above equation with equation (11), we have

$$R(0,z) = \alpha z^K \int_0^\infty P(x,z)dx = \alpha z^K P(z) \quad (20)$$

Integrating equation (13) partially with respect to 'x' with the limits from '0' to 'x', we have

$$P(x,z) = P(0,z)e^{-ax-\int_0^x \mu(x)dx} dx \quad (21)$$

Integrating equation (21) partially with respect to 'x' with the limits from '0' to '∞', we have

$$P(z) = \frac{P(0,z)[1-G^*(a)]}{a} \quad (22)$$

Multiplying equation (21) by $\mu(x)$ and integrating partially with respect to 'x', with the limits from '0' to '∞', we have

$$\int_0^\infty \mu(x)P(x,z)\partial x = P(0,z)G^*(a), \quad (23)$$

Substituting equation (23) in (19), we have

$$V(0,z) = P(0,z)G^*(a), \quad (24)$$

Integrating equation (14) partially with respect to 'x', with the limits from '0' to 'x', we have

$$V(x,z) = V(0,z)e^{-mx-\int_0^x \beta(x)dx} \quad (25)$$

where $m = \lambda - \lambda C(z)$

Substituting equation (25) in equation (24), we have

$$V(x,z) = P(0,z)G^*(a)e^{-mx-\int_0^x \beta(x)dx} \quad (26)$$

Integrating equation (26) partially with respect to 'x', with the limits form '0' to '∞', we have

$$V(z) = \frac{P(0,z)G^*(a)[1-B^*(m)]}{m} \quad (27)$$

Multiplying equation (26) by $\beta(x)$ and integrating partially with respect to x, with limits from 0 to ∞.

$$\int_0^\infty V(x,z)\beta(x)dx = P(0,z)G^*(a)B^*(m) \quad (28)$$

Integrating equation (15) partially with respect to 'x', with the limits from '0' to 'x', we have

$$R(x,z) = R(0,z)e^{-mx-\int_0^x \gamma(x)dx} \quad (29)$$

Substituting equation (20),(22) in (29), we have

$$R(x,z) = \frac{\alpha z^K P(0,z)[1-G^*(a)]e^{-mx-\int_0^x \gamma(x)dx}}{a} \quad (30)$$

Table 5.1 Arrival rate versus performance measures Q and E(N)
($\alpha = 1, \beta = 2, \gamma = 1, \mu = 3, K = 15, s = 0.7, p = 0.5$)

λ	Q	E(N)					
		Z_0 values					
		1.0001	**1.00001**	**1.5**	**5**	**10**	**15**
1	0.9048	952.4	9523	0.2857	0.1190	0.1058	0.1020
2	0.8095	1905	19048	0.5714	0.2381	0.2116	0.2041
3	0.7143	2857.4	28572	0.8571	0.3571	0.3175	0.3061
4	0.6190	3809.9	38096	1.1429	0.4762	0.4233	0.4082
5	0.5238	4763.4	47620	1.4286	0.5952	0.5291	0.5102
6	0.4286	5714.9	57143	1.7143	0.7143	0.6349	0.6122
7	0.3333	6667.3	66667	2.0000	0.8333	0.7407	0.7143
8	0.2381	7619.8	76191	2.2857	0.9524	0.8466	0.8163
9	0.1429	8572.3	85715	2.5714	1.0714	0.9524	0.9184
10	0.0476	9524.8	95239	2.8571	1.1905	1.0582	1.0204

Integrating equation (30) partially with respect to 'x', with the limits from 0 to ∞, we have

$$R(z)=\frac{\alpha z^K P(0,z)[1-G^*(a)][1-H^*(m)]}{am} \quad (31)$$

Multiplying equation (30) by $\gamma(x)$ and integrating partially with respect to ' ', with the limits from 0 to ∞, we have

$$\int_0^\infty R(x,z)\gamma(x)dx=\frac{\alpha z^K P(0,z)[1-G^*(a)]H^*(m)}{a} \quad (32)$$

Now using equation (23), (28) and (32) in equation (18), we have

$$P(0,z)=\frac{aQ(z)m}{D} \quad (33)$$

(3)

where

$$D=\alpha z^K[1-G^*(a)]H^*(m)-a[z^K-B^*(m)G^*(a)]$$

Substituting $P(0,z)$ in the equation (22), (27) and (31), we have

$$P(z)=\frac{mQ(z)[1-G^*(a)]}{D} \quad (34)$$

$$V(z)=\frac{aG^*(a)[1-B^*(m)]Q(z)}{D} \quad (35)$$

$$R(z)=\frac{\alpha z^K Q(z)[1-G^*(a)][1-H^*(m)]}{D} \quad (36)$$

Now adding (34) ,(35) and (36), we have

$$S(z)=P(z)+V(z)+R(z), \quad (37)$$

Here S(z) represent the probability generating function of number of customer in the queue and independent of the server state.

$$S(z)=\frac{N}{D} \quad (38)$$

where m= λ - λ C(z), a= λ - λ C(z)+ α and

$$N=Q(z)\{[m+\alpha z^K(1-H^*(m))][1-G^*(a)]+aG^*(a)[1-B^*(m)]\}$$

We know that S(z) is probability generating function, it surely converge in the region $|z|<1$. Here it can be seen that the denominator D has 'K' zero. By applying Rouches theorem, we can conclude that K-1 zero's of D lies in the region $|z|<1$, and this must coinside with K-1 zero's N, and exactly one zero lies in the region $|z|>1$. Let z_0 be the zero which lies in the region $|z|>1$. Since S(z) converges, So K-1 zero's of N and D of S(z) will be cancelled.

Therefore we have

$$S(z)=\frac{B}{z-z_0} \quad (39)$$

By substituting z=1 ,we have $B=(1-z_0)S(1)$, But since

Table 5.2 Arrival rate versus performance measures Q and E(N) ($\alpha=10, \beta=20, \gamma=10, \mu=30, K=10, s=0.9$)

λ	Q	E(N)					
		Z_0 values					
		1.0001	1.00001	1.5	5	10	15
1	0.9870	129.6	1296.3	0.0389	0.0162	0.0144	0.0139
2	0.9741	259.2	2592.6	0.0778	0.0324	0.0288	0.0278
3	0.9611	388.9	3888.9	0.1167	0.0486	0.0432	0.0417
4	0.9481	518.5	5185.2	0.1556	0.0648	0.0576	0.0556
5	0.9352	648.2	6481.5	0.1944	0.0810	0.0720	0.0694
6	0.9222	777.8	7777.9	0.2333	0.0972	0.0864	0.0833
7	0.9093	907.4	9074.2	0.2722	0.1134	0.1008	0.0972
8	0.8963	1037.1	10370	0.3111	0.1296	0.1152	0.1111
9	0.8833	1166.8	11667	0.3500	0.1458	0.1296	0.1250
10	0.8704	1296.4	12963	0.3889	0.1620	0.1440	0.1389

$$S(1)=\frac{I_1}{I_2} \tag{40}$$

where

$$I_1=Q\lambda E(X)\{[1-G^*(\alpha)][1+\alpha E(R)]+\alpha G^*(\alpha)E(V)\}$$
$$I_2=\alpha G^*(\alpha)[K-\lambda E(X)E(V)]-\lambda E(X)[1-G^*(\alpha)][1+\alpha E(R)]$$

By substituting the equation (40) , we have

$$B=\frac{(1-z_0)I_1}{I_2} \tag{41}$$

Substituting the equation (41) in (38), we have

$$S(z)=\frac{(z_0-1)I_1}{z_0 I_2}\sum_{n=0}^{\infty}\left(\frac{z}{z_0}\right)^n \tag{42}$$

Which is probability generating function of number of customer in the queue.

3. System Performance Measures

In this section, the system performance measures, the mean number of customers in the queue and idle probability have been calculated.

(i) The Expected number of customers in the queue

$$E(N)=S'(1)=\frac{I_1}{(z_0-1)I_2} \tag{43}$$

(ii) The idle probability

Since $Q+S(1)=1$, where $Q=\sum_{n=0}^{K-1}Q_n$, which leads to

$$Q=1-\lambda E(X)\left\{\frac{1}{\alpha KG^*(\alpha)}-\frac{1}{\alpha K}+\frac{E(R)}{KG^*(\alpha)}-\frac{E(R)}{K}+\frac{E(V)}{K}\right\} \tag{44}$$

4. Some Particular Case

Here there are three particular case have been obtained by assigning particular forms to the parameters and to the distribution functions.

Particular Case-01

In the above model, we assume that batch arrival size random variable X follows geometric distributin with

probablilty $C_n=(1-s)^{n-1}s$ for $n\geq 1$ and s=1-t, then

$E(X)=\frac{1}{s}$. Also we assume that the service time random

Table 5.3 Arrival rate versus performance measures Q and E(N)
($\beta=20,\ \alpha=10, \gamma=10, \mu=30, K=5, s=0.5$)

λ	Q	E(N)					
		Z_0 values					
		1.0001	1.00001	1.5	5	10	15
1	0.9533	466.7	4666.7	0.1400	0.0583	0.0519	0.0500
2	0.9067	933.4	9333.4	0.2800	0.1167	0.1037	0.1000
3	0.8600	1400.1	14000	0.4200	0.1750	0.1556	0.1500
4	0.8133	1866.9	18667	0.5600	0.2333	0.2074	0.2000
5	0.7667	2333.6	23334	0.7000	0.2917	0.2593	0.2500
6	0.7200	2800.3	28000	0.8400	0.3500	0.3111	0.3000
7	0.6733	3267.0	32667	0.9800	0.4083	0.3630	0.3500
8	0.6267	3733.7	37334	1.1200	0.4667	0.4148	0.4000
9	0.5800	4200.4	42000	1.2600	0.5250	0.4667	0.4500
10	0.5333	4667.1	46667	1.4000	0.5833	0.5185	0.5000

variables follows exponential distribution with mean $\frac{1}{\mu}$ then $G^*(\alpha)=\frac{\mu}{\alpha+\mu}$, and the repair time random variable R follows exponential distribution with mean $\frac{1}{\gamma}$, In addition, we assume that vacation time random variable V follows exponential distribution with mean $\frac{1}{\beta}$.

Now equations (42), (43) and (44) becomes

$$S(z)=\frac{(z_0-1)Q\lambda[\beta(\alpha+\gamma)+\mu\gamma]}{z_0\{\mu\gamma(Ks\beta-\lambda)-\lambda\beta(\alpha+\gamma)\}}\sum_{n=0}^{\infty}(\frac{z}{z_0})^n$$

The idle probability

$$Q=\frac{\beta\mu\gamma Ks-\lambda[\beta(\alpha+\gamma)+\mu\gamma]}{\beta\mu\gamma Ks}$$

$$E(N)=\frac{Q\lambda[(\alpha+\gamma)\beta+\mu\gamma]}{(z_0-1)\{\mu\gamma[Ks\beta-\lambda]-\beta\lambda(\alpha+\gamma)\}}$$

Particular Case-02

If we put K=1,we get a model with services are given singly.

$$S(z)=\frac{QJ_1}{J_2}$$

where

$$J_1=\{[m+\alpha z(1-H^*(m))][1-G^*(a)]+aG^*(a)[1-B^*(m)]\}$$

$$J_2=\alpha z[1-G^*(a)]H^*(m)-a[z-B^*(m)G^*(a)]$$

and m= λ - λ C(z), a= λ - λ C(z)+ α

The Idle probability

$$Q=1-\lambda E(X)\{\frac{1}{\alpha G^*(\alpha)}-\frac{1}{\alpha}+\frac{E(R)}{G^*(\alpha)}-E(R)+E(V)\}$$

$$E(N)=\frac{Q\lambda E(X)L_1}{(z_0-1)L_2}$$

where $L_1=\{[1-G^*(\alpha)][1+\alpha E(R)]+\alpha G^*(\alpha)E(V)\}$

$$L_2=\{\alpha G^*(\alpha)[1-\lambda E(X)E(V)]-\lambda E(X)[1-G^*(\alpha)][1+\alpha E(R)]\}$$

Particular Case-03

If we put K=1,and X=1, we get a model with services are given singly.

$$S(z)=\frac{QJ_1}{J_2}$$

where

$$J_1=\{[m+\alpha z(1-H^*(m))][1-G^*(a)]+aG^*(a)[1-B^*(m)]\}$$

$$J_2=\alpha z[1-G^*(a)]H^*(m)-a[z-B^*(m)G^*(a)]$$

and m= λ - λ z, a= λ - λ z+ α

The idle probability

$$Q=1-\lambda\{\frac{1}{\alpha G^*(\alpha)}-\frac{1}{\alpha}+\frac{E(R)}{G^*(\alpha)}-E(R)+E(V)\}$$

$$E(N)=\frac{Q\lambda L_1}{(z_0-1)L_2}$$

where

$$L_1=\{[1-G^*(\alpha)][1+\alpha E(R)]+\alpha G^*(\alpha)E(V)\}$$

$$L_2=\{\alpha G^*(\alpha)[1-\lambda E(V)]-\lambda[1-G^*(\alpha)][1+\alpha E(R)]\}$$

5. Numerical Illustration

Here we have given numerical illustrations related to the models in section 4. We fix the values of the parameters α, β, γ, μ, K, s, p and we vary the values of arrival rate λ. For various values of z_0, we find the values of E(N) and also the vlaues of Q for the particular model 01. The corresponding results are presented in Table 5.1, Table 5.2 and Table 5.3. From the table values, it is clear that, as the arrival rate increases, the idle probability decreases. Which is very much coincide with our expectations. Also the expected number of customers in the queue increases, for increasing values of λ. Again, which is very much coincide with our expectation. Surprisingly in all the models, If the zero z_0 increases from 1.00001 to 15, the expected number of customers in the queue considerably decreases.

6. Conclustion

In this article, arrival of random size X with a single server, batch service (fixed) with Compulsory vacation and with unreliable server has been completely analysed. To illustrate the analytical compatability of the model, we present some numerical examples by taking particular values to the parameters and particular form to the probability distribution. The model can be extended by taking the break down period as generally distributed.

7. References

1. Bacot JB, Dshalalow JH. A bulk input queueing system with batch gated service and multiple vacation policy. Mathematical and Computer Modelling. 2001; 34:873-86.
2. Choudhary G. A batch arrival queue with a vacation time under single vacation policy. Computers and operation research. 2002; 29:1941-55.
3. Choudhury G, Deka M. A single server queueing system with two phases of service subject to server break down and Bernoulli vacation. Applied Mathematical Modelliing. 2012; 36:6050-60.
4. Choudhury G, Deka M. A batch arrival unreliable server Bernoulli vacation queue with two phases of service delayed repair. International Journal of Operations Research. 2013; 10(3):134-52.
5. Doshi BT. Single server queue with vacation; a survey. Queuing systems. 1986; I:29-66.
6. Grey WL, Wang PP, Scatt MK. A vacation queueing model with service break down. Applied Mathematical Modelling. 2000; 24:391-400.
7. Haridass M, Arrumuganathan R. Analysis of a bulk queue with unreliable server and single vacation. International Journal of Open Problems Compt.Math.2008; 1(2):130-48.
8. Kalyanaraman R, Nagarajan P. Bulk arrival, fixed batch service queue with unreliable server and with Bernoulli vacation. International Journal of Applied Engineering Research. 2016; 11(1):421-29.
9. Ke JC. Modified T Vacation policy for an M/G/1 queuing system with an unreliable server and start up. Mathematical and Computer Modelling. 2005; 41:1267-77.
10. Ke JC. Operation characteristics analysis of $M^X/G/1$ system with varient vacation policy and balking. Applied Mathematical Modelling. 2007; 31:1321-37.
11. Ke JC, Huang KB, Peam WL. A batch arrival queue under randomized multple vacation policy with unreliable server and repair. International Journal of System Science. 2012; 13(2):552-65.
12. Lee SS, Lew HW, Yoon SH, Chae KC. Batch arrival queue with N Policy and single vacation. Computer and Operations Research. 1995; 22:173-89.
13. Li W, Shi D, Chao X. Reliablity analysis of M/G/1 queueing system with server breakdown and vacations. Journal of Applied Probability. 1997; 34:546-55.
14. Takagi H. Amsterdam, North-Holland: Queueing Analysis vacation and priority system. 1991; 29.
15. Wang KH. Optimal operation of a Markovian queuing system with a removable and non-reliable server. Micro Electron Reliab. 1995; 35:1131-36.
16. Wang KH. Optimal control of an $M/E_k/1$ queuing system with removable service station subject to breakdow. Operation Research Society. 1997; 48:936-42.
17. Wang KH, Chang KW, Sivazlian BD. Optimal control of removable and non-reliable server in an infinite and finite $M/H_Z/1$ queueing system. Applied Mathematical Modelling. 1999; 23:651-66.