

EOQ Model with Inventory Level Dependent Demand Rate under Permissible Delay in Payments with Cash Discount

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Abstract

In this paper, we develop an EOQ model for power demand under the condition of permissible delay in payment by considering four different situations. Mathematical formulation is derived under these four different situations. The main objective of this work is to obtain minimum total relevant cost. Next, we derive optimal solution optimal cycle time, order quantity and total relevant cost for the proposed model. The theoretical results are illustrated with numerical examples. The sensitivity analysis of the optimal solution is provided with respect to key parameters of the system. Mathematica 5.1 software is used for finding numerical results.

Keywords: Cash Discount, Demand Rate, EOQ Model, Permissible Delay, Total Relevant Cost

1. Introduction

In practice EOQ model, the demand rate is considered to be either constant or time-dependent or stock-dependent. However, in practice the demand for consumer goods may be depends on inventory level, that is the demand rate may increase or decrease with the inventory level. Several EOQ models have established by researchers considering the varying and probabilistic demand. The varying demand was first established by Silver and Meal¹ which is simple modification of the inventory model. Donaldson² was the first researcher who developed EOQ (Economic Order Quantity) model for linearly time dependent demand. Khanna et al.³ established an economic order quantity model for deteriorating items having time dependent demand when delay in payments is permissible. Large number of research papers/articles done by researchers in direction like Ritchie⁴, Mitra⁵, Tripathi and Misra^{6,7}, Tripathi and Kumar⁸. Recently, Tripathi and Panday⁹ presented an inventory model for deteriorating items with Weibull

distribution time-dependent demand rate under permissible delay in payments.

Deterioration is a natural phenomenon for any type of goods. During the last few decades, large number of articles/research papers presented by researchers on inventory models for deteriorating items. Ghare and Schrader¹⁰ first established an inventory model for deteriorating items. Goyal¹¹ developed an EOQ model under condition of permissible delay in payments. Chang¹² proposed an inventory model under a situation in which the supplier provides the purchaser a permissible delay in payments if the purchaser orders a large quantity. Covert and Philip¹³ extended Ghare and Schrader's¹⁰ model for constant deterioration rate to a two-parameter Weibull distribution. Yang et al.¹⁴ developed an optimal replenishment policy for deteriorating items with time-varying demand and partial backlogging. Shah and Jaswal¹⁵ and Aggarwal¹⁶ discussed the EOQ model with constant rate of deterioration. Goyal and Giri¹⁷ established an EOQ model on the recent trends in modeling of deteriorating inventory. Teng et al.¹⁸ establish an EOQ model for stock-dependent

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demand under progressive payment scheme for deteriorating items. Chung and Liao¹⁹ considered an inventory model for deteriorating items under the conditions of using the Discounted Cash-Flows (DCF) approach to the permissible delay in payments. Other many related articles/research papers can be found by Aggarwal and Jaggi²⁰, Jamal et al.²¹, Huang²², Ouyang et al.²³, Heng et al.²⁴, Sarkar et al.²⁵ and Misra et al.²⁶. Roy²⁷ developed a deterministic inventory model when the deterioration rate is time dependent, demand rate is a function of selling price and holding cost is time proportional. Liao²⁸ derived a production model for a lot-size inventory model with finite production rate taking into consideration the effect of decay and the condition of permissible delay in payments, in which the restriction assumption of a permissible delay is relaxed to that at the end of the credit period, the retailer will make a partial payment on total purchasing cost to the supplier and payoff the remaining balance by loan from the bank. Dye et al.²⁹ developed a deterministic inventory model for deteriorating items with price-dependent demand. Liao³⁰ considered the impact of the trade credit policy on the classical Economic Production Quantity (EPQ) model for an item subjected to exponential decays. Chung³¹ established an EOQ model for deteriorating items under credit linked to the ordering quantity. Several research paper published by Sharma and Singh³², Tayal, Singh and Sharma³³, Shahraki et al.³⁴ and Bhagoria et al.³⁵ in this direction.

The rest of the paper is organized as follows. In section 2, the assumption and notations are given. In section 3, we develop the mathematical models under four different circumstances. In section 4, optimal solutions are derived. In section 5, we provide three numerical examples to illustrate the results. In addition, the sensitivity analysis of the optimal solution with respect to parameters of the system is carried out in section 6. Finally, we draw the conclusions and future research in section 7.

2. Assumptions and Notation

2.1 Notations

p : Selling price per unit.
 c : The unit purchasing cost with $p > c$.
 I_c : The interest charged per dollar in stock per year by the supplier.
 I_d : The interest earned per dollar per year.
 s : The ordering cost per order.

Q : The order quantity.
 r : The cash discount rate $0 < r < 1$.
 h : The unit holding cost per year excluding the interest charges.
 M_1 : The period of cash discount.
 M_2 : The period of permissible delay in setting account with $M_2 > M_1$.
 T : The replenishment time interval.
 $I(T)$: The level of inventory at time t , $0 \leq t \leq T$.
 T_1, T_2, T_3 and T_4 : The optimal replenishment time for case (1), (2), (3) and (4) respectively.
 $Z(T)$: The total relevant cost per year.
 $Z_1(T), Z_2(T), Z_3(T)$ and $Z_4(T)$: The total relevant cost per year for case (1), (2), (3) and (4) respectively.
 $Z^*(T_1), Z^*(T_2), Z^*(T_3)$ and $Z^*(T_4)$: Optimal total relevant cost per year for case (1), (2), (3) and (4) respectively.
 $Q^*(T_1), Q^*(T_2), Q^*(T_3)$ and $Q^*(T_4)$ the optimal order quantity case (1), (2), (3) and (4) respectively.

2.2 Assumptions

- The demand rate is power inventory dependent, i.e. $D = D\{I(t)\} = a\{I(t)\}^\beta$, $a > 0$, $0 \leq \beta < 1$.
- Lead time is negligible.
- Time horizon is infinite.
- Shortages are not allowed.
- The generated sales revenue is deposited when the account is not settled in an interest bearing account. The account is settled as well as the buyer pays of units sold at the end of M_1 or, M_2 and starts paying for the interest charges on the items in the stock.

3. Mathematical Formulation

According to assumption, the variation of inventory with respect to time can be represented by the following differential equation

$$\frac{dI(t)}{dt} = -a\{I(t)\}^\beta, \quad a > 0, 0 < \beta < 1 \quad (1)$$

The solution of (1) with the condition $I(T) = 0$ is given by

$$I(t) = a(T-t)^{1/(1-\beta)}, \quad \text{where } a = \{a(1-\beta)\}^{1/(1-\beta)}, 0 \leq t \leq T \quad (2)$$

$$\text{Also the order quantity } Q = I(0) = aT^{1/(1-\beta)} \quad (3)$$

The total demand during one cycle is $= aT^{\beta/(1-\beta)}$

The total relevant cost per cycle time contains of the following elements:

$$\text{Cost of placing order} = \frac{s}{T} \quad (4)$$

$$\text{Cost of purchasing} = acT^{\beta/(1-\beta)} \quad (5)$$

$$\text{Cost of carrying inventory} = \frac{ah(1-\beta)}{(2-\beta)} T^{1/(1-\beta)} \quad (6)$$

Case (1). $T \geq M_1$,

The customer save rcQ per cycle due to price discount, since the payment is made at time M_1 .

$$\text{The discount per year} = \frac{rcQ}{T} = racT^{\beta/(1-\beta)} \quad (7)$$

The customer pays off all units ordered at time M_1 to obtain the cash discount according to the assumption. Consequently, the items in stock have to financed (at the rate I_c) after time M_1 . Therefore,

The interest payable per year

$$= \frac{ac(1-r)(1-\beta)I_c(T-M_1)^{(2-\beta)/(1-\beta)}}{(2-\beta)T} \quad (8)$$

Also in the period $[0, M_1]$ the customer sells the product and deposits the revenue into an account that earns interest I_d per dollar per year. Therefore,

The interest earned per year

$$= \frac{pI_d aa^\beta}{T} \left[-(1-\beta)M_1(T-M_1)^{\frac{1}{1-\beta}} - \frac{(1-\beta)^2}{(2-\beta)} \left\{ (T-M_1)^{\frac{(2-\beta)}{(1-\beta)}} - T^{\frac{(2-\beta)}{(1-\beta)}} \right\} \right] \quad (9)$$

Total relevant cost $Z_1(T)$ per year is given by

$$\begin{aligned} Z_1(T) = & \frac{s}{T} + racT^{\beta/(1-\beta)} + \frac{ah(1-\beta)}{(2-\beta)} T^{1/(1-\beta)} \\ & + \frac{ac(1-r)(1-\beta)I_c(T-M_1)^{(2-\beta)/(1-\beta)}}{(2-\beta)T} \\ & - \frac{pI_d aa^\beta}{T} \left[-(1-\beta)M_1(T-M_1)^{\frac{1}{1-\beta}} \right. \\ & \left. - \frac{(1-\beta)^2}{(2-\beta)} \left\{ (T-M_1)^{\frac{(2-\beta)}{(1-\beta)}} - T^{\frac{(2-\beta)}{(1-\beta)}} \right\} \right] \quad (10) \end{aligned}$$

Case (2). $T < M_1$

In this situation the customer sells $a\{I(t)\}^\beta$. T units in total at time T and has $c(1-r)a\{I(t)\}^\beta T$ to pay the supplier in full at time M_1 . Consequently there is no interest payable, while the cash discount is the same as that case (1) discussed above. However,

The interest earned per year =

$$\begin{aligned} & \frac{pI_d}{T} \left[\int_0^T a\{I(t)\}^\beta .tdt + (M_1 - T) \int_0^T a\{I(t)\}^\beta dt \right] \\ & = \frac{aa^\beta pI_d(1-\beta)T^{1/(1-\beta)}}{T} \left\{ \frac{(1-\beta)T}{(2-\beta)} - (T-M_1) \right\} \quad (11) \end{aligned}$$

Now the total relevant cost per year $Z_2(T)$ per year is given by

$$\begin{aligned} Z_2(T) = & \frac{s}{T} + racT^{\beta/(1-\beta)} + \frac{ah(1-\beta)}{(2-\beta)} T^{1/(1-\beta)} \\ & - \frac{aa^\beta pI_d(1-\beta)T^{1/(1-\beta)}}{T} \left\{ \frac{(1-\beta)T}{(2-\beta)} - (T-M_1) \right\} \quad (12) \end{aligned}$$

Case (3). $T < M_2$,

In this situation, there is no discount i.e. $r = 0$, (since the payment is made at time M_2), Now

The interest payable per year

$$= \frac{ac(1-\beta)I_c(T-M_2)^{(2-\beta)/(1-\beta)}}{T(2-\beta)} \quad (13)$$

The interest earned per year

$$\begin{aligned} & = \frac{pI_d aa^\beta}{T} \left[-(1-\beta)M_2(T-M_2)^{\frac{1}{1-\beta}} \right. \\ & \left. - \frac{(1-\beta)^2}{(2-\beta)} \left\{ (T-M_2)^{\frac{(2-\beta)}{(1-\beta)}} - T^{\frac{(2-\beta)}{(1-\beta)}} \right\} \right] \quad (14) \end{aligned}$$

Thus, the total relevant cost $Z_3(T)$ per year is given by

$$\begin{aligned} Z_3(T) = & \frac{s}{T} + racT^{\beta/(1-\beta)} + \frac{ah(1-\beta)}{(2-\beta)} T^{1/(1-\beta)} \\ & + \frac{ac(1-\beta)I_c(T-M_2)^{(2-\beta)/(1-\beta)}}{(2-\beta)T} - \frac{pI_d aa^\beta}{T} \end{aligned}$$

$$\left[-(1-\beta)M_2(T-M_2)^{\frac{1}{(1-\beta)}} - \frac{(1-\beta)^2}{(2-\beta)} \left\{ (T-M_2)^{\frac{(2-\beta)}{(1-\beta)}} - T^{\frac{(2-\beta)}{(1-\beta)}} \right\} \right] \quad (15)$$

Case (4). $T < M_2$. In this situation there is no interest charged, but supplier earned interest. Thus

The interest earned per year by the supplier

$$= \frac{aa^\beta pI_d(1-\beta)T^{\frac{1}{(1-\beta)}}}{T} \left\{ \frac{(1-\beta)T}{(2-\beta)} - (T-M_2) \right\} \quad (16)$$

Now the total relevant cost per year $Z_4(T)$ per year is given by

$$Z_4(T) = \frac{s}{T} + racT^{\frac{\beta}{(1-\beta)}} + \frac{ah(1-\beta)}{(2-\beta)} T^{\frac{1}{(1-\beta)}} - \frac{aa^\beta pI_d(1-\beta)T^{\frac{1}{(1-\beta)}}}{T} \left\{ \frac{(1-\beta)T}{(2-\beta)} - (T-M_2) \right\} \quad (17)$$

4. Determination of Optimal Solution

To determine optimal solution, differentiating (10), (12), (15) and (17) with respect to T two times, we get

$$\begin{aligned} \frac{dZ_1(T)}{dT} &= -\frac{s}{T^2} + \frac{rac\beta T^{\frac{(2\beta-1)}{(1-\beta)}}}{1-\beta} + \frac{ah}{(2-\beta)} T^{\frac{\beta}{(1-\beta)}} \\ &\quad + ac(1-r)I_c \left\{ \frac{(T-M_1)^{\frac{1}{(1-\beta)}}}{T} - \frac{(1-\beta)(T-M_1)^{\frac{(2-\beta)}{(1-\beta)}}}{(2-\beta)T^2} \right\} \\ &\quad - \frac{pI_d aa^\beta}{T} \left[-M_1(T-M_1)^{\frac{\beta}{(1-\beta)}} - (1-\beta)(T-M_1)^{\frac{1}{(1-\beta)}} \right. \\ &\quad \left. + (1-\beta)T^{\frac{1}{(1-\beta)}} \right] + \frac{pI_d aa^\beta}{T^2} \left[-(1-\beta)M_1(T-M_1)^{\frac{1}{(1-\beta)}} \right. \\ &\quad \left. - \frac{(1-\beta)^2}{(2-\beta)} \left\{ (T-M_2)^{\frac{(2-\beta)}{(1-\beta)}} - T^{\frac{(2-\beta)}{(1-\beta)}} \right\} \right] \quad (18) \end{aligned}$$

$$\begin{aligned} \frac{dZ_2(T)}{dT} &= -\frac{s}{T^2} + \frac{rac\beta T^{\frac{(2\beta-1)}{(1-\beta)}}}{1-\beta} + \frac{ah}{(2-\beta)} T^{\frac{\beta}{(1-\beta)}} \\ &\quad + \frac{pI_d aa^\beta}{T} (T-M_1)T^{\frac{\beta}{(1-\beta)}} \end{aligned}$$

$$+ \frac{pI_d aa^\beta}{T^2} \left\{ \frac{(1-\beta)^2}{(2-\beta)} T^{\frac{(2-\beta)}{(1-\beta)}} - (1-\beta)(T-M_1)T^{\frac{1}{(1-\beta)}} \right\} \quad (19)$$

$$\begin{aligned} \frac{dZ_3(T)}{dT} &= -\frac{s}{T^2} + \frac{rac\beta T^{\frac{(2\beta-1)}{(1-\beta)}}}{1-\beta} + \frac{ah}{(2-\beta)} T^{\frac{\beta}{(1-\beta)}} \\ &\quad + \frac{acI_c(T-M_2)^{\frac{1}{(1-\beta)}}}{T} \left\{ 1 - \frac{(1-\beta)(T-M_2)}{(2-\beta)T} \right\} - \\ &\quad \frac{pI_d aa^\beta}{T} \left\{ -M_2(T-M_2)^{\frac{\beta}{(1-\beta)}} - (1-\beta)(T-M_2)^{\frac{1}{(1-\beta)}} + (1-\beta)T^{\frac{1}{(1-\beta)}} \right\} \\ &\quad + \frac{pI_d aa^\beta}{T^2} \left[-(1-\beta)M_2(T-M_2)^{\frac{1}{(1-\beta)}} \right. \\ &\quad \left. - \frac{(1-\beta)^2}{(2-\beta)} \left\{ (T-M_2)^{\frac{(2-\beta)}{(1-\beta)}} - T^{\frac{(2-\beta)}{(1-\beta)}} \right\} \right] \quad (20) \end{aligned}$$

$$\begin{aligned} \frac{dZ_4(T)}{dT} &= -\frac{s}{T^2} + \frac{rac\beta T^{\frac{(2\beta-1)}{(1-\beta)}}}{1-\beta} + \frac{ah}{(2-\beta)} T^{\frac{\beta}{(1-\beta)}} \\ &\quad + \frac{pI_d aa^\beta}{T} (T-M_2)T^{\frac{\beta}{(1-\beta)}} \\ &\quad + \frac{pI_d aa^\beta}{T^2} \left\{ \frac{(1-\beta)^2}{(2-\beta)} T^{\frac{(2-\beta)}{(1-\beta)}} - (1-\beta)(T-M_2)T^{\frac{1}{(1-\beta)}} \right\} \quad (21) \end{aligned}$$

$$\begin{aligned} \frac{d^2 Z_1}{dT^2} &= \frac{2s}{T^3} + \frac{rac\beta(2\beta-1)T^{\frac{(3\beta-2)}{(1-\beta)}}}{(1-\beta)^2} + \frac{ah\beta T^{\frac{(2\beta-1)}{(1-\beta)}}}{(2-\beta)(1-\beta)} \\ &\quad + ac(1-r)I_c \left[\frac{(T-M_1)^{\frac{\beta}{(1-\beta)}}}{(1-\beta)T} - \right. \\ &\quad \left. \frac{2(T-M_1)^{\frac{1}{(1-\beta)}}}{T^2} + \frac{2(1-\beta)(T-M_1)^{\frac{(2-\beta)}{(1-\beta)}}}{(2-\beta)T^3} \right] \\ &\quad - \frac{pI_d aa^\beta}{T} \left[-\frac{\beta M_1(T-M_1)^{\frac{(2\beta-1)}{(1-\beta)}}}{(1-\beta)} - (T-M_1)^{\frac{\beta}{(1-\beta)}} \right. \\ &\quad \left. + T^{\frac{\beta}{(1-\beta)}} \right] + \frac{2pI_d aa^\beta}{T^2} \left[-M_1(T-M_1)^{\frac{\beta}{(1-\beta)}} \right. \\ &\quad \left. - (1-\beta)(T-M_1)^{\frac{1}{(1-\beta)}} + (1-\beta)T^{\frac{1}{(1-\beta)}} \right] \end{aligned}$$

$$-\frac{2pI_d aa^\beta}{T^3} \left[-(1-\beta)M_1(T-M_1)^{1/(1-\beta)} - \frac{(1-\beta)^2}{(2-\beta)} \left\{ (T-M_1)^{(2-\beta)/(1-\beta)} - T^{(2-\beta)/(1-\beta)} \right\} \right] \quad (22)$$

$$\begin{aligned} \frac{d^2 Z_2(T)}{dT^2} = & \frac{2s}{T^3} + \frac{rac\beta(2\beta-1)T^{(3\beta-2)/(1-\beta)}}{(1-\beta)^2} \\ & + \frac{ah\beta T^{(2\beta-1)/(1-\beta)}}{(2-\beta)(1-\beta)} + \frac{pI_d aa^\beta}{T} \left[\frac{\beta(T-M_1)T^{(2\beta-1)/(1-\beta)}}{(1-\beta)} \right. \\ & \left. + T^{\beta/(1-\beta)} \right] - \frac{2pI_d aa^\beta (T-M_1)T^{\beta/(1-\beta)}}{T^2} \\ & - \frac{2pI_d aa^\beta}{T^3} \left[\frac{(1-\beta)^2 T^{(2-\beta)/(1-\beta)}}{(2-\beta)} - (1-\beta)(T-M_1)T^{1/(1-\beta)} \right] \end{aligned} \quad (23)$$

$$\begin{aligned} \frac{d^2 Z_3}{dT^2} = & \frac{2s}{T^3} + \frac{rac\beta(2\beta-1)T^{(3\beta-2)/(1-\beta)}}{(1-\beta)^2} + \frac{ah\beta T^{(2\beta-1)/(1-\beta)}}{(2-\beta)(1-\beta)} \\ & + acI_c \left[\frac{(T-M_2)^{\beta/(1-\beta)}}{(1-\beta)T} - \frac{2(T-M_2)^{1/(1-\beta)}}{T^2} \right. \\ & \left. + \frac{2(1-\beta)(T-M_2)^{(2-\beta)/(1-\beta)}}{(2-\beta)T^3} \right] - \frac{pI_d aa^\beta}{T} \\ & \left[-\frac{\beta M_2(T-M_2)^{(2\beta-1)/(1-\beta)}}{(1-\beta)} - (T-M_2)^{\beta/(1-\beta)} + T^{\beta/(1-\beta)} \right] \\ & + \frac{2pI_d aa^\beta}{T^2} \left[-M_2(T-M_2)^{\beta/(1-\beta)} - (1-\beta)(T-M_2)^{1/(1-\beta)} \right. \\ & \left. + (1-\beta)T^{1/(1-\beta)} \right] \\ & - \frac{2pI_d aa^\beta}{T^3} \left[-(1-\beta)M_2(T-M_2)^{1/(1-\beta)} - \frac{(1-\beta)^2}{(2-\beta)} \left\{ (T-M_2)^{(2-\beta)/(1-\beta)} - T^{(2-\beta)/(1-\beta)} \right\} \right] \end{aligned} \quad (24)$$

$$\begin{aligned} \frac{d^2 Z_4(T)}{dT^2} = & \frac{2s}{T^3} + \frac{rac\beta(2\beta-1)T^{(3\beta-2)/(1-\beta)}}{(1-\beta)^2} \\ & + \frac{ah\beta T^{(2\beta-1)/(1-\beta)}}{(2-\beta)(1-\beta)} + \frac{pI_d aa^\beta}{T} \left[\frac{\beta(T-M_2)T^{(2\beta-1)/(1-\beta)}}{(1-\beta)} \right. \\ & \left. + T^{\beta/(1-\beta)} \right] - \frac{2pI_d aa^\beta (T-M_2)T^{\beta/(1-\beta)}}{T^2} \\ & - \frac{2pI_d aa^\beta}{T^3} \left[\frac{(1-\beta)^2 T^{(2-\beta)/(1-\beta)}}{(2-\beta)} - (1-\beta)(T-M_2)T^{1/(1-\beta)} \right] \end{aligned} \quad (25)$$

The necessary and sufficient condition for finding minimum value of $Z_i(T)$ is given by

$$\begin{aligned} \frac{dZ_i(T)}{dT} = 0, \text{ and } \frac{d^2 Z_i(T)}{dT^2} > 0, \quad (i = 1, 2, 3, 4) \\ \frac{s}{T^2} - \frac{rac\beta T^{(2\beta-1)/(1-\beta)}}{1-\beta} - \frac{ah}{(2-\beta)} T^{\beta/(1-\beta)} \\ - ac(1-r)I_c \left\{ \frac{(T-M_1)^{1/(1-\beta)}}{T} - \frac{(1-\beta)(T-M_1)^{(2-\beta)/(1-\beta)}}{(2-\beta)T^2} \right\} \\ + \frac{pI_d aa^\beta}{T} \left[-M_1(T-M_1)^{\beta/(1-\beta)} - (1-\beta)(T-M_1)^{1/(1-\beta)} \right. \\ \left. + (1-\beta)T^{1/(1-\beta)} \right] + \frac{pI_d aa^\beta}{T} \left[(1-\beta)M_1(T-M_1)^{1/(1-\beta)} \right. \\ \left. - \frac{s}{T^2} - \frac{rac\beta T^{(2\beta-1)/(1-\beta)}}{1-\beta} - \frac{ah}{(2-\beta)} T^{\beta/(1-\beta)} - \frac{pI_d aa^\beta}{T} (T-M_1)T^{\beta/(1-\beta)} \right. \\ \left. - \frac{pI_d aa^\beta}{T^2} \left\{ \frac{(1-\beta)^2}{(2-\beta)} T^{(2-\beta)/(1-\beta)} - (1-\beta)(T-M_1)T^{1/(1-\beta)} \right\} \right] = 0 \end{aligned} \quad (26)$$

$$\begin{aligned} \frac{s}{T^2} - \frac{rac\beta T^{(2\beta-1)/(1-\beta)}}{1-\beta} - \frac{ah}{(2-\beta)} T^{\beta/(1-\beta)} \\ - \frac{acI_c (T-M_2)^{1/(1-\beta)}}{T} \left\{ 1 - \frac{(1-\beta)(T-M_2)}{(2-\beta)T} \right\} \\ + \frac{pI_d aa^\beta}{T} \left\{ -M_2(T-M_2)^{\beta/(1-\beta)} - (1-\beta)(T-M_2)^{1/(1-\beta)} \right. \\ \left. + (1-\beta)T^{1/(1-\beta)} \right\} \end{aligned} \quad (27)$$

$$+ \frac{pI_d a a^\beta}{T^2} \left[(1-\beta)M_2(T-M_2)^{1/(1-\beta)} + \frac{(1-\beta)^2}{(2-\beta)} \left\{ (T-M_2)^{(2-\beta)/(1-\beta)} - T^{(2-\beta)/(1-\beta)} \right\} \right] = 0 \quad (28)$$

$$\frac{s}{T^2} - \frac{rac\beta T^{(2\beta-1)/(1-\beta)}}{1-\beta} - \frac{ah}{(2-\beta)} T^{\beta/(1-\beta)} - \frac{pI_d a a^\beta}{T} (T-M_2) T^{\beta/(1-\beta)} - \frac{pI_d a a^\beta}{T^2} \left\{ \frac{(1-\beta)^2}{(2-\beta)} T^{(2-\beta)/(1-\beta)} - (1-\beta)(T-M_2) T^{1/(1-\beta)} \right\} = 0 \quad (29)$$

5. Numerical Examples

Example 1 (case 1). Let us take the parameter values as $a = 200$; $\beta = 0.02$; $h = \$8/\text{unit/year}$; $I_c = 0.10/\text{year}$; $I_d = 0.04/\text{year}$; $s = \$30/\text{order}$; $c = \$35/\text{unit}$; $p = \$15/\text{unit}$; $r = 0.05$; $M_1 = 0.020134 \text{ year}$. Substituting these values in equation (26) and solving; we get optimal $T = T_1 = 0.155313 \text{ years}$; and optimal ordered quantity $Q^*(T_1) = 32.6391 \text{ units}$; and total relevant cost per year $Z^*(T_1) = \$730.568$, which verified case 1 i.e. $T \geq M_1$ Where $\frac{d^2 Z_1}{dT^2} = 15941.0 > 0$.

Example 2 (case 2). Let us take the parameter values as $a = 600$; $\beta = 0.04$; $h = \$5/\text{unit/year}$; $I_d = 0.08/\text{year}$; $s = \$10/\text{order}$; $c = \$15/\text{unit}$; $p = \$40/\text{unit}$; $r = 0.05$; $M_1 = 0.151125 \text{ year}$. Substituting these values in equation (27) and solving; we get optimal $T = T_1 = 0.0581032 \text{ years}$; and optimal ordered quantity $Q^*(T_2) = 38.739 \text{ units}$; and total relevant cost $Z^*(T_2) = \$507.842$, which verified case 2 i.e. $T \geq M_1$. Where $\frac{d^2 Z_2}{dT^2} = 84840.22 > 0$.

Example 3 (case 3). Let us take the parameter values as $a = 700$; $\beta = 0.03$; $h = \$6/\text{unit/year}$; $I_c = 0.08/\text{year}$; $I_d = 0.07/\text{year}$; $s = \$60/\text{order}$; $c = \$25/\text{unit}$; $p = \$45/\text{unit}$; $r = 0.04$; $M_2 = 0.09524 \text{ year}$. Substituting these values in equation (28) and solving; we get optimal $T = T_3 = 0.130425 \text{ years}$; and optimal ordered quantity $Q^*(T_3) = 101.731 \text{ units}$; and total relevant cost $Z^*(T_3) = \$1461.44$, per year which verified case 3 i.e. $T \geq M_2$, where $\frac{d^2 Z_3}{dT^2} = 51085.5 > 0$.

Example 4 (case 4). Let us take the parameter values as $a = 400$; $\beta = 0.05$; $h = \$7/\text{unit/year}$; $I_d = 0.09/\text{year}$; $s = \$25/\text{order}$; $c = \$30/\text{unit}$; $p = \$30/\text{unit}$; $r = 0.02$; $M_2 = 0.19336 \text{ year}$. Substituting these values in equation (29) and solving; we get optimal $T = T_4 = 0.103255 \text{ year}$; and optimal ordered

quantity $Q^*(T_4) = 47.5963 \text{ units}$; and total relevant cost $Z^*(T_4) = \$506.259$, which verified case 4, i.e. $T \geq M_2$. Where $\frac{d^2 Z_4}{dT^2} = 38550.534 > 0$.

6 Sensitivity Analysis

We have discussed sensitivity analysis by varying the parameters s , I_c , I_d and r and keeping the remaining parameters at their original values as in numerical examples 1 to 4. The corresponding changes for the replenishment cycle time, economic order quantity and total relevant cost are provided in the following Tables 1 to 4.

Case (1) Table 1.

Table 1(a). The sensitivity analysis of s keeping all the parameters same as in Example 1

S	T_1 (in years)	$Q^*(T_1)$	$Z^*(T_1)$	$\frac{d^2 Z_1}{dT^2}$
35	0.167812	35.3216	761.515	14756.0
40	0.17944	37.8207	790.313	13802.9
45	0.190357	40.1701	817.355	13014.6
50	0.200678	42.3938	842.928	12348.6

Table 1(b). The sensitivity analysis of I_c keeping all the parameters same as in Example 1

I_c	T_1 (in years)	$Q^*(T_1)$	$Z^*(T_1)$	$\frac{d^2 Z_1}{dT^2}$
0.11	0.153194	32.1848	734.588	16625.1
0.12	0.151162	31.7492	738.538	17318.3
0.13	0.149212	31.3314	742.418	18020.2
0.14	0.147338	30.9299	746.233	18731.0

Table 1(c). The sensitivity analysis of r keeping all the parameters same as in Example 1

r	T_1 (in years)	$Q^*(T_1)$	$Z^*(T_1)$	$\frac{d^2 Z_1}{dT^2}$
0.06	0.154935	32.5581	803.692	15995.3
0.07	0.154556	32.4768	876.814	16050.4
0.08	0.154177	32.3956	949.934	16106.0
0.09	0.153798	32.3143	1023.05	16162.2

Table 1(d). The sensitivity analysis of I_d keeping all the parameters same as in Example 1

I_d	T_1 (in years)	$Q^*(T_1)$	$Z^*(T_1)$	$\frac{d^2 Z_1}{dT^2}$
0.05	0.155296	32.6355	730.526	15942.9
0.06	0.15528	32.6321	730.484	15944.5
0.07	0.155263	32.6284	730.442	15946.3
0.08	0.155247	32.625	730.4	15947.9

Table 2(a). The sensitivity analysis of s keeping all the parameters same as in Example 2

s	T_2 (in years)	$Q^*(T_2)$	$Z^*(T_2)$	$\frac{d^2 Z_2}{dT^2}$
11	0.0609397	40.711	524.643	80240.321
12	0.0636475	42.597	540.696	76213.738
13	0.0662424	44.4076	556.093	72644.021
14	0.0687374	46.1512	570.91	69444.066

Table 2(b). The sensitivity analysis of I_d keeping all the parameters same as in Example 2

I_d	T_2 (in years)	$Q^*(T_2)$	$Z^*(T_2)$	$\frac{d^2 Z_2}{dT^2}$
0.09	0.0570322	37.9955	475.383	89089.675
0.10	0.0560287	37.2993	442.808	93414.486
0.11	0.0550858	36.6457	410.125	97811.844
0.12	0.0541976	36.0304	377.34	102278.889

Table 3(a). The sensitivity analysis of s keeping all the parameters same as in Example 3

s	T_3 (in years)	$Q^*(T_3)$	$Z^*(T_3)$	$\frac{d^2 Z_3}{dT^2}$
65	0.136063	106.268	1498.97	48868.5
70	0.141486	110.637	1535.0	46933.9
75	0.146713	114.853	1569.69	45223.9
80	0.151764	118.932	1603.2	43694.9

Table 3(b). The sensitivity analysis of I_c keeping all the parameters same as in Example 3

I_c	T_3 (in years)	$Q^*(T_3)$	$Z^*(T_3)$	$\frac{d^2 Z_3}{dT^2}$
0.09	0.129589	101.059	1462.3	52934.6
0.10	0.128794	100.42	1463.12	54766.3
0.11	0.128037	99.8115	1463.91	56613.4
0.12	0.127316	99.2321	1464.67	58463.3

Table 3(c). The sensitivity analysis of I_d keeping all the parameters same as in Example 3

I_c	T_3 (in years)	$Q^*(T_3)$	$Z^*(T_3)$	$\frac{d^2 Z_3}{dT^2}$
0.08	0.128695	100.34	1449.06	51938.0
0.09	0.126952	98.9397	1436.52	52844.7
0.10	0.125196	97.5291	1423.82	53814.2
0.11	0.123428	96.1095	1410.95	54855.8

Table 4(a). The sensitivity analysis of s keeping all the parameters same as in Example 4

s	T_4 (in years)	$Q^*(T_4)$	$Z^*(T_4)$	$\frac{d^2 Z_4}{dT^2}$
30	0.112882	52.2787	552.524	34477.712
35	0.121717	56.5944	595.149	31120.707
40	0.129925	60.6187	634.887	28199.86
45	0.137622	64.4047	672.264	25526.348

Table 4(b). The sensitivity analysis of I_d keeping all the parameters same as in Example 4

I_d	T_4 (in years)	$Q^*(T_4)$	$Z^*(T_4)$	$\frac{d^2 Z_4}{dT^2}$
0.10	0.101969	46.9725	486.803	39624.993
0.12	0.0995579	45.8041	447.664	41831.99
0.13	0.0984248	45.2555	427.988	42963.81
0.14	0.0973357	44.7286	408.246	44113.576

All the above observations from Tables 1–4, can be explained as follows:

- If the ordering cost s increases, slight decrease in economic order quantity (except case 2)
- And increase in total relevant cost $Z(T)$. That is, change in s leads slight negative change in Q and total relevant cost $Z(T)$.
- If the interest charge I_c per dollar increases, the economic order quantity slightly changes and slight increase in total relevant cost $Z(T)$. That is, change in I_c positive change in Q and slight positive change in total relevant cost $Z(T)$.
- If the cost discount rate r increases, the economic order quantity slightly decreases and the total relevant cost increases. That is, change in r leads slight negative in Q and increase in total relevant cost $Z(T)$.

- If the interest earned I_d per dollar increases, the economic order quantity slightly decreases and the total relevant cost decreases except case 1.

7. Conclusion and Future Research

In this paper we have developed an inventory model under trade credit with power inventory dependent demand, considering four different cases. We have proved several managerial phenomena.

- The higher value of s , I_c , I_d , and r caused higher value of the total relevant cost except case 1 (for I_d), case 2 (for I_d), case 3 (for I_d) and case 4 (for I_d). We have also provided some examples to validate the proposed model and its optimal solution. Finally sensitivity analysis of the system is also discussed.
- The model proposed in this manuscript can be extended for several ways. For instance, we may extend the model for time dependent deterioration as well as two parameters weibull distribution deterioration. We may also extend the model for adding freight charges and others.

8. References

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