ISSN (Print) : 0974-6846
ISSN (Online) : 0974-5645
Indian Journal of Science and Technology, *Vol 9(43), DOI: 10.17485/ijst/2016/v9i43/104756, November 2016*

Fuzzy Graph Structures and Its Properties

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Abstract

Objectives: To find the vertex cohesive number and edge cohesive number of Gear and Bistar fuzzy graph structure. **Methods/ Statistical Analysis:** Gear graph and Bistar graph is converted into a fuzzy graph by assigning membership function for vertices and edges. The edges with same membership function are grouped to get a gear and Bistar fuzzy graph structure. For this Gear and Bistar fuzzy graph structure, vertex and edge cohesive number are computed. **Findings**: The vertex and edge cohesive number of Gear and Bistar fuzzy graph structure are found. **Application**: In any organisation, the employees can be treated as vertices. Keeping in mind how one employee co-ordinates with other employee, one can study how employees can work in groups.

Keywords: Bistar Graph, Gear Graph, Graph Structures, Hamacheer Product, Vertex and Edge Cohesive Numbers

1. Introduction

Graph theory began with finding a walk linking seven bridges in Konigsberg. Later, this field has developed enormously in all spheres of sciences and in humanities too with wide applications. The notion of fuzzy sets was introduced by [1] in 1965 which paved way to develop the new subject called Fuzzy graph theory. The first definition of Fuzzy Graph was introduced 1973, and then it was developed in 1975. Various concepts on fuzzy graphs were discussed authors 2002and soon. Graph structure concept was introduced by [2]. LaterFuzzy graph structures were introduced by [3–5]. Fuzzy graph structures for star, wheel and Helm graphs were studied by [6]. The fuzzy graph structures have various applications in the field of networks both in computers and social analysis.

In an organisation, assume each employee to be the set of vertices and connect any two persons with the particular work they perform by edges. A particular type of work is grouped as R_i's. Here, the independence of work and how cohesive they are can be studied and interpreted.

Preliminaries:

Definition 1.1: A graph[7] G(V,E) consist of vertices V and set of edges E which connect some or all the vertices of V.

Definition 1.2: Let V be a non-empty set. A fuzzy graph[8,9] is a pair of function $G(\sigma,\mu)$ where $\sigma: V\to[0,1]$ and $\mu: V\times V\to[0,1]$ such that $\mu(u,v)\leq\sigma(u)\wedge\sigma(v)\ \forall u,v$ in V.

Definition 1.3: A graph structure[2] $G=(V,R_1,R_2,\ldots,R_k)$ consists of a non-empty set V together with relations $R_1,R_2,\ldots,R_k$ on V which are mutually disjoint such that each R_i , $1\leq i\leq k$, is symmetric and irreflexive.

Definition 1.4: The capacity of a vertex[2] c(v) in a graph structure $G=(V,R_1,R_2,\ldots,R_k)$ is the numberof different R_i edges incident at v.

Definition 1.5: A graph structure $G=(V,R_1,R_2,\ldots,R_k)$ is t-saturated[2] for some $t\leq k$, if there exists a set of t-edges say of $\{R_1,R_2,\ldots,R_k\}$ which appear at each vertex of G.

Definition 1.6[3]**:** Saturation number[2] S(G) of a graph structure is maximum t, $1\leq t\leq k$ such that G is t-saturated.

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Definition 1.7: A set S of vertices in $R_1, R_2 \ldots R_k$ structure G is R_i – cohesive for some i, $1 \le i \le k$, if S is R_i – connected. The vertex cohesive number[2] $C_v(G)$ of a graph structure $G=(V,R_1,R_2 \ldots R_k)$ is the minimum order of a partition of V into cohesive sets. The edge cohesive number[2] $C_e(G)$ of G is the minimum order of a partition of the edge set E of G into cohesive sets. A set of vertices in a graph structure $G=(V,R_1,R_2 \ldots R_k)$ is R_i– connected for some i if any two vertices in S are connected by a R_i path.

Definition 1.8: Zadeh defined operations for fuzzy sets. Later extended using t-norm .One such function is Hamacheer product[1]: For $x, y \in [0,1]$

$$t(x,y) = \frac{xy}{x+y-xy} \in [0,1]$$

2. Example

First we give example of graph structure in crisp case and find its vertex and edge cohesive numbers.

Let the graph G(V,E) be(Figure1(a))

$V = \{v_1, v_2, v_3, v_4, v_5\}$

$E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_5, v_1), (v_5, v_2), (v_5, v_3), (v_5, v_4)\}$

The graph structure $G(V,R_1,R_2)$[5–7] is

$V = \{v_1, v_2, v_3, v_4, v_5\}$

$R_1 = \{(v_5, v_1), (v_5, v_2), (v_5, v_3), (v_5, v_4)\}$

$R_2 = \{(v_1, v_2), (v_2, v_3)\ (v_3, v_4)\ (v_4, v_1)\ \}$

Now, Consider the graph structure, $G(V, R_1,R_2)$ (Figure 1(b))

Here $c(v_1) = 2, c(v_2) = 2, c(v_3) = 2, c(v_4) = 2, c(v_5) = 1$.

Here, this part of the graph structure has one entity with all R_1 which includes all vertices.

$\mathbf{C_v(G) = 1}$ (Figure 1(c))

Here, Figure 1(c)is part of the graph structure which has one entity with all R_1 and Figure1(d)) is part of the graph structure which has one entity with all R_2. Both together which includes all edges.

$\mathbf{C_e(G) = 2}$ (Figure 1(c) and Figure1(d))

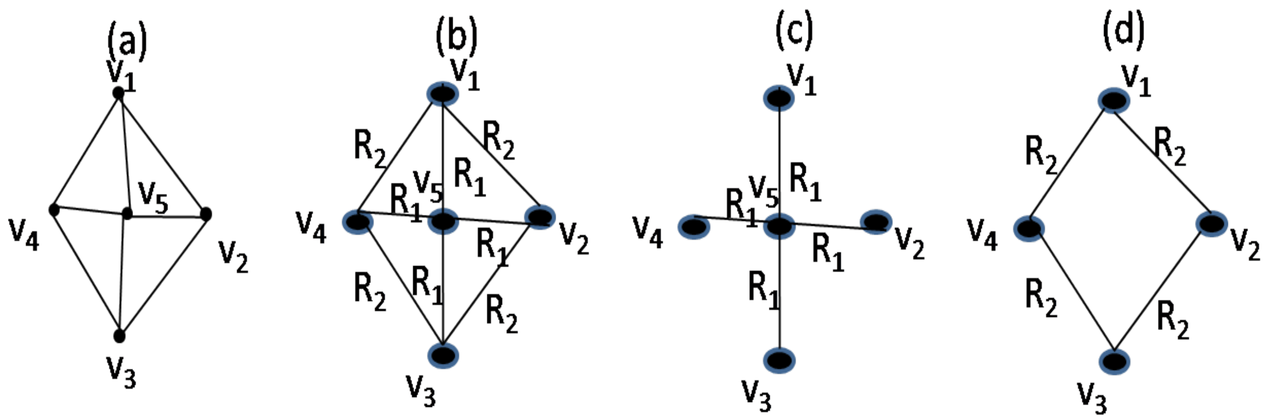

Figure 1. Graph and fuzzy graph structure.

3. Fuzzy Graph Structures

Here, a graph is considered for which membership function for vertices are defined. This graph becomes a fuzzy graph. So, using Hamacheer product we define the membership function for edges to define Fuzzy Graph from the crisp graph.

For this fuzzy graph, fuzzy graph structure concept is developed by grouping the edges with same membership function. Later, the cohesive number for this fuzzy graph structure is obtained. We have derived the same for Star, Wheel and Helm graphs[6].

Theorem 3.1: The vertex Cohesive number and edge Cohesive number of fuzzy graph structure of Gear graph is 2 and 2 respectively ie., $c_v(G_n) = 2, c_e(G_n) = 2.$

Proof: Consider G_n(Figure 2(a)) Let $Wn= Cn+ K_1$ be the wheel graph with apex vertex v and the rim vertices $v_1; v_2; :::; v_n$. To obtain the gear graph Gn, subdivide each rim edge of wheel Wn by the vertices $u_1; u_2; :::; u_n$ where each u_i subdivides the edge $v_i\, v_{i+1}$ for $i= 1; 2; ::; n - 1$ and u_n subdivides the edge $v_1 v_n$.

$d(v_i)$ - degree of vertex v_i

The vertex membership functions are defined as follows:

Here, $\rho(v_i) = \dfrac{1}{d(v_i)} = \dfrac{1}{3} = 0.3$

$$\rho(v_i) = 0.3 \ , \forall i = 1 to n$$

Here, $\rho(u_i) = \dfrac{1}{d(u_i)} = \dfrac{1}{2} = 0.5$

$$\rho(u_i) = 0.5, \forall i = 1 to n$$

$$\rho(v) = \frac{d(v)}{\sum d(v_i)}, \ v_i \text{ ranges over all vertices in } G_n$$

$$\rho(v) = \frac{n}{6n} = \frac{1}{6} = 0.16$$

$$\rho(v) = 0.16$$

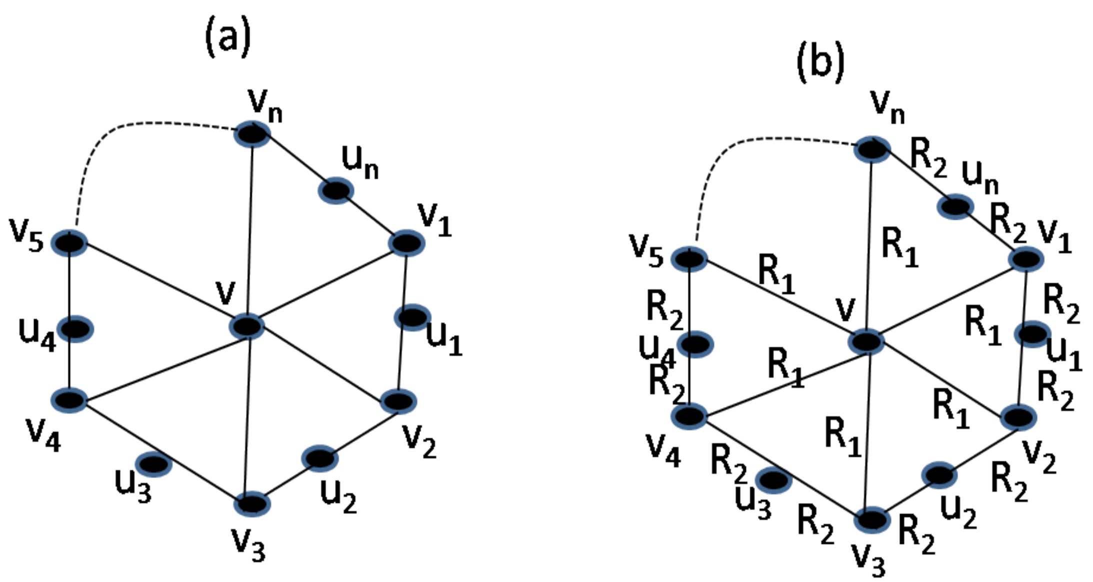

Figure 2. Gear graph and gear fuzzy graph structure.

Using Hamacheer product the edge membership functions are computed,

$$\mu(v,v_i)=\frac{\rho(v)\rho(v_i)}{\rho(v)+\rho(v_i)-\rho(v)\rho(v_i)}=0.1207$$

$$\mu(v_i,u_i)=\frac{\rho(v_i)\rho(u_i)}{\rho(v_i)+\rho(u_i)-\rho(v_i)\rho(u_i)}=0.2481$$

Group the same edge membership functions as R_1 and R_2

$R_1 = \{(v,v_1),(v,v_2),..., (v,v_{n-1}),(v,v_n)\}$

$R_2 = \{(v_1,u_1), (u_1,v_2), (v_2,u_2), (u_2,v_3), (v_3,u_3)..., (v_n,u_n), (u_n,v_1)\}$

Graph Structure of G_n is (G_n, R_1, R_2) (Figure2(b))

Capacity of the vertex is the number of different R_i incident on the vertex.

Capacities of vertices in are

$c(v_i) = 2$, i = 1 to n

$c(v) = 1$

$c(u_i) = 1$, i = 1 to n

A set S of vertices in $R_1,R_2,\ldots,R_k$ structure G is R_i – cohesive for some i, $1\le i\le k$, if S is R_i – connected. The vertex cohesive number $C_v(G)$ of a graph structure $G=(V,R_1,R_2,\ldots,R_k)$ is the minimum order of a partition of V into cohesive sets.

The edge cohesive number $C_e(G)$ of G is the minimum order of a partition of the edge set E of G into cohesive sets.

The vertex cohesive number and the edge cohesive number are

C_v $((G_n)=$ **2**(Figure 3(a)& Figure3(b)).

C_e $((G_n) =$ **2**(Figure 3(a)& Figure3(b)).

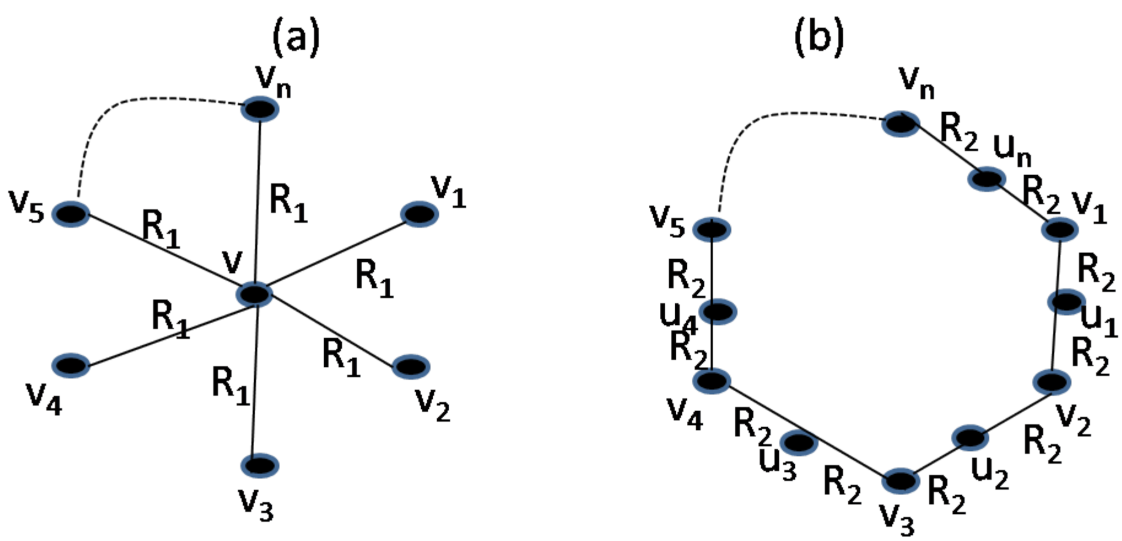

Figure 3. Components of gear fuzzy graph structure.

Theorem 3.2: The vertex Cohesive number and edge Cohesive number of fuzzy graph structure of Bistar graph is 2 and 3 respectively ie., $c_v(B_{n,n}) = 2, c_e(B_{n,n}) = 3$.

Proof: Consider $B_{n,n}$ (Figure4(a)) The graph *Bn;n; n* > 2 is a bistar obtained from two disjoint copies of $K_{1,n}$ by joining the centre vertices by an edge. It has 2*n*+2 vertices and 2*n*+1edges.

$d(v_i)$ - degree of vertex v_i

The vertex membership functions are defined as follows:

Here, $\rho(u_i) = \dfrac{1}{d(u_i)} = \dfrac{1}{1} = 1.$

$$\rho(u_i) = 1 \forall i = 1 ton$$

Here, $\rho(v_i) = \dfrac{1}{d(v_i)} = \dfrac{1}{1} = 1.$

$$\rho(v_i) = 1 \forall i = 1 ton$$

$\rho(u) = \dfrac{d(u)-1}{\sum d(v_i)-2}$, v_i ranges over all vertices in $B_{n,n}$

$$\rho(u) = \frac{n+1-1}{4n+2-2}$$

$$\rho(u) = 0.25$$

$\rho(v) = \dfrac{d(u)-1}{\sum d(v_i)-2}$, v_i ranges over all vertices in $B_{n,n}$

$$\rho(v) = \frac{n+1-1}{4n+2-2}$$

$$\rho(u) = 0.25$$

Using Hamacheer product the edge membership functions are computed,

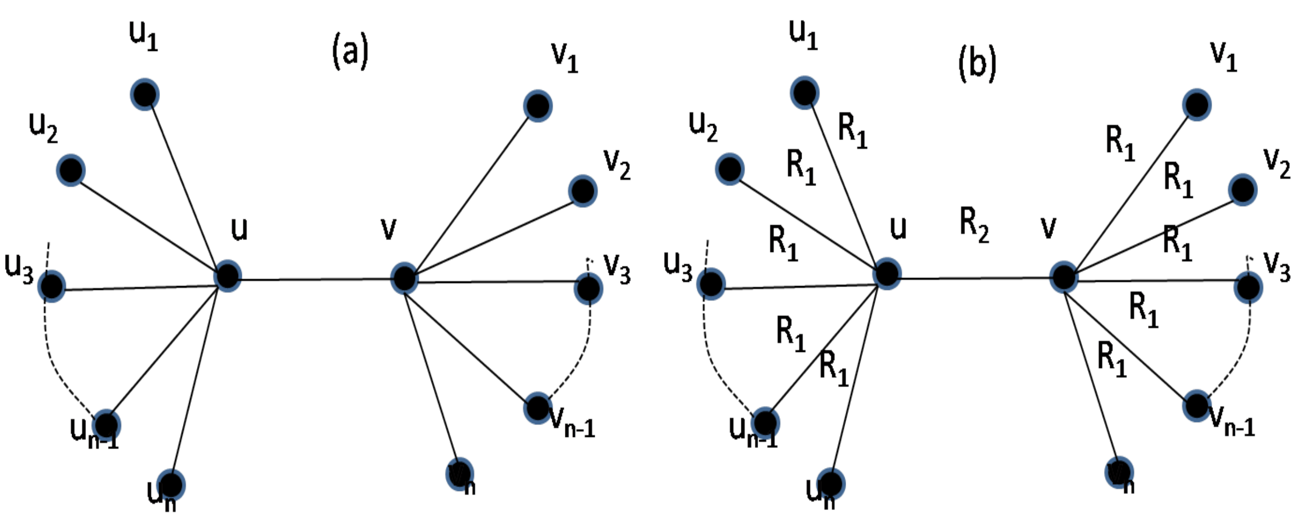

Figure 4. Bistar graph and Bistar fuzzy graph structure.

$$\mu(u,u_i)=\frac{\rho(u)\rho(u_i)}{\rho(u)+\rho(u_i)-\rho(u)\rho(u_i)}=0.25, i=1ton$$

$$\mu(v,v_i)=\frac{\rho(v)\rho(v_i)}{\rho(v)+\rho(v_i)-\rho(v)\rho(v_i)}=0.25, i=1ton$$

$$\mu(u,v)=\frac{\rho(u)\rho(v)}{\rho(u)+\rho(v)-\rho(u)\rho(v)}=0.1428$$

Group the same edge membership functions as R_1 and R_2

R_1 = {(u,u_1), (u,u_2),..., (u,u_{n-1}),(u,v_n), (v,v_1), (v,v_2),..., (v,v_{n-1}),(v,v_n)}

R_2 = {(u,v)}

Graph Structure of $B_{n,n}$ is ($B_{n,n}$, R_1, R_2)(Figure4(b))

Capacity of the vertex is the number of different R_i incident on the vertex.

Capacity of vertices in (V, R_1, R_2) are

c(u_i) = 1, i = 1 to n

c(v_i) = 1, i = 1 to n

c(u) = 2

c(v) = 2

A set of vertices in a graph structure G=(V,R_1,R_2...... R_k) is R_i– connected for some i if any two vertices in S are connected by a R_i path.

Therefore, Graph Structure of $B_{n,n}$ is R_1 connected and R_2 connected.

(The vertex cohesive number C_v(G) of a graph structure G=(V,R_1,R_2...... R_k) is the minimum order of a partition of V into cohesive sets. The edge cohesive number C_e(G) of G is the minimum order of a partition of the edge set E of G into cohesive sets.).

Here, $B_{n,n}$ has two entity with all R_1 includes all vertices, The vertex cohesive number **C_v($B_{n,n}$) = 2** (Figure 5(a)).

[Figure 5]

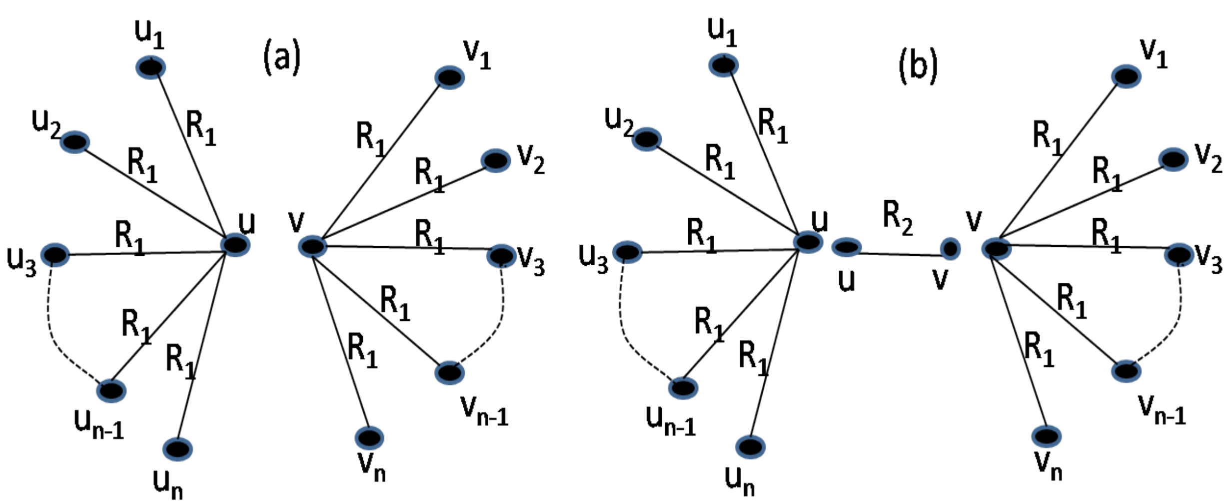

Figure 5. Components of Bistar fuzzy graph structure.

Here, $B_{n,n}$ has two entity with all R_1 and one entity with all R_2 includes all edges, the edge cohesive number

$\mathbf{C_e(B_{n,n}) = 3}$(Figure 5(b)).

4. Result

Fuzzy graph structure	Vertex Cohesive number	Edge Cohesive number
G_n	2	2
$B_{n,n}$	2	3

5. Conclusion

Fuzzy graph structures for Gear graph and Bistar graph are constructed and their properties are studied.

6. References

1. Zimmermann HJ. Fuzzy set theory-and applications. 2nd Revised edition, Kluwer Academic Publishers; 2001.
2. Kumar ES. Generalised graph structures. Bulletin of Kerala Mathematics Association. 2006; 3(2):67–123.
3. Ramakrishnan RV, Dinesh T. On generalised fuzzy graph structures. Applied Mathematical Sciences. 2011; 5(4):173–80.
4. Ramakrishnan RV, Dinesh T. On generalised fuzzy graph structures II. Advances in Fuzzy Mathematics. 2011; 6(1):5–12.
5. Ramakrishnan RV, Dinesh T. On generalised fuzzy graph structures III. Bulletin of Kerala Mathematics Association. 2011 Jun; 8(1):57–66
6. Harinath P, Lavanya S. Fuzzy graph structures. International Journal of Applied Engineering and Research. 2015; 10(80).
7. Harary F. Graph Theory. Addison-Wesley Publishing Co.: New York; 1969.
8. Nagoorgani, Chandrasekaran VT. A first look at fuzzy graph theory. Allied Publishers Pvt. Ltd: India; 2010.
9. Basha SS, Karthik E. Laplacian energy of an intiutionistic fuzzy graph. Indian Journal of Science and Technology. 2015; 8(33):1–7.