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Measuring Efficiency and Ranking Fully Fuzzy DEA

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Abstract

In this current literature, there are several models of Fully Fuzzy Data Envelopment Analysis (FFDEA) where inputs-outputs data and weights were fuzzy numbers. The main purpose of this study is to evaluate the performance of a set of Decision Making Units (DMUs) and fully ranking DMUs in fully fuzzified environment. In this paper, an FFDEA problem is discussed, and a method for solving this FFDEA problem is also proposed. This method based on the multi-objective linear programming and the simplex method is proposed for computing an optimal fuzzy solution to a FFDEA problem in which fuzzy ranking functions are not used. We acquire fuzzy efficiency scores with solving FFDEA. Afterwards, we have used a ranking function to rank these fuzzy scores. A numerical example is used to demonstrate and compare the results with those obtained using alternative approaches.

Keywords: Data Envelopment Analysis, Efficiency, Fuzzy Numbers, Multi-Objective

1. Introduction

Data Envelopment Analysis (DEA) is systematic approach for the analyzing the performance of organizations and operational processes. It can evaluate the relative efficiencies of homogeneous Decision Making Units (DMUs) without knowing production functions, only by using input and output data. The first model od DEA introduced by charnes et al[4]. To see the other classic models of DEA, the readers can see[3,5,7,15]. The basic DEA results group the DMUs into two sets: one set is efficient DMUs and the other is inefficient DMUs. In many cases, it is necessary to give a full ranking of the DMUs. For this purpose, different methods with different properties to achieve full ranking of DMUs have been proposed. Sexton et al. proposed the ranking of DMUs based on a cross efficiency[17]. The benchmarking, initially developed by Torgersen was employed to rank all the efficient DMUs[18]. Andersen and petersen first developed the most popular ranking method called super-efficiency[19]. Cooper and Tone ranked the DMUs according to scalar measuring of inefficiency in DEA, based on the slack variables[6]. Liu and Peng proposed Common Weights Analysis (CWA) to determine a set of indices for common weights to rank efficient DMUs of DEA[9]. In many situations, such as manufacturing system, a production process or service system, inputs and outputs are volatile and complex so that it is difficult to measure them in an accurate way. Instead the data can be given as a fuzzy variable. Many fuzzy approaches have been introduced in the DEA literature. Guo and Tanaka and Lertsirkul et al. applied possibility measure proposed by Zadeh to the fuzzy DEA model[14,20–25]. Lertsirkul et al. show that for the special case, in which fuzzy membership functions of fuzzy data are trapezoidal types, possibility DEA models become LP models. Liu and Liu presented credibility measure in 2002[26]. This paper will extend the CCR model to a fuzzy DEA model based on cedibility measure and then give a fuzzy ranking method all the DMUs with fuzzy inputs and outputs. The fuzzy ranking method was developed by Guo and Tanaka[23]. This approach provides fuzzy efficiency for an evaluated DMU at a specified α-level. Meilin Wen et al. proposed a new fuzzy DEA model based on credibility measure as well as a ranking method provided, they designed a hybrid algorithm combined with fuzzy simulation and genetic algorithm to compute the fuzzy model DEA[12]. Hatami and et al. proposed a DEA Method by constructing a FFLP model. They applied the FFLP model developed by Allahviranloo et al. (2008) and transform the DEA model into a fully fuzzified DEA model[33]. Zerafat Angil et al. proposed a six-stage algorithm to rank

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the efficient DMUs using fuzzy concept[13]. Kazemi and et al. used the proposed method of Lumar et al. to solve the mentioned problems[31,32]. To see the other FDEA models, the readers can see[8,10,11,16,19]. This paper is organized as follows: In section 2 we present the basic definitions of fuzzy arithmetic. In the next section, is defined ranking function. In section 4 the FFDEA is introduced and solve using the above mentioned multiplication and the properties of the presented linear ranking function with related theorems, are in introduced. Section 5, is devoted to a numerical example.

2. Preliminaries

Definition 1. A fuzzy number $\tilde{A} = (a^{(1)}, a^{(2)}, a^{(3)})$ is said to be a triangular fuzzy number if it, s membership function is given by,

$$u_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a^{(1)}}{a^{(2)}-a^{(1)}} & a^{(1)} \le x \le a^{(2)} \\ \dfrac{x-a^{(3)}}{a^{(2)}-a^{(3)}} & a^{(1)} \le x \le a^{(2)} \\ 0 & otherwise \end{cases}$$

Parametric form of a fuzzy number has been introduced and presented by $\tilde{A} = (\underline{A}(r), \bar{A}(r))$, whose $\underline{A}(r)$= $(\bar{A}(r)$, $0 \le r \le 1$ satisfying the following requirements:

1. $\underline{A}(r)$ is monotonically increasing left continuous function.
2. $\bar{A}(r)$ is monotonically decreasing left continuous function.
3. $\underline{A}(r) \le (\bar{A}(r), 0 \le r \le 1$
4. $\underline{A}(r) = (\bar{A}(r) = 0, r \le 0, 1 \le r$

If $\tilde{A} = (a^{(1)}, a^{(2)}, a^{(3)})$ then in parametric form is

$$\tilde{A} = (a^{(1)} + r(a^{(2)} - a^{(1)}), a^{(3)} - r(a^{(3)} - a^{(2)}))$$

Definition 2. A triangular fuzzy number $\tilde{A} = (a^{(1)}, a^{(2)}, a^{(3)})$ is said to be nonnegative fuzzy number, if and only if $a^{(1)} \ge 0$.

Theorem 1. Let $\tilde{A} = (a^{(1)}, a^{(2)}, a^{(3)})$ and $\tilde{B} = (b^{(1)}, b^{(2)}, b^{(3)})$ be two triangular fuzzy number, then

(i) $\tilde{A} \oplus \tilde{B} = (a^{(1)} + b^{(1)}, a^{(2)} + b^{(2)}, a^{(3)} + b^{(3)})$
(ii) $-\tilde{A} = (-a^{(3)}, -a^{(2)}, -a^{(1)})$
(iii) $\tilde{A} \ominus \tilde{B} = (a^{(1)} - b^{(3)}, a^{(2)} - b^{(2)}, a^{(3)} - b^{(1)})$
(iv) If $\tilde{B} = (b^{(1)}, b^{(2)}, b^{(3)})$ be a nonnegative triangular fuzzy number, and then

$$\tilde{A} \otimes \tilde{B} = \begin{cases} \left(a^{(1)}b^{(1)}, a^{(2)}b^{(2)}, a^{(3)}b^{(3)}\right) & a^{(1)} \ge 0 \\ \left(a^{(1)}b^{(3)}, a^{(2)}b^{(2)}, a^{(3)}b^{(3)}\right) & a^{(1)} < 0, a^{(3)} \ge 0 \\ \left(a^{(1)}b^{(3)}, a^{(2)}b^{(2)}, a^{(3)}b^{(1)}\right) & a^{(1)} < 0 \end{cases}$$

Proff[28,29].

Definition 3. Let $\tilde{A} = (a^{(1)}, a^{(2)}, a^{(3)})$ and $\tilde{B} = (b^{(1)}, b^{(2)}, b^{(3)})$ be two triangular fuzzy number, then

(i) $\tilde{A} \asymp \tilde{B}$ if $a^{(i)} = b^{(i)}$, i = 1, 2, 3
(ii) $\tilde{A} \preccurlyeq \tilde{B}$ if $a^{(i)} \le b^{(i)}$, i = 1, 2, 3
(iii) $\tilde{A} \succcurlyeq \tilde{B}$ if $a^{(i)} \ge b^{(i)}$, i = 1, 2, 3
(iv) $\tilde{A} \succ \tilde{B}$ if $a^{(i)} \ge b^{(i)}$, i = 1, 2, 3 and $a^{(r)} > b^{(r)}$, for some $r \varepsilon \{1, 2, 3\}$[34].

3. Ranking Function

A simple method for ordering fuzzy numbers consists in the definition of a ranking function F, mapping each fuzzy number to the real number R, where a natural order exists. Suppose $S = \{\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, ..., \tilde{A}_n\}$ is a set of n fuzzy numbers,and the ranking function F is a mapping from S to the real numbers R, i.e. $F: S \to R$ then for any distinct pair of fuzzy numbers $\tilde{A}_i, \tilde{A}_j \in S$, the ranking function can be defined as,

If $F(\tilde{A}_i) < F(\tilde{A}_k)$; then $\tilde{A}_i <^* \tilde{A}_k$
If $F(\tilde{A}_i) > F(\tilde{A}_k)$; then $\tilde{A}_i >^* \tilde{A}_k$
If $F(\tilde{A}_i) = F(\tilde{A}_k)$; then $\tilde{A}_i =^* \tilde{A}_k$

This implies for example, that if $F(\tilde{A}_i) > F(\tilde{A}_k)$, the fuzzy number $\tilde{A}_i$ is numerically greater than fuzzy number $\tilde{A}_k$ the higher $\tilde{A}_i$ is, the larger $F(\tilde{A}_i)$ is.

Here we introduce a linear ranking function that is similar to the ranking function[27]. For any arbitrary fuzzy number $\tilde{A} = (\underline{A}(r), \bar{A}(r))$, we use ranking function as follows:

$$D(\tilde{A}) = 1/2(\int_0^1 \underline{A}(r)\, dr + \int_0^1 \bar{A}(r)\, dr) \tag{1}$$

For triangular fuzzy number this reduces to:

$$D(\tilde{A}) = a^{(2)} + 1/4(a^{(3)} - a^{(1)}).$$

4. Fully Fuzzy DEA Model

Suppose the evaluation of the efficiency of n DMUs is desirable. Each DMU_j, j = 1, 2, …, n consumes m inputs x_{ij}(i = 1, 2, …, m) to produces outputs y_{rj}(r = 1, 2, …, s).

The following multiplier from of CCR model can be applied to assess the efficiency score of DMU_o[4]:

$$
\begin{aligned}
\text{Max} \quad & \theta = \sum_{r=1}^{s} u_r y_{ro} \\
\text{s.t} \quad & \sum_{i=1}^{m} v_i x_{io} = 1 \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \ j = 1, \ldots n \\
& u_r \geq 0, \ r = 1, \ldots, s \\
& v_i \geq 0, \ i = 1, \ldots, m
\end{aligned} \tag{1}
$$

Where $u_r (r = 1, 2, \ldots, s)$ and $v_i (i = 1, 2, \ldots, m)$ are the weights assigned to the rth output and ith input, respectively.

Consider the following fully fuzzy DEA model with fuzzy inputs, fuzzy outputs and fuzzy weights may be formulated as follows:

$$
\begin{aligned}
\text{Max} \quad & \tilde{\theta} = \sum_{r=1}^{s} \tilde{u}_r \otimes \tilde{y}_{ro} \\
\text{s.t} \quad & \sum_{i=1}^{m} \tilde{v}_i \otimes \tilde{x}_{io} \preccurlyeq \tilde{1} \\
& \sum_{r=1}^{s} \tilde{u}_r \otimes \tilde{y}_{rj} \ominus, \sum_{i=1}^{m} \tilde{v}_i \otimes \tilde{x}_{ij} \preccurlyeq \tilde{0} \ j = 1, .., n \\
& \tilde{u}_r \succcurlyeq 0, \ r = 1, \ldots, s \\
& \tilde{v}_i \succcurlyeq 0, \ i = 1, \ldots, m
\end{aligned} \tag{2}
$$

Let the parameters $\tilde{u}_r$, $\tilde{v}_i$, $\tilde{y}_{rj}$, $\tilde{x}_{ij}$, $\tilde{1}$ and $\tilde{0}$ be the triangular fuzzy numbers $\tilde{u}_r = (u_r^{(1)}, u_r^{(2)}, u_r^{(3)})$, $\tilde{v}_i = (v_i^{(1)}, v_i^{(2)}, v_i^{(3)})$, $\tilde{y}_{rj} = (y_{rj}^{(1)}, y_{rj}^{(2)}, y_{rj}^{(3)})$, $\tilde{x}_{ij} = (x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)})$, $i = 1, \ldots, m$, $r = 1, \ldots, s$, $j = 1, \ldots, n$, $\tilde{1} = (1, 1, 1)$, $\tilde{0} = (0, 0, 0)$ respectively. Then the problem (2) can be written as follows:

$$
\begin{aligned}
\text{Max} \quad & \tilde{\theta} = \sum_{r=1}^{s} \left(u_r^{(1)} y_{ro}^{(1)}, u_r^{(2)} y_{ro}^{(2)}, u_r^{(3)} y_{ro}^{(3)} \right) \\
\text{s.t} \quad & \sum_{i=1}^{m} \left(v_i^{(1)} x_{io}^{(1)}, v_i^{(2)} x_{io}^{(2)}, v_i^{(3)} x_{io}^{(3)} \right) \preccurlyeq (1, 1, 1)
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
& \sum_{r=1}^{s} \left(u_r^{(1)} y_{rj}^{(1)}, u_r^{(2)} y_{rj}^{(2)}, u_r^{(3)} y_{rj}^{(3)} \right) \\
& \quad + \sum_{i=1}^{m} \left(-v_i^{(3)} x_{ij}^{(3)}, -v_i^{(2)} x_{ij}^{(2)}, -v_i^{(1)} x_{ij}^{(1)} \right) \preccurlyeq (0, 0, 0), \ j = 1, \ldots, n \\
& \quad (u_r^{(1)}, u_r^{(2)}, u_r^{(3)}) \succcurlyeq 0, \ r = 1, \ldots, s \\
& \quad (v_i^{(1)}, v_i^{(2)}, v_i^{(3)}) \succcurlyeq 0, \ i = 1, \ldots, m
\end{aligned}
$$

Now, we apply the FFLP model developed by Pandian[35], namely level-sum method and transform the fuzzy DEA model to the following model:

$$
\begin{aligned}
\text{Max} \quad & \theta^1 = \sum_{r=1}^{s} u_r^{(1)} y_{r0}^{(1)} \\
\text{Max} \quad & \theta^2 = \sum_{r=1}^{s} u_r^{(2)} y_{r0}^{(2)} \\
\text{Max} \quad & \theta^3 = \sum_{r=1}^{s} u_r^{(3)} y_{r0}^{(3)} \\
\text{s.t} \quad & \sum_{i=1}^{m} v_i^{(1)} x_{i0}^{(1)} \leq 1 \\
& \sum_{i=1}^{m} v_i^{(2)} x_{i0}^{(2)} \leq 1 \\
& \sum_{i=1}^{m} v_i^{(3)} x_{i0}^{(3)} \leq 1 \\
& \sum_{r=1}^{s} u_r^{(1)} y_{rj}^{(1)} - \sum_{i=1}^{m} v_i^{(3)} x_{ij}^{(3)} \leq 0, \ j = 1, 2, \ldots, n \\
& \sum_{r=1}^{s} u_r^{(2)} y_{rj}^{(2)} - \sum_{i=1}^{m} v_i^{(2)} x_{ij}^{(2)} \leq 0, \ j = 1, 2, \ldots, n \\
& \sum_{r=1}^{s} u_r^{(3)} y_{rj}^{(3)} - \sum_{i=1}^{m} v_i^{(1)} x_{ij}^{(1)} \leq 0, \ j = 1, 2, \ldots, n \\
& \theta^1 \leq \theta^2 \\
& \theta^2 \leq \theta^3 \\
& u_r^{(1)} \leq u_r^{(2)}, \ r = 1, \ldots, s \\
& u_r^{(2)} \leq u_r^{(3)}, \ r = 1, \ldots, s \\
& v_i^{(1)} \leq v_i^{(2)}, \ i = 1, \ldots, m \\
& v_i^{(2)} \leq v_i^{(3)}, \ i = 1, \ldots, m \\
& u_r^{(1)} \geq 0, \ r = 1, \ldots, s \\
& v_i^{(1)} \geq 0, \ i = 1, \ldots, m
\end{aligned} \tag{4}
$$

Based on Pandian[35], if $\widetilde{u^*}_r = (u_r^{(1)^*}, u_r^{(2)^*}, u_r^{(3)^*})$, $\widetilde{v^*}_i = (v_i^{(1)^*}, v_i^{(2)^*}, v_i^{(3)^*})$, $r = 1, 2, .., s$, $i = 1, 2, \ldots, m$ is an efficient solution to the MOLP problem(4). Then, $\widetilde{u^*}_r = (u_r^{(1)^*}, u_r^{(2)^*}, u_r^{(3)^*})$, $\widetilde{v^*}_i = (v_i^{(1)^*}, v_i^{(2)^*}, v_i^{(3)^*})$, $r = 1, 2, ..., s$, $i = 1, 2, \ldots, m$ is an optimal solution to the problem(3)[35]. We run MOLP model(4) and obtain the following optimal fuzzy solution to the FDEA problem(3): $\widetilde{u^*}_r = (u_r^{(1)^*}, u_r^{(2)^*}, u_r^{(3)^*})$, $\widetilde{v^*}_i = (v_i^{(1)^*}, v_i^{(2)^*}, v_i^{(3)^*})$, $r = 1, 2, ..., s$, $i = 1, 2, \ldots, m$. Finally, the objective function of model (4) is used to computing the optimal fuzzy efficiency of each DMU_o as follows:

$$
\left(\theta^{*1}, \theta^{*2}, \theta^{*3} \right) = \left(\sum_{r=1}^{s} u_r^{(1)*} y_{ro}^{(1)}, \sum_{r=1}^{s} u_r^{(2)*} y_{ro}^{(2)}, \sum_{r=1}^{s} u_r^{(3)*} y_{ro}^{(3)} \right)
$$

5. Numerical Example

In this section we have a numerical example from Guo and Tanaka[23] study that its data are on Table 1. By putting the data of Table 1 on the proposed model, we will have model (5). This model is written to calculate the first decision making units efficiency and 4 other models have to be written to calculate the efficiency of other units. Now, the MOLP problem to the given fully fuzzy DEA problem is given below:

$$
\begin{aligned}
&\text{Max} \quad \theta^1 = 2.4\, u_1^{(1)} + 3.8\, u_2^{(1)}\\
&\text{Max} \quad \theta^2 = 2.6\, u_1^{(2)} + 4.1\, u_2^{(2)}\\
&\text{Max} \quad \theta^3 = 2.8\, u_1^{(3)} + 4.4\, u_2^{(3)}\\
&\text{s.t} \quad 2.4\, u_1^{(1)} + 3.8\, u_2^{(1)} - 4.5\, v_1^{(3)} - 2.3\, v_2^{(3)} \le 0; \qquad (5)\\
&\quad 2.2\, u_1^{(1)} + 3.3\, u_2^{(1)} - 2.9\, v_1^{(3)} - 1.6\, v_2^{(3)} \le 0;\\
&\quad 2.7\, u_1^{(1)} + 4.3\, u_2^{(1)} - 5.4\, v_1^{(3)} - 3\, v_2^{(3)} \le 0;\\
&\quad 2.5\, u_1^{(1)} + 5.5\, u_2^{(1)} - 4.8\, v_1^{(3)} - 2.4\, v_2^{(3)} \le 0;\\
&\quad 4.4\, u_1^{(1)} + 6.5\, u_2^{(1)} - 7.1\, v_1^{(3)} - 4.6\, v_2^{(3)} \le 0;\\
&\quad 2.\, u_1^{(2)} + 4.1\, u_2^{(2)} - 4\, v_1^{(2)} - 2.1\, v_2^{(2)} \le 0;\\
&\quad 2.2\, u_1^{(2)} + 3.5\, u_2^{(2)} - 2.9\, v_1^{(2)} - 1.5\, v_2^{(2)} <= 0;\\
&\quad 3.2\, u_1^{(2)} + 5.1\, u_2^{(2)} - 4.9\, v_1^{(2)} - 2.6\, v_2^{(2)} \le 0;\\
&\quad 2.9\, u_1^{(2)} + 5.7\, u_2^{(2)} - 4.1\, v_1^{(2)} - 2.3\, v_2^{(2)} \le 0;\\
&\quad 5.1\, u_1^{(2)} + 7.4\, u_2^{(2)} - 6.5\, v_1^{(2)} - 4.1\, v_2^{(2)} \le 0;\\
&\quad 2.8\, u_1^{(3)} + 4.4\, u_2^{(3)} - 3.5\, v_1^{(1)} - 1.9\, v_2^{(1)} \le 0;\\
&\quad 2.2\, u_1^{(3)} + 3.7\, u_2^{(3)} - 2.9\, v_1^{(1)} - 1.9\, v_2^{(1)} \le 0;\\
&\quad 3.7\, u_1^{(3)} + 5.9\, u_2^{(3)} - 4.4\, v_1^{(1)} - 1.9\, v_2^{(1)} \le 0;\\
&\quad 3.3\, u_1^{(3)} + 5.9\, u_2^{(3)} - 3.4\, v_1^{(1)} - 1.9\, v_2^{(1)} \le 0;\\
&\quad 5.8\, u_1^{(3)} + 8.3\, u_2^{(3)} - 5.9\, v_1^{(1)} - 1.9\, v_2^{(1)} \le 0;\\
&\quad 3.5\, v_1^{(1)} + 1.9\, v_2^{(1)} \le 1;\\
&\quad 4\, v_1^{(2)} + 2.1\, v_2^{(2)} \le 1;\\
&\quad 4.5\, v_1^{(3)} + 2.3\, v_2^{(3)} \le 1;\\
&\quad u_1^{(1)} - u_1^{(2)} \le 0;\\
&\quad u_1^{(2)} - u_1^{(3)} \le 0;\\
&\quad u_2^{(1)} - u_2^{(2)} \le 0;\\
&\quad u_2^{(2)} - u_2^{(3)} \le 0;\\
&\quad v_1^{(1)} - v_1^{(2)} \le 0;\\
&\quad v_1^{(2)} - v_1^{(3)} \le 0;\\
&\quad v_2^{(1)} - v_2^{(2)} \le 0;\\
&\quad v_2^{(2)} - v_2^{(3)} \le 0;\\
&\quad \theta^1 \le \theta^2\\
&\quad \theta^2 \le \theta^3\\
&\quad u_1^{(1)} \ge 0,\ u_2^{(1)} \ge 0,\ v_1^{(1)} \ge 0,\ v_2^{(1)} \ge 0,
\end{aligned}
$$

Table 1. Example of Guo and Tanaka

DMU	Input 1	Input 2	Output 1	Output 2
1	(3.5,4,4.5)	(1.9,2.1,2.3)	(2.4,2.6,2.8)	(3.8,4.1,4.4)
2	(2.9,2.9,2.9)	(1.4,1.5,1.6)	(2.2,2.2,2.2)	(3.3,3.5,3.7)
3	(4.4,4.9,5.4)	(2.2,2.6,3)	(2.7,3.2,3.7)	(4.3,5.1,5.9)
4	(3.4,4.1,4.8)	(2.2,2.3,2.4)	(2.5,2.9,3.3)	(5.5,5.7,5.9)
5	(5.9,6.5,7.1)	(3.6,4.1,4.6)	(4.4,5.1,5.8)	(6.5,7.4,8.3)

We consider the following LP problem (6) related to the above MOLP problem:

$$
\begin{aligned}
&\text{Max } 2.4\, u_1^{(1)} + 2.6\, u_1^{(2)} + 2.8\, u_1^{(3)} + 3.8\, u_2^{(1)} + 4.1\, u_2^{(2)} + 4.4\, u_2^{(3)}\\
&\text{s.t} \quad 2.4\, u_1^{(1)} + 3.8\, u_2^{(1)} - 4.5\, v_1^{(3)} - 2.3\, v_2^{(3)} \le 0;\\
&\quad 2.2\, u_1^{(1)} + 3.3\, u_2^{(1)} - 2.9\, v_1^{(3)} - 1.6\, v_2^{(3)} \le 0;\\
&\quad 2.7\, u_1^{(1)} + 4.3\, u_2^{(1)} - 5.4\, v_1^{(3)} - 3\, v_2^{(3)} \le 0; \qquad (6)\\
&\quad 2.5\, u_1^{(1)} + 5.5\, u_2^{(1)} - 4.8\, v_1^{(3)} - 2.4\, v_2^{(3)} \le 0;\\
&\quad 4.4\, u_1^{(1)} + 6.5\, u_2^{(1)} - 7.1\, v_1^{(3)} - 4.6\, v_2^{(3)} \le 0;\\
&\quad 2.\, u_1^{(2)} + 4.1\, u_2^{(2)} - 4\, v_1^{(2)} - 2.1\, v_2^{(2)} \le 0;\\
&\quad 2.2\, u_1^{(2)} +3.5\, u_2^{(2)} - 2.9\, v_1^{(2)} - 1.5\, v_2^{(2)} <= 0;\\
&\quad 3.2\, u_1^{(2)} + 5.1\, u_2^{(2)} - 4.9\, v_1^{(2)} - 2.6\, v_2^{(2)} \le 0;\\
&\quad 2.9\, u_1^{(2)} + 5.7\, u_2^{(2)} - 4.1\, v_1^{(2)} - 2.3\, v_2^{(2)} \le 0;\\
&\quad 5.1\, u_1^{(2)} + 7.4\, u_2^{(2)} - 6.5\, v_1^{(2)} - 4.1\, v_2^{(2)} \le 0;\\
&\quad 2.8\, u_1^{(3)} + 4.4\, u_2^{(3)} - 3.5\, v_1^{(1)} - 1.9\, v_2^{(1)} \le 0;\\
&\quad 2.2\, u_1^{(3)} + 3.7\, u_2^{(3)}) - 2.9\, v_1^{(1)} - 1.9\, v_2^{(1)} \le 0;\\
&\quad 3.7\, u_1^{(3)} + 5.9\, u_2^{(3)} - 4.4\, v_1^{(1)} - 1.9\, v_2^{(1)} \le 0;\\
&\quad 3.3\, u_1^{(3)} + 5.9\, u_2^{(3)} - 3.4\, v_1^{(1)} - 1.9\, v_2^{(1)} \le 0;\\
&\quad 5.8\, u_1^{(3)} + 8.3\, u_2^{(3)} - 5.9\, v_1^{(1)} - 1.9\, v_2^{(1)} \le 0;\\
&\quad 3.5\, v_1^{(1)} + 1.9\, v_2^{(1)} \le 1;\\
&\quad 4\, v_1^{(2)} + 2.1\, v_2^{(2)} \le 1;\\
&\quad 4.5\, v_1^{(3)} + 2.3\, v_2^{(3)} \le 1;\\
&\quad u_1^{(1)} - u_1^{(2)} \le 0;\\
&\quad u_1^{(2)} - u_1^{(3)} \le 0;\\
&\quad u_2^{(1)} - u_2^{(2)} \le 0;\\
&\quad u_2^{(2)} - u_2^{(3)} \le 0;\\
&\quad v_1^{(1)} - v_1^{(2)} \le 0;\\
&\quad v_1^{(2)} - v_1^{(3)} \le 0;\\
&\quad v_2^{(1)} - v_2^{(2)} \le 0;\\
&\quad v_2^{(2)} - v_2^{(3)} \le 0;\\
&\quad u_1^{(1)} \ge 0,\ u_2^{(1)} \ge 0,\ v_1^{(1)} \ge 0,\ v_2^{(1)} \ge 0,
\end{aligned}
$$

Table 2. Comparison of result of different methods

DMU	Lertwo Rasirikul (2002)	Zerafat and et al. (2010)	Wen et.al (2010)	Hatami - Marbini And et al. (2011)	Kazemi and Et al. (2014)	Efficiencies Our study
1	0.855	0.915	0.907	(0.789,0.855,0.921)	(0.624,0.747,0.899)	(0.5453,0.5906,0.6360)
2	1	1	1	(0.981,0.999,1.017)	(0.834,0.882,0.935)	(0.7515,0.7760,0.7786)
3	0.861	0.948	0.907	(0.726,0.861,0.996)	(0.555,0.747,1)	(0.5117,0.6065,0.7013)
4	1	1	1	(0.964,0.999,1.034)	(0.854,0.924,1)	(0.6930,0.7182,0.7434)
5	1	0.991	1	(0.867,0.999,1.132)	(0.631,0.799,1)	(0.6322,0.7229,0.8319)

Table 3. Ranking of fuzzy efficiency scores

DMU	Hatami And et al. (2011)	Rank	Kazemi and Et al. (2014)	Rank	Efficiencies Our study	Rank
1	0.888	5	0.8157	5	0.6133	5
2	1.008	3	0.9072	2	0.782	1
3	0.9285	4	0.8582	4	0.6539	4
4	1.0165	2	0.9605	1	0.7308	3
5	1.0652	1	0.8912	3	0.781	2

By solving the problem (5), based on the level-sum method proposed by Pandian et al[35],

$\widetilde{u}^*{}_1 = (0.2144, 0.2144, 0.2144)$, $\widetilde{u}^*{}_2 = (0.0081, 0.0081, 0.0081)$, $\widetilde{v}^*{}_1 = (0.2222, 0, 0.2222)$, $\widetilde{v}^*{}_2 = (0, 0.222, 0)$ and $\widetilde{\theta}^* \approx (0.5453, 0.5906, 0.6360)$ is an optimal fuzzy solution to the given initial fully fuzzy DEA problem.

By model (4), fuzzy efficiency of DMUs is calculated and its result is on column 7 of Table 2. In all of methods of Lertworasirkul[24], Zerafat Angiz and et al[30], Wen et al[2], units two, four and five have the highest efficiency and after the units three and one have the highest efficiency.

We applied fuzzy ranking function (1) to the results of Hatami-Marbini approach[33], Kazemi and et al. approach[31] and our approach, the results are presented in Table 3.

If we consider the decimal numbers in seventh column of third Table, we will notice that DMU_2, DMU_5 will be classified in first rank and DMU_4 in second rank. Also the DMU_3, DMU_1 are located in third, fourth ranks, respectively. As regards all presented methods for evaluating of efficiency DMUs differ together, thus it's naturalized that the results of methods are nuanced together. But generally, as for Table 2 and Table 3 all methods are assessed DMU_2, DMU_4, DMU_5 best decision making units and after the units three and one take the score efficiency orders .Table 2 shown to conform both the results of this study and the results of other methods.

6. Conclusion

In this paper, an FFDEA problem has been presented. Also, an approach has been given to solve it. We are using the proposed method of Pandian[35] to find an optimal fuzzy solution to a FFDEA problem. The main advantage of the proposed method is that the FFDEA problems can be solved by any LP solver using the level-sum method since it's based on only simplex method. With using of this method, we obtain fuzzy efficiency scores and DMUs are ranked.

7. Reference

1. Khodabakhshi M, Aryavash K. Ranking units with fuzzy data in DEA. Data Envelopment Analysis and Decision Science. 2014; 14:1–10.
2. Wen M, You C, Kang R. A new ranking method to fuzzy data envelopment analysis. Computers and Mathematics with Applications. 2010; 59:3398–404.
3. Banker RD, Charnes A, Cooper WW. Some models for estimating technical and scale inefficiencies in data envelopment analysis. Management Science. 1984; 30:1078–92.
4. Charnes A, Cooper WW, Rhodes E. Measuring the efficiency of DMUs. European Journal of Operational Research. 1978; 2:429–44.
5. Charnes A, Cooper WW, Golany B, Seiford L, Stutz VA. Foundations of data envelopment analysis for Pareto-Koopman efficient empirical production frontiers. Journal of Econometrics. 1985; 30:91–107.
6. Cooper WW, Tone K. Measuring of inefficiency in data envelopment analysis ans stochastic frontier estimation. European Journal of Operational Research. 1997; 99:72–88.
7. Deprins D, Simar L, Tulkens H. Measuring lador-efficiency in post offices. In: Marchand M, Pestieau P, Tulkens H editors. The Performance of Public Enterprises. Amsterdam: Elsevier Science Publishers; 1984. p. 243–67.
8. Jahanshahloo GR, Hosseinzadeh lotfi F, Shahverdi R, Adabitabar M, Rostamy-Malkhalifeh M, Sohraiee S.

Ranking DMUs by l_1-norm with fuzzy data in DEA. Chas, Solitons and Fractals. 2009; 39:2294–302.
9. Liu FHF, Peng HH. Ranking of units on the DEA frontier with common weights. Computers and Operations Research. 2008; 35:1624–37.
10. Ma LC, Li HL. A fuzzy ranking method with range reduction techniques. European Journal of Operation Research. 2008; 184:1032–43.
11. Wen M, Li H. Fuzzy Data Envelopment Analysis (DEA): Model and Ranking method. Journal of Computational and Applied Mathematics. 2009; 233:872–8.
12. Wen M, You C, Rui K. A new Ranking method to fuzzy data envelopment analysis. Computers and Mathematics with Applications. 2010; 59:3398–404.
13. Zerafat Angiz LM, Mustafa A, Emrouznejad A. Ranking efficient decision making units in data envelopment analysis using fuzzy concept. Computers and Industrial Engineering. 2010; 59:712–9.
14. Kao C, Liu ST. Fuzzy efficiency measure in data envelopment analysis. Fuzzy Set and Systems. 2000; 113:427–37.
15. Tone K. A slacks-based measure of efficiency in data envelopment analysis. European Journal of Operation Research. 2001; 130:498–509.
16. Zhu J. Imprecise Data Envelopment Analysis (IDEA): A review and improvement with an application. European Journal of Operation Research. 2003; 14:513–29.
17. Sexton TR, Silkman RH, Hogan AJ. Data envelopment analysis: critique and extensions, In: Silkman RH, editor. Measuring Efficiency: An Assessment of data envelopment analysis. San Francisco, CA: Jossey-Bass; 1986. p. 73–105.
18. Torgersen AM, Forsund FR, Kittelsen SAC. Slack-adjusted efficiency measures and ranking of efficient units. The Journal of Productivity Analysis. 1996; 7:379–98.
19. Andersen P, Petersen NC. A procedure for ranking efficient units in data envelopment analysis. Management Science. 1993; 39(10):1261–94.
20. Cooper WW, Park KS, Yu G. IDEA and AR-IDEA: models for dealing with imprecise data in DEA. Management Science. 1999; 45:597–607.
21. Cooper WW, Park KS, Yu G. IDEA (Imprecise Data in Data Envelopment Analysis) with CMDs (Column Maximum Decision Making Units). European Journal of Operation Research. 2001; 52:176–81.
22. Entani T, Maeda Y, Tanaka H. Dual models of interval DEA and its extension to interval data. European Journal of Operation Research. 2002; 136:32–45.
23. Guo P, Tanaka H. Fuzzy DEA: a perceptual evaluation method. Fuzzy Sets and Systems 2001; 119:149–60.
24. Lertworasirkul S, Feng SC, Joines JA, Nuttle HLW. Fuzzy Data Envelopment Analysis (DEA): A possibility approach. Fuzzy Set and Systems. 2003; 139:379–94.
25. Zadeh LA. Fuzzy sets as a basis for a theory of possibility. Fuzzy Set and systems. 1978; 1:3–28.
26. Liu B, Liu YK. Expected value of fuzzy variable and fuzzy expected value models. IEEE Transactions on Fuzzy Systems. 2002; 10(4):445–50.
27. Bortolan G, Degani R. A review of some methods for ranking fuzzy subsets. Fuzzy Set and Systems. 1985; 15:1–19.
28. Liou TS, Wang MJ. Ranking fuzzy numbers with integral value. Fuzzy Set and Systems. 1992; 50:247–55.
29. Maleki HR, Mishmast H, Mashinchi M. Fuzzy number linear programming: A probabilistic approach. In: Pal NR, Sugeno M, editors. AFSS 2002, LNAI 2275. 2002. p. 21–33.
30. Zerafat Angil LM, Emrouznejad A, Mustafa A. Fuzzy assessment of performance of a decision making units using DEA: A non-radial approach. Expert System with Applications. 2010; 37(7):5153–7.
31. Kazemi M, Alimi A. A fully fuzzy approach to data envelopment analysis. Journal of Mathematics and Science. 2014; 11:238–45.
32. Kumar A, Kaur J, Singh P. Fuzzy optimal solution of fully fuzzy linear programming problems with inequality constraints. International Journal of Mathematical and Computer Science. 2010; 6:37–41.
33. Hatami-Marbini A, Tavana M, Ebrahimi A. A fully fuzzified data envelopment analysis model. Int J Information and Decision Sciences. 2011; 3:252–64.
34. Zimmerman HJ. Fuzzy programming and linear programming with several objective functions. Fuzzy Sets and Systems. 1978; 1:45–55.
35. Pandian P. Multi-objective programming approach for fuzzy linear programming problems. Applied Mathematics Sciences. 2013; 7:1811–7.