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Generalizing Modified Homotopy Perturbation Method to Study the Large Amplitude Vibration of Beams Subjected to an External Harmonic Excitation

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Abstract

Objectives: In present paper, large amplitude vibration behavior of an Euler-Bernoulli beam with immovable clamped-clamped boundary conditions subjected to an external harmonic excitation resting on Pasternak foundation is investigated. **Methods:** Assuming the mid-plane stretching in the beam and using the Newton's second law and then implementing the Galerkin's method, the ordinary nonlinear differential equation is derived. Because of the large coefficient of the nonlinear term, the traditional perturbation methods based on the small coefficient of the nonlinear term lead to an invalid solution. **Results:** To solve the obtained strongly nonlinear non-homogeneous equation, the Modified Homotopy Perturbation Method (MHPM) is generalized. In order to validate the results of MHPM, some experimental tests carried out. **Conclusion:** The results show a good agreement between analytical and experimental data. Moreover, the time response of the first and second order generalized MHPM follows accurately the time response obtained by numerical solution.

Keywords: Euler-Bernoulli Beam, Generalized MHPM, Harmonic Excitation, Large Amplitude Vibration of Beams, Pasternak Foundation

1. Introduction

There are many difficulties encountered in the application of perturbation techniques to the study of nonlinear problems. All classical perturbation techniques rely on the assumption of the small parameter. To overcome the limitations[1–4] presented some approximate analytical methods to solve the nonlinear equations. There are many approximate analytical methods for solving the nonlinear equations, including the perturbation techniques[5,6], the homotopy methods[7–9], frequency-amplitude formulation[10], energy balance method[11,12], harmonic balance method[13], modified variational approach[14] and max-min method[15]. In spite of the other perturbation techniques, the homotopy methods are applicable to strongly nonlinear systems. In[16] employed the HAM to obtain analytical expressions for the nonlinear fundamental frequency and deflection of Euler-Bernoulli beams. In[17] used this method for studying the vibration behavior of beams with damping nonlinearity. In[18] studied various finite element formulations to the large amplitude vibration of a hinged-hinged beam with immovable ends and presented an analytical formulation based on the Rayleigh-Ritz method. In[19] presented the Modified Homotopy Perturbation Method (MHPM) to study the large amplitude free vibration behavior of a pretensioned beam with clamped–clamped immovable ends. They modified He's new perturbation technique[20] and shown that their new presented method has higher accuracy than HPM and VIM. In this paper the Modified Homotopy Perturbation Method which can be used only for studying the free vibration analysis of beams is generalized to analyze the forced vibration cases. To this end, an Euler-Bernoulli clamped-clamped beam subjected to an external harmonic excitation which is rested on a Pasternak foundation is assumed. Applying the Von-

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Karman nonlinear strain-displacement relation and the Newton's second law and by implementing the Galerkin's Method, the nonlinear equation of motion is derived. To solve this nonhomogeneous strongly nonlinear equation, the MHPM which is already presented by is generalized. For validating the results of MHPM, some experimental tests are carried out. Moreover, the time response of the first and second order generalized MHPM follows accurately the time response obtained by the Runge-Kutta Method.

2. Equation of Motion

A schematic of Euler-Bernoulli beam with a length of L, cross-sectional area of A, density of ρ, area moment of inertia of I and the elasticity modulus of E, resting on a Pasternak foundation is shown in Figure 1. Considering an element of the beam as Figure 2 and using the Newton's second law, one can obtain the equation of motion. $\mathbf{f}$ represents the reaction force of the foundation.

$$f = k_L w + k_{NL} w^3 \tag{1}$$

Where k_L and k_{NL} are linear and nonlinear foundation stiffness, respectively.

The equilibrium of moments around point O is written

$$\sum M_O = 0 \rightarrow -V\mathbf{d}x + M - \left(M + \frac{\partial M}{\partial x}\mathbf{d}x\right) - f\frac{(\mathbf{d}x)^2}{2} = 0 \rightarrow V \approx -\frac{\partial M}{\partial x} \tag{2}$$

The strain-displacement relations for a beam undergoing large deflections are as:

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \qquad \kappa_x = -\frac{\partial^2 w}{\partial x^2} \tag{3}$$

Where u is the longitudinal displacement, w is the lateral displacement, and x is the longitudinal coordinate. The bending moment will be as:

$$M = -EI\kappa_x = EI\frac{\partial^2 w}{\partial x^2} \tag{4}$$

The value of inertial force is $\rho A \frac{\partial^2 w}{\partial t^2}\mathbf{d}x$ and its direction will be downward.

The sum of all forces in the y-direction or vertical is as:

$$\sum F_y = -V + \left(V + \frac{\partial V}{\partial x}\mathbf{d}x\right) - P\sin\theta + \left(P + \frac{\partial P}{\partial x}\mathbf{d}x\right)\left(\theta + \frac{\partial \theta}{\partial x}\mathbf{d}x\right) - f\mathbf{d}x = \rho A\frac{\partial^2 w}{\partial t^2}\mathbf{d}x \tag{5}$$

With some simplifications, the Equation (5) is rewritten as:

$$\frac{\partial V}{\partial x}\mathbf{d}x + P\frac{\partial \theta}{\partial x}\mathbf{d}x - f\mathbf{d}x = \frac{\partial^2 M}{\partial x^2}\mathbf{d}x + P\frac{\partial^2 w}{\partial x^2}\mathbf{d}x - k_L w\mathbf{d}x - k_{NL}w^3\mathbf{d}x = \rho A\frac{\partial^2 w}{\partial t^2}\mathbf{d}x \tag{6}$$

The value of force P is assumed as:

$$P = P_0 + P_1 \tag{7}$$

Where P_0 is the initial pretension force in the beam and P_1 is the initial force due to mid-plane stretching and its value is:

$$P_1 = \varepsilon_x EA \tag{8}$$

Integrating from strain relation in Equations (3) and assuming immovable boundary conditions, one obtains:

$$u(l,t) - u(0,t) = \int_0^l \varepsilon_x\,\mathbf{d}x - \int_0^l \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \mathbf{d}x = 0 \tag{9}$$

Consequently from Equations (9), we have:

$$\varepsilon_x = \frac{1}{2l}\int_0^l \left(\frac{\partial w}{\partial x}\right)^2 \mathbf{d}x \tag{10}$$

From Equations (7), (8) and (10), one obtains:

$$P = P_0 + \frac{EA}{2l}\int_0^l \left(\frac{\partial w}{\partial x}\right)^2 \mathbf{d}x \tag{11}$$

If a concentrated vertical force, F, is applied on the beam at distance x_0 from the left side of the beam, and using Equations (4), (6) and (11) yields:

$$EI\frac{\partial^4 w}{\partial x^4} - P_0\frac{\partial^2 w}{\partial x^2} - \frac{EA}{2l}\left(\int_0^l \left(\frac{\partial w}{\partial x}\right)^2 \mathbf{d}x\right)\frac{\partial^2 w}{\partial x^2} + \rho A\frac{\partial^2 w}{\partial t^2} + k_L w + k_{NL}w^3 = F\delta(x - x_0)\cos\Omega t \tag{12}$$

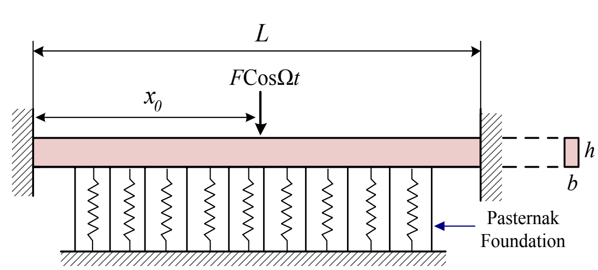

Figure 1. A clamped-clamped beam subjected to an external harmonic excitation and rested on Pasternak foundation.

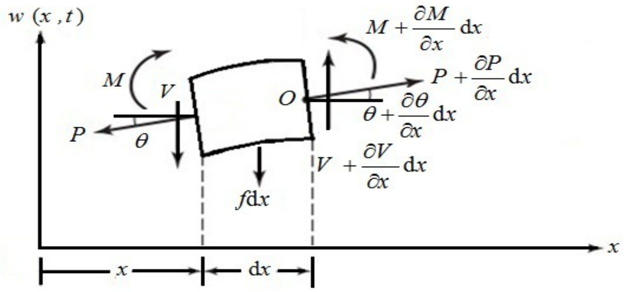

Figure 2. An element of the beam with applied forces and moments.

3. Non-Dimensionalization of Equation of Motion

It is common and efficient to work with the dimensionless antities are defined as:

$$\hat{t} = \omega_i t\,, \qquad \hat{x} = \frac{x}{l}\,, \qquad \hat{w} = \frac{w}{l} \tag{13}$$

Where i is the number of excited mode and ω_i is the corresponding linear natural frequency and it is defined as:

$$\omega_i = (\beta_i L)^2 \sqrt{\frac{EI}{\rho A L^4}} \tag{14}$$

$\beta_i L$ is the eigenvalue of the beam with clamped-clamped boundary conditions. Substitution of Equation (13) and Equation (14) in to Equation (12) yields:

$$\frac{\partial^4 \hat{w}}{\partial \hat{x}^4} - \frac{P_0}{EA}\left(\frac{L}{r}\right)^2 \frac{\partial^2 \hat{w}}{\partial \hat{x}^2} - \frac{1}{2}\left(\frac{L}{r}\right)^2 \left(\int_0^1 \left(\frac{\partial \hat{w}}{\partial \hat{x}}\right)^2 \mathrm{d}\hat{x}\right) \frac{\partial^2 \hat{w}}{\partial \hat{x}^2}$$

$$+ (\beta_i L)^4 \frac{\partial^2 \hat{w}}{\partial \hat{t}^2} \tag{15}$$

$$+ \frac{k_L L^2}{EA}\left(\frac{L}{r}\right)^2 \hat{w} + \frac{k_{NL} L^4}{EA}\left(\frac{L}{r}\right)^2 \hat{w}^3 = \frac{F L^3}{EI}\delta(\hat{x} - 0.5)\cos\frac{\Omega}{\omega_i}\hat{t}$$

Where $r = \sqrt{\frac{I}{A}}$ is the radius of gyration of the beam cross section. The solution of Equation (15) can be assumed as $\hat{w}(\hat{x},\hat{t}) = \varphi_i(\hat{x}) q(\hat{t})$ where $\varphi_i(\hat{x})$ is the comparison function of mode shape i of the beam. For the clamped-clamped beam $\varphi_i(\hat{x})$ can be considered as follows:

$$\varphi_i(\hat{x}) = \cosh(\beta_i L \hat{x}) - \cos(\beta_i L \hat{x}) -$$

$$\frac{\cosh(\beta_i L) - \cos(\beta_i L)}{\sinh(\beta_i L) - \sin(\beta_i L)}\left(\sinh(\beta_i L \hat{x}) - \sin(\beta_i L \hat{x})\right) \tag{16}$$

Applying Galerkin technique to Equation (16) results in a second order nonlinear ordinary differential equation as:

$$(\beta_i L)^4 \left(\int_0^1 \varphi^2 \,\mathrm{d}\hat{x}\right)\ddot{q} + \left\{\left(\left(\int_0^1 \varphi \varphi^{(4)} \,\mathrm{d}\hat{x}\right) - \frac{P_0}{EA}\left(\frac{L}{r}\right)^2\right.\right.$$

$$\left.\left(\int_0^1 \varphi \varphi'' \,\mathrm{d}\hat{x}\right) + \frac{k_L L^2}{EA}\left(\frac{L}{r}\right)^2 \left(\int_0^1 \varphi^2 \,\mathrm{d}\hat{x}\right)\right\} q + \tag{17}$$

$$\left\{-\frac{1}{2}\left(\frac{L}{r}\right)^2 \left(\int_0^1 \varphi \square^2 \,\mathrm{d}\hat{x}\right)\left(\int_0^1 \varphi \varphi'' \,\mathrm{d}\hat{x}\right) + \frac{k_{NL} L^4}{EA}\left(\frac{L}{r}\right)^2\right.$$

$$\left.\left(\int_0^1 \varphi^4 \,\mathrm{d}\hat{x}\right)\right\} q^3 = \frac{F L^3}{EI}\varphi(0.5)\cos\frac{\Omega}{\omega_i}\hat{t}$$

Equation (17) can be rewritten as:

$$\ddot{q} + \omega_0^2 q + \hat{\beta} q^3 = \hat{F}\cos\hat{\Omega}\hat{t} \tag{18}$$

Where:

$$\omega_0^2 = \frac{f_4 EI - f_2 P_0 L^2 + f_1 k_L L^4}{f_1 EI \beta_i^4 L^4} \tag{19}$$

$$\hat{\beta} = \frac{-EA f_2 f_3 + 2 f_5 k_{NL} L^4}{2 f_1 EI \beta_i^4 L^2}$$

$$\hat{F} = \frac{f_5 F L^3}{f_1 EI \beta_i^4 L^4}$$

$$\hat{\Omega} = \frac{\Omega}{(\beta_i L)^2 \sqrt{\frac{EI}{\rho A L^4}}}$$

And:

$$f_1 = \int_0^1 \varphi^2 \,\mathrm{d}\hat{x}\,, \qquad f_2 = \int_0^1 \varphi \varphi'' \,\mathrm{d}\hat{x}\,, \qquad f_3 = \int_0^1$$

$$\varphi \square^2 \,\mathrm{d}\hat{x}\,, \qquad f_4 = \int_0^1 \varphi \varphi^{(4)} \,\mathrm{d}\hat{x}\,, \qquad f_5 = \varphi(0.5)$$

(20)

The initial conditions of the beam are considered as:

$$q(0) = 0\,, \qquad \dot{q}(0) = 0 \tag{21}$$

From Equation (18) and Equation (21), to take account of the external loading and the initial conditions, the response of the system is assumed as:

$$q(\hat{t}) = \hat{Y}(\cos\hat{\Omega}\hat{t} - \cos\hat{\sigma}\hat{t}) \tag{22}$$

In which $\hat{Y}$ is the amplitude of vibration and $\hat{\sigma}$ is the correction frequency. These both unknown coefficients will be obtained using MHPM. The initial approximation is assumed as:

$$q_0(\hat{t}) = \hat{Y}\cos\hat{\Omega}\hat{t} \tag{23}$$

Based on the MHPM, the terms q and $\hat{\Omega}^2$ are considered as below:

$$q = q_0 + \hat{\beta}q_1 + \hat{\beta}^2 q_2 + \cdots \tag{24}$$

$$\hat{\Omega}^2 = \omega_0^2 + \hat{\beta}\omega_1 + \hat{\beta}^2\omega_2 + \cdots \tag{25}$$

Considering $\hat{F} = \hat{\beta}\bar{F}$ and substituting the Equation (24) and Equation (25) in to Equation (18) yields:

$$\hat{\beta}^1: \frac{d^2 q_1}{d\hat{t}^2} + \omega_0^2 q_1 = \omega_1 q_0 - q_0^3 + \bar{F}\cos\hat{\Omega}\hat{t} \tag{26}$$

$$\hat{\beta}^2: \frac{d^2 q_2}{d\hat{t}^2} + \omega_0^2 q_2 = \omega_1 q_1 + \omega_2 q_0 - 3q_0^2 q_1 \tag{27}$$

By substituting Equation (23) in to Equation (26) we have:

$$\frac{d^2 q_1}{d\hat{t}^2} + \omega_0^2 q_1 = \left(\omega_1\hat{Y} - \frac{3}{4}\hat{Y}^3 + \bar{F}\right)\cos\hat{\Omega}\hat{t} - \frac{1}{4}\hat{Y}^3 \cos 3\hat{\Omega}\hat{t} \tag{28}$$

In order to avoid the secular term, the coefficient of $\cos\hat{\Omega}\hat{t}$ will be equated to zero, so:

$$\omega_1\hat{Y} - \frac{3}{4}\hat{Y}^3 + \bar{F} = 0 \tag{29}$$

Then:

$$\omega_1 = \frac{3}{4}\hat{Y}^2 - \frac{\bar{F}}{\hat{Y}} \tag{30}$$

Using Equation (25), assuming only the first order approximate solution and neglecting $O(\hat{\beta}^2)$, we obtain:

$$\omega_1 = \frac{\hat{\Omega}^2 - \omega_0^2}{\hat{\beta}} \tag{31}$$

From Equation (30) and Equation (31), one reaches:

$$\left(\frac{\hat{\Omega}^2 - \omega_0^2}{\hat{\beta}}\right)\hat{Y} - \frac{3}{4}\hat{Y}^3 + \bar{F} = 0 \tag{32}$$

Then:

$$\frac{3}{4}\hat{\beta}\hat{Y}^3 - \left(\hat{\Omega}^2 - \omega_0^2\right)\hat{Y} - \hat{F} = 0 \tag{33}$$

Solving the equation, the value of $\hat{Y}$ will be defined for the first approximation. In addition, the frequency response can be determined by defining the value of $\hat{Y}$ for varying excitation frequencies. It is also worth mentioning that the Equation (18) has cubic nonlinearity; therefore, super harmonic resonance at $\omega_0 = \frac{1}{3}\hat{\Omega}$ with sub harmonic resonance at $\omega_0 = 3\hat{\Omega}$ will be occurred. Next, in order to determine the correction frequency, $\hat{\sigma}$, a same procedure as one demonstrated for obtaining the nonlinear resonance frequency is used. To this aim, the initial conditions are assumed to be the same as one considered for free vibration analysis with $q(0) = a_0 = \hat{Y}$, $\dot{q}(0) = 0$, in which $\hat{Y}$ is the amplitude of vibration obtained from Equation (33) previously[21]. Using MHPM and considering the first order approximation, one can obtain:

$$\hat{\sigma} = \sqrt{\omega_0^2 + \frac{3}{4}\hat{\beta}\hat{Y}^2} \tag{34}$$

Therefore, knowing $\hat{Y}$ and $\hat{\sigma}$, the time response of the beam under the harmonic load is defined from Equation (22) for the first order approximation. To obtain the second order approximate solution, q_1 will be obtained by solving the Equation (28):

$$q_1(t) = \frac{\hat{Y}^3}{4\left(9\hat{\Omega}^2 - \omega_0^2\right)}\left(\cos 3\hat{\Omega}\hat{t} - \cos\hat{\Omega}\hat{t}\right) \tag{35}$$

Substituting Equation (23) and Equation (35) into Equation (27) yields:

$$\frac{d^2 q_2}{d\hat{t}^2} + \omega_0^2 q_2 = \left(\frac{3}{4}\hat{Y}^2 - \frac{\bar{F}}{\hat{Y}}\right)\left(\frac{\hat{Y}^3}{4\left(9\hat{\Omega}^2 - \omega_0^2\right)}\left(\cos 3\hat{\Omega}\hat{t} - \cos\hat{\Omega}\hat{t}\right)\right)$$

$$+\omega_2\hat{Y}\cos\hat{\Omega}\hat{t} - 3(\hat{Y}^2\cos^2\hat{\Omega}\hat{t})\left(\frac{\hat{Y}^3}{4(9\hat{\Omega}^2 - \omega_0^2)}(\cos 3\hat{\Omega}\hat{t} - \cos\hat{\Omega}\hat{t})\right) \tag{36}$$

Then:

$$\frac{d^2 q_2}{d\hat{t}^2} + \omega_0^2 q_2 = \left(\frac{3\hat{Y}^5}{16\left(9\hat{\Omega}^2 - \omega_0^2\right)} + \frac{\bar{F}\hat{Y}^2}{4\left(9\hat{\Omega}^2 - \omega_0^2\right)} + \omega_2\hat{Y}\right)\cos\hat{\Omega}\hat{t} -$$

$$\left(\frac{\bar{F}\hat{Y}^2}{4(9\hat{\Omega}^2 - \omega_0^2)}\right)\cos 3\hat{\Omega}\hat{t} - \left(\frac{3\hat{Y}^5}{16(9\hat{\Omega}^2 - \omega_0^2)}\right)\cos 5\hat{\Omega}\hat{t}$$

$$+\omega_2\hat{Y}\cos\hat{\Omega}\hat{t} - 3(\hat{Y}^2\cos^2\hat{\Omega}\hat{t})\left(\frac{\hat{Y}^3}{4(9\hat{\Omega}^2 - \omega_0^2)}(\cos 3\hat{\Omega}\hat{t} - \cos\hat{\Omega}\hat{t})\right) \tag{37}$$

To avoid the secular term, the coefficient of $\cos\hat{\Omega}\hat{t}$ will be equated to zero, therefore:

$$\frac{3\hat{Y}^5}{16\left(9\hat{\Omega}^2-\omega_0^2\right)}+\frac{\bar{F}\hat{Y}^2}{4\left(9\hat{\Omega}^2-\omega_0^2\right)}+\omega_2\hat{Y}=0 \tag{38}$$

Then:

$$\omega_2=-\frac{3\hat{Y}^4}{16\left(9\hat{\Omega}^2-\omega_0^2\right)}-\frac{\bar{F}\hat{Y}}{4\left(9\hat{\Omega}^2-\omega_0^2\right)} \tag{39}$$

Substituting Equation (30) and Equation (39) in to the Equation (25), assuming only the second order approximate solution and neglecting $O(\hat{\beta}^3)$, we obtain:

$$\hat{\Omega}^2=\omega_0^2+\frac{3}{4}\hat{\beta}\hat{Y}^2-\frac{\bar{F}\hat{\beta}}{\hat{Y}}-\frac{3\hat{\beta}^2\hat{Y}^4}{16\left(9\hat{\Omega}^2-\omega_0^2\right)}-\frac{\bar{F}\hat{\beta}^2\hat{Y}}{4\left(9\hat{\Omega}^2-\omega_0^2\right)} \tag{40}$$

After some mathematical manipulations, we reach:

$$3\hat{\beta}^2\hat{Y}^5+\left(-108\hat{\beta}\hat{\Omega}^2+12\hat{\beta}\omega_0^2\right)\hat{Y}^3+4\hat{\beta}^2\bar{F}\hat{Y}^2 \tag{41}$$

$$+(-160\omega_0^2\hat{\Omega}^2+16\omega_0^4+144\hat{\Omega}^4)\hat{Y}-16\hat{\beta}\bar{F}\omega_0^2+144\hat{\beta}\bar{F}\hat{\Omega}^2=0$$

As mentioned before, the correction frequency, $\hat{\sigma}$, is the same as the nonlinear resonance frequency for the free vibration system with initial conditions $q(0)=a_0=\hat{Y}$, $\dot{q}(0)=0$ [21]. Where the value of $\hat{Y}$ can be calculated from Equation (41). Using MHPM and considering the second order approximation, one can obtain:

$$\hat{\sigma}=\frac{1}{4}\sqrt{8\omega_0^2+6\hat{\beta}\hat{Y}^2+\sqrt{64\omega_0^4+96\hat{\beta}\hat{Y}^2\omega_0^2+30\hat{\beta}^2\hat{Y}^4}} \tag{42}$$

So, knowing $\hat{Y}$ and $\hat{\sigma}$, the time response of the beam under the harmonic load is defined from Equation (22) for the second order approximation. In all of these formulations, the time response is considered to be dependent on just one frequency, i.e. only the primary resonance is studied.

4. Validating the Results of MHPM with the Experiments

To validate the MHPM results, some experimental tests were carried out on the clamped-clamped steel beam with the given characteristics in Table 1. The beam is subjected to various initial displacements at its mid-point and the acceleration response of the beam was captured using a 4507 B&K accelerometer and a 3109 B&K signal analyzer. The test setup is shown in Figure 3. In Table 2, the measured linear and nonlinear natural frequencies of the beam for various values of initial displacements are compared with the nonlinear natural frequencies obtained by the MHPM. As it can be seen, for various values of vibration amplitudes, there is a good agreement between the results obtained from the MHPM and the experiments. Figure 4 shows the variation of the nonlinear fundamental frequency of the beam against the maximum displacement at its mid-span. As it is seen, as the initial displacement at the beam mid-span increases, the nonlinear fundamental frequency rises. Moreover, this figure shows that the nonlinear fundamental frequencies obtained by the MHPM closely match with the corresponding experiments and the relative error is lower than 0.76%.

Table 1. The characteristics of the beam

3.9	b (mm)
6.4	h (mm)
485	L (mm)
7860	ρ (kg/m³)
190	E (GPa)
0.3	ν

Table 2. Frequencies of the beam obtained from MHPM method and experiments

Error Percentage (%)	Experimental Nonlinear Frequency (Hz)	Theoretical Nonlinear Frequency (Hz)	Experimental Linear Frequency (Hz)	Theoretical Linear Frequency (Hz)	W (mm)
0.44	137.5	136.9	136.0	136.7	1
0.29	139.2	139.6			2
0.76	142.8	143.9			3
0.20	150.0	149.7			4

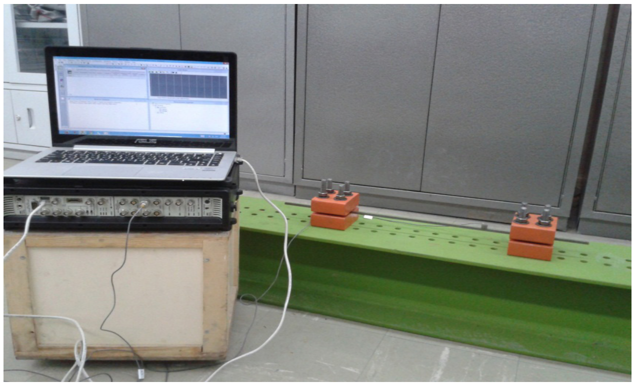

Figure 3. Set up for the experimental tests.

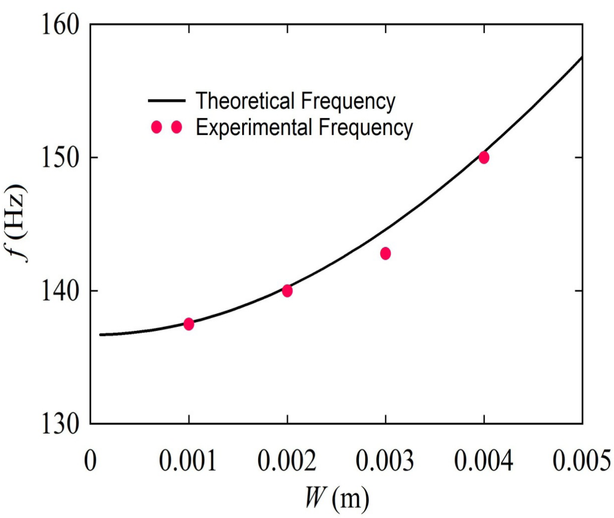

Figure 4. Variation of nonlinear frequency versus displacement.

5. Validating the Results of Generalized MHPM with Numerical Solution

After validating the MHPM results with the experiments, here we decide to validate the generalized MHPM results with the numerical solution. Figure 5 indicates the comparison between the time responses of forced vibration of the beam obtained by the first order of generalized MHPM with those obtained by the RK45. Moreover, in Figure 6, the time responses of forced vibration of the beam obtained by the second order of generalized MHPM with those obtained by the RK45 are compared. As these figures reveal, the results are in a very good agreement with each other. In addition, these figures show that the second order of generalized MHPM follows the RK45 more accurate than the first order of generalized MHPM. Consequently, using this new presented method, the strongly nonlinear vibrational response of the beam under an external harmonic excitation is accurately derived.

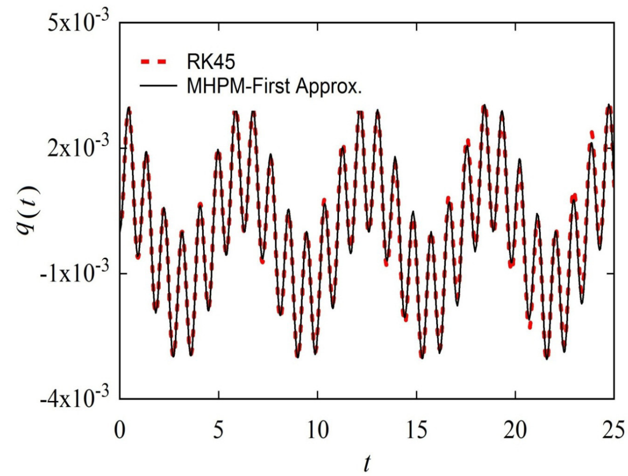

Figure 5. Comparing the responses obtained by the first order MHPM and the numerical method for the clamped-clamped beam under external harmonic excitation.

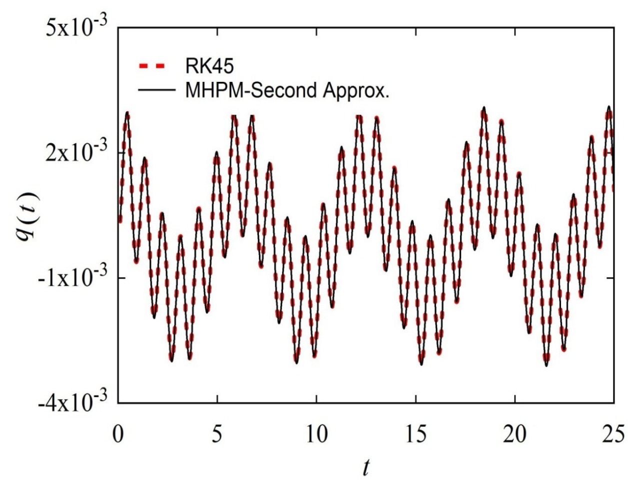

Figure 6. Comparing the responses obtained by the second order MHPM and the numerical method for the clamped-clamped.

6. Conclusion

In this study, large amplitude vibration behavior of an Euler-Bernoulli beam subjected to an external harmonic excitation which is rested on a Pasternak foundation is investigated. It is assumed that the beam vibrates only in a single mode. Employing the Von-Karman strain-displacement relations, using the Newton's second law and then implementing the Galerkin's method, the nonlinear ODE is derived. The coefficient of the nonlinear term would be very larger than unity; therefore, the traditional perturbation methods which are based on the small coefficient of the nonlinear term lead to an invalid solution. To solve the obtained strongly nonlinear non-homogeneous equation, the Modified Homotopy Perturbation Method (MHPM) is generalized. Also, to validate the results of MHPM, an experimental test was carried out. The results of analytical and experimental investigation were in a very good agreement. Moreover, the time response of the first and second order generalized MHPM follows accurately the time response obtained by Runge-Kutta solution.

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