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A Partial Backlogging Inventory Model for Non-Instantaneous Decaying Items under Trade Credit Financing Facility

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Abstract

Background/Objectives: In this model we investigate and analyzing an inventory model for non-instantaneous decaying items are considered under permissible delay in payments for retailer's optimal replenishment policy. **Statistical Analysis:** Inventory models play most influential role in analyzing many realistic situations into the picture like, vegetable and food markets, oil exploration industries, warehouses, market yards, etc. In the most inventory organizations are usually formed without considering the effect of deterioration of items but here we have considered variable deterioration, linear demand and unsatisfied demand is partially backlogged. **Findings:** The model is solved analytically by minimize the retailer's total cost w.r.to specific value of the parameters. **Application/Improvements:** Three conditions of allowable delay in payments are discussed. Sensitivity analysis of the model with different parameter is use to discover the most favorable solution.

Keywords: Linear Demand, Non-Instantaneous Deterioration, Partial Backlogging, Permissible Delay in Payments

1. Introduction

In the both deterministic and probabilistic inventory systems of classical type, for many businesses the rate of deterioration has very important role on the optimal ordering and pricing scheme of the inventory models. Normally it is observed that payments are made to the supplier immediately after receiving the stock but in practice life, the supplier offers to the retailer a delay period in payment for the amount of purchase to increase the demand and for financing growth known as trade credit period, that means any time retailer can have delivery of stock without paying cash on the spot. Offering such a credit period to the retailer will encourage the supplier's selling and reduce on hand inventory level. Simultaneously, without a initial payment the retailer can have the advantages of a credit period to reduce cost and increase profit. The retailer doesn't need to pay any interest during the contracted period but if the payment gets delayed beyond the period interest will be charged by the dealer. Hence, trade credit can have an significant role in integrated inventory system.

Formulated a first economic order quantity model under trade credit facility[1]. Developed an ordering policies of deteriorating items under permissible delay in payments[2]. Presented a joint price and lot size model

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under conditions of permissible delay in payments and quantity discounts for freight cost[3]. Addressed an retailer's pricing and lot sizing policy for exponentially deteriorating product under the condition of permissible delay in payments[4]. Derived an optimal payment time for a retailer under permitted delay of payment by the wholesaler[5]. Introduced an optimal replenishment and payment policies in the EOQ model under cash discount and trade credit[6]. Investigated an optimal pricing and ordering policy under permissible delay in payments[7]. Elaborated an EOQ model in fuzzy environment and trade credit[8]. Discussed an economic order quantity model under conditions of permissible delay in payments[9].

Partial backlogging is a shortage of products in which some customers can wait for the items till the next cycle of products to come and some can't. In actuality, for electronic components and fashionable commodities with short life cycles, during a shortage period some but not all customers will wait for backlogged products. Normally customers may purchase the goods from another store if longer the waiting time for the goods that is stock-out will be less likely to purchase again from the supplier. Overlong the waiting time to the consumer for the items leads to the smaller backlogging rate. According to such situations, considering the backlogging rate is requisite. Formulated an ordering policy for deteriorating items with allowable shortages and permissible delay in payments[10]. Investigated an inventory model for deteriorating items with partial backlogging and trade credit facility[11].

Deterioration is a realistic process of overall decrease in value of goods over a period of time due to expiry, decaying, destroying, spoilage etc. Elaborated an optimal replenishment policy for non-instantaneous deteriorating items. Other articles related to this research[14-17]. Developed a multi item inventory model for deteriorating items with expiration date and allowable shortages[18]. Investigated a model for Weibull deteriorate items with price dependent demand rate and inflation[19]. Eluciated an optimal payment policy with preservation technology investment and shortages under trade credit[20].

In this paper we have considered a non-instantaneous-deteriorating item that means value of product will decrease over the period of time such as fashion clothes, electronic items etc. Shortages of products are allowed and are partially backlogged.

2. Assumptions and nomenclature

2.1 Assumptions

- The demand rate for the item is considered to be time dependent.
- The goods are deteriorating in nature and the rate of deterioration is time dependent.
- The occurring shortages are partially backlogged and backlogging rate is supposed to be a function of time.
- Life time deterioration is considered.
- Delay of payment is allowed.
- T is the cycle time.

2.2 Nomenclature

- $D(t) = a + bt$: demand rate where $a, b > 0$
- $B(t) = e^{-\delta(T-t)}$: backlogging rate where $0 < \delta < 1$
- $\theta(t) = \theta t$: rate of deterioration where $0 < \theta < 1$
- p: per unit selling price
- C: per unit purchase cost
- A: fixed ordering cost of inventory
- h : per unit inventory holding cost
- C_1 : per unit deterioration cost
- C_2 : per unit shortage cost
- C_3 : per unit lost sales cost
- M: permissible period of delay in settling the accounts with the supplier
- I_e : the interest which can be earned
- I_c : the interest charges which are invested in inventory $I_c \geq I_e$
- t_μ : the life time deteriorating period of inventory system
- t_1 : time when inventory level comes down to zero
- $I_1(t)$: the inventory level at time t with $t \in (0, t_\mu)$

- $I_2(t)$: the inventory level at time t with $t \in (t_\mu, t_1)$
- $I_3(t)$: the inventory level at time t with $t \in (t_1, T)$

3. Formulation of the Model

The inventory cycle starts at $t=0$ with the initial inventory level of $I_{max.}$ units. During the time interval $(0,t_\mu)$ the inventory level reduces only owing to demand. The demand for the product is time dependent. After that the time period (t_μ, t_1) inventory is dropping to zero due to demand rate and deterioration both and duration of (t_1, T) shortage starts and due to partial backlogging some sales are lost. The differential equations governing the transition of the system are

$$\frac{dI_1(t)}{dt} = -(a+bt), \qquad 0 \le t \le t_\mu \tag{1}$$

$$\frac{dI_2(t)}{dt} + \theta t I_2(t) = -(a+bt), \qquad t_\mu \le t \le t_1 \tag{2}$$

$$\frac{dI_3(t)}{dt} = -(a+bt)\, e^{-\delta(T-t)} \qquad t_1 \le t \le T \tag{3}$$

With boundary conditions

$$I_1(0) = I_{max.}, I_2(t_1) = 0 \;\&\; I_3(t_1) = 0 \tag{4}$$

Solutions of these equations are

$$I_1(t) = \left\{ I_{max.} - (at + \frac{bt^2}{2}) \right\} \tag{5}$$

$$I_2(t) = (1 - \frac{\theta t^2}{2})\left\{ a\left(t_1 - t\right) + \frac{b}{2}\left(t_1^2 - t^2\right) + \frac{a\theta}{6}\left(t_1^3 - t^3\right) + \frac{b\theta}{8}\left(t_1^4 - t^4\right) \right\} \tag{6}$$

$$I_3(t) = \left\{ (1-\delta T)\left((at_1 + \frac{bt_1^2}{2}) - (at + \frac{bt^2}{2}) \right) + \frac{a\delta}{2}(t_1^2 - t^2) + \frac{b\delta}{6}(t_1^3 - t^3) \right\} \tag{7}$$

Considering the continuity at $t = t_\mu$ it follows from equation (5) & (6) such that $I_1(t_\mu) = I_2(t_\mu)$

$$I_{max.} - (at_\mu + \frac{bt_\mu{}^2}{2}) = (1 - \frac{\theta t_\mu^2}{2})\left\{ a\left(t_1 - t_\mu\right) + \frac{b}{2}\left(t_1^2 - t_\mu^2\right) + \frac{a\theta}{6}\left(t_1^3 - t_\mu^3\right) + \frac{b\theta}{8}\left(t_1^4 - t_\mu^4\right) \right\} \tag{8}$$

We get maximum inventory level Q_1 is

$$Q_1 = I_{max.} = (a\, t_\mu + \frac{bt_\mu{}^2}{2}) + (1 - \frac{\theta t_\mu^2}{2})\left\{ a\left(t_1 - t_\mu\right) + \frac{b}{2}\left(t_1^2 - t_\mu^2\right) + \frac{a\theta}{6}\left(t_1^3 - t_\mu^3\right) + \frac{b\theta}{8}\left(t_1^4 - t_\mu^4\right) \right\} \tag{9}$$

Now equation (5) becomes

$$I_1(t) = \left[(at_\mu + \frac{bt_\mu{}^2}{2}) + (1 - \frac{\theta t_\mu^2}{2})\left\{ a\left(t_1 - t_\mu\right) + \frac{b}{2}\left(t_1^2 - t_\mu^2\right) + \frac{a\theta}{6}\left(t_1^3 - t_\mu^3\right) + \frac{b\theta}{8}\left(t_1^4 - t_\mu^4\right) \right\} - (at + \frac{bt^2}{2}) \right] \tag{10}$$

At time T, we get max. backordered inventory

$$Q_2 = -I_3(T) = \left[(\delta T - 1)\{(at_1 + \frac{bt_1^2}{2}) - (aT + \frac{bT^2}{2})\} - \frac{a\delta}{2}(t_1^2 - T^2) - \frac{b\delta}{6}(t_1^3 - T^3) \right] \tag{11}$$

Order quantity $Q = Q_1 + Q_2$ is

$$Q = ae^{mt_d}\left[\left(t_1 - t_d\right) + \frac{\theta}{6}\left(t_1^3 - t_d{}^3\right) + \frac{m}{2}\left(t_1^2 - t_d{}^2\right) + \frac{\theta}{2} t_d{}^2 (t_d - t_1) + \frac{\theta m}{4} t_d{}^2 (t_d{}^2 - t_1^2) \right.$$

$$+mt_d\left(t_1-t_d\right)+\frac{\theta m}{6}t_d\left(t_1^3-t_d^3\right)\Bigg]+at_d\left(1+\frac{m}{2}t_d\right)$$

$$-a\left(T-\delta\frac{T^2}{2}\right)+a\left(t_1-\delta\left(Tt_1-\frac{t_1^2}{2}\right)\right) \tag{12}$$

3.1 Ordering Cost

OC = A (13)

3.2 Present worth Holding Cost

$$HC=h\left[\int_0^{t_\mu}I_1(t)dt+\int_{t_\mu}^{t_1}I_2(t)dt\right]$$

$$HC=h\Bigg[\int_0^{t_\mu}\Bigg((at_\mu+\frac{bt_\mu^{\,2}}{2})+(1-\frac{\theta t_\mu^2}{2})\Bigg(a\left(t_1-t_\mu\right)+\frac{b}{2}\left(t_1^2-t_\mu^2\right)+$$

$$\frac{a\theta}{6}\left(t_1^3-t_\mu^3\right)+\frac{b\theta}{8}\left(t_1^4-t_\mu^4\right)-(at+\frac{bt^2}{2})\Bigg)dt$$

$$+\int_{t_\mu}^{t_1}\Bigg((1-\frac{\theta t^2}{2})\Bigg(a\left(t_1-t\right)+\frac{b}{2}\left(t_1^2-t^2\right)+$$

$$+\frac{a\theta}{6}\left(t_1^3-t^3\right)+\frac{b\theta}{8}\left(t_1^4-t^4\right)\Bigg)\Bigg)dt\Bigg] \tag{14}$$

3.3 Present worth Deterioration Cost

$$DC=C_1\left(I_2(t_\mu)-\int_{t_\mu}^{t_1}(a+bt)\,dt\right)$$

$$DC=C_1\left(I_2(t_\mu)-a\left(t_1-t_\mu\right)-\frac{b}{2}\left(t_1^2-t_\mu^2\right)\right) \tag{15}$$

3.4 Present worth Shortage Cost

$$SC=-C_2\int_{t_1}^{T}I_3(t)dt$$

$$SC=-C_2\int_{t_1}^{T}\Bigg((1-\delta T)\{(at_1+\frac{bt_1^2}{2})-(at+\frac{bt^2}{2})\}$$

$$+\frac{a\delta}{2}(t_1^2-t^2)+\frac{b\delta}{6}(t_1^3-t^3)\Bigg)dt \tag{16}$$

3.5 Present worth Lost Sales Cost

$$LS=C_3\int_{t_1}^{T}(a+bt)(1-e^{-\delta(T-t)})dt$$

$$LS=C_3\delta\Bigg(aT(T-t_1)+(bT-a)$$

$$\frac{(T^2-t_1^{\,2})}{2}-\frac{b}{3}(T^3-t_1^{\,3})\Bigg) \tag{17}$$

3.6 Now three cases are being considered here to evaluate the interest earned from sale and interest charged on inventory in stock. Therefore total inventory cost per unit time during the cycle (0, T) is given by

$$TC(t_1,T)=\begin{cases}TC_1(t_1,T), & if \quad M\le t_\mu\\ TC_2(t_1,T), & if \quad t_\mu\le M<t_1\\ TC_3(t_1,T), & if \quad M\ge t_1\end{cases} \tag{18}$$

Case 1: When $M\le t_\mu$

In this case interest is charged on the inventory for the duration of (t_1-M) at the rate of I_c. But interest will be earned till the day of account settlement.

Interest Earned

$$IE_1 = pI_e\int_0^{M}t(a+bt)dt=\frac{pI_eM^2}{6}(3a+2bM) \tag{19}$$

Interest Charged $IC_1 = CI_c\left(\int_M^{t_\mu}I_1(t)dt+\int_{t_\mu}^{t_1}I_2(t)dt\right)$

$$IC_1=CI_c\Bigg[\int_M^{t_\mu}\Bigg\{(at_\mu+\frac{bt_\mu^{\,2}}{2})+(1-\frac{\theta t_\mu^2}{2})\Bigg(a\left(t_1-t_\mu\right)+\frac{b}{2}\left(t_1^2-t_\mu^2\right)$$

$$+\frac{a\theta}{6}\left(t_1^3-t_\mu^3\right)+\frac{b\theta}{8}\left(t_1^4-t_\mu^4\right)\Bigg)-(at+\frac{bt^2}{2})\Bigg\}dt$$

$$+\int_{t_\mu}^{t_1}\Bigg\{(1-\frac{\theta t^2}{2})\Bigg(a\left(t_1-t\right)+\frac{b}{2}\left(t_1^2-t^2\right)+$$

$$\left\{\frac{a\theta}{6}\left(t_1^3-t^3\right)+\frac{b\theta}{8}\left(t_1^4-t^4\right)\Bigg)\right\}dt\Bigg] \tag{20}$$

The total inventory cost per unit time is given by

$$TC_1(t_1,T)=\frac{1}{T}[DC+OC+HC+SC+LS+IC_1-IE_1] \quad (21)$$

$$TC_1(t_1,T)=\frac{1}{T}\Bigg[C_1\Bigg((1-\frac{\theta t_\mu^2}{2})\Big\{a(t_1-t_\mu)+\frac{b}{2}(t_1^2-t_\mu^2)$$

$$\frac{a\theta}{6}(t_1^3-t_\mu^3)+\frac{b\theta}{8}(t_1^4-t_\mu^4)\Big\}-a(t_1-t_\mu)$$

$$-\frac{b}{2}(t_1^2-t_\mu^2)\Bigg)+A+C_3\delta\Bigg(aT(T-t_1)+(bT-a)\cdot$$

$$\frac{(T^2-t_1^2)}{2}-\frac{b}{3}(T^3-t_1^3)\Bigg)-C_2\Bigg(\frac{a\delta}{6}(t_1^3-T^3)$$

$$+\frac{b\delta}{24}(t_1^4-T^4)+(T-t_1)\delta\left(\frac{at_1^2}{2}+\frac{bt_1^3}{6}\right)+(1-\delta T)\Bigg((T-t_1)$$

$$\left(at_1+\frac{bt_1^2}{2}\right)-\frac{a}{2}(T^2-t_1^2)-\frac{b}{6}(T^3-t_1^3)\Bigg)\Bigg)$$

$$+h\Bigg(\left(\frac{at_\mu^2}{2}+\frac{bt_\mu^3}{3}\right)+t_d\Big(a(t_1-t_\mu)+\frac{b}{2}(t_1^2-t_\mu^2)+\frac{a\theta}{6}$$

$$(t_1^3-t_\mu^3)+\frac{b\theta}{8}(t_1^4-t_\mu^4)\Big)-\frac{a}{2}(t_1^2-t_\mu^2)-\frac{b}{6}(t_1^3-t_\mu^3)$$

$$-\frac{\theta t_\mu^3}{2}\left(a(t_1-t_\mu)+\frac{b}{2}(t_1^2-t_\mu^2)\right)+\frac{a\theta}{12}(t_1^4-t_\mu^4)-\frac{\theta}{6}$$

$$(t_1^3-t_\mu^3)\left(at_1+\frac{bt_1^2}{2}\right)+\frac{b\theta}{40}(t_1^5-t_\mu^5)+(t_1-t_\mu)$$

$$\left(at_1+\frac{bt_1^2}{2}+\frac{a\theta}{6}t_1^3+\frac{b\theta}{8}t_1^4\right)\Bigg)+CI_c\Bigg(\Big(at_1+\frac{bt_1^2}{2}+\frac{a\theta}{6}t_1^3+$$

$$\frac{b\theta}{8}t_1^4\Big)(t_1-t_\mu)-\frac{a}{2}(t_1^2-t_\mu^2)-\frac{b}{6}(t_1^3-t_\mu^3)-\frac{a\theta}{24}$$

$$(t_1^4-t_\mu^4)-\frac{b\theta}{40}(t_1^5-t_\mu^5)-\frac{\theta}{6}(t_1^3-t_\mu^3)\left(at_1+\frac{bt_1^2}{2}\right)+\frac{a\theta}{8}$$

$$(t_1^4-t_\mu^4)+\frac{b\theta}{20}(t_1^5-t_\mu^5)+(t_\mu-M)\left(at_\mu+\frac{bt_\mu^2}{2}\right)$$

$$+(t_\mu-M)(1-\frac{\theta t_\mu^2}{2})\Big\{a(t_1-t_\mu)+\frac{b}{2}(t_1^2-t_\mu^2)+\frac{a\theta}{6}(t_1^3-t_\mu^3)+$$

$$\frac{b\theta}{8}(t_1^4-t_\mu^4)\Big\}-\frac{a}{2}(t_\mu^2-M^2)-\frac{b}{6}(t_\mu^3-M^3)\Bigg)$$

$$-\frac{pI_eM^2}{6}(3a+2bM)\Bigg] \quad (22)$$

Thus necessary conditions for minimizing the cost are

$$\frac{\partial TC_1(t_1,T)}{\partial t_1}=0,\frac{\partial TC_1(t_1,T)}{\partial T}=0 \quad (23)$$

And the sufficient conditions for minimizing the $TC_1(t_1,T)$ are

$$\frac{\partial^2 TC_1(t_1,T)}{\partial t_1^2}>0,\frac{\partial^2 TC_1(t_1,T)}{\partial T^2}>0 \;\; \&$$

$$\left[\frac{\partial^2 TC_1(t_1,T)}{\partial t_1^2}\right]\left[\frac{\partial^2 TC_1(t_1,T)}{\partial T^2}\right]$$

$$-\left[\frac{\partial^2 TC_1(t_1,T)}{\partial T\partial t_1}\right]^2>0 \quad (24)$$

Case 2: When $t \quad \leq M \quad t$

In this case interest is charged on the inventory for the period of (t_1-M) at the rate of I_c. But interest will be

earned till the day of account settlement.

Interest Earned

$$IE_2=pI_e\int_0^M t(a+bt)dt=\frac{pI_eM^2}{6}(3a+2bM) \quad (25)$$

Interest Charged $IC_2 = CI_c\int_M^{t_1} I_2(t)dt$

$$IC_2=CI_c\int_M^{t_1}\Bigg[(1-\frac{\theta t^2}{2})\Big\{a(t_1-t)+\frac{b}{2}(t_1^2-t^2)$$

$$+\frac{a\theta}{6}(t_1^3-t^3)+\frac{b\theta}{8}(t_1^4-t^4)\Big\}\Bigg]dt \quad (26)$$

The total inventory cost per unit time is given by

$$TC_2(t_1,T)=\frac{1}{T}[DC+OC+HC+SC$$

$$+LS+IC_2-IE_2] \quad (27)$$

$$TC_2(t_1,T)=\frac{1}{T}\Bigg[C_1\Bigg((1-\frac{\theta t_\mu^2}{2})\Big\{a\left(t_1-t_\mu\right)+\frac{b}{2}\left(t_1^2-t_\mu^2\right)+$$

$$\frac{a\theta}{6}\left(t_1^3-t_\mu^3\right)+\frac{b\theta}{8}\left(t_1^4-t_\mu^4\right)\Big\}-a\left(t_1-t_\mu\right)$$

$$-\frac{b}{2}\left(t_1^2-t_\mu^2\right)\Bigg)+A+C_3\delta\Bigg(aT(T-t_1)+(bT-a)$$

$$\frac{(T^2-t_1^{\,2})}{2}-\frac{b}{3}(T^3-t_1^{\,3})\Bigg)-C_2\left(\frac{a\delta}{6}\left(t_1^3-T^3\right)\right.$$

$$+\frac{b\delta}{24}\left(t_1^4-T^4\right)+(T-t_1)\delta\left(\frac{at_1^{\,2}}{2}+\frac{bt_1^{\,3}}{6}\right)+(1-\delta T)\Bigg((T-t_1)$$

$$\left(at_1+\frac{bt_1^{\,2}}{2}\right)-\frac{a}{2}(T^2-t_1^{\,2})-\frac{b}{6}(T^3-t_1^{\,3})\Bigg)\Bigg)$$

$$+h\Bigg(\left(\frac{at_\mu^{\,2}}{2}+\frac{bt_\mu^{\,3}}{3}\right)+t_d\left(a\left(t_1-t_\mu\right)+\frac{b}{2}\left(t_1^2-t_\mu^2\right)+\frac{a\theta}{6}\right.$$

$$\left.\left(t_1^3-t_\mu^3\right)+\frac{b\theta}{8}\left(t_1^4-t_\mu^4\right)\right)-\frac{a}{2}\left(t_1^2-t_\mu^2\right)-\frac{b}{6}\left(t_1^3-t_\mu^3\right)$$

$$-\frac{\theta t_\mu^{\,3}}{2}\left(a\left(t_1-t_\mu\right)+\frac{b}{2}\left(t_1^2-t_\mu^2\right)\right)+\frac{a\theta}{12}(t_1^4-t_\mu^4)-\frac{\theta}{6}$$

$$\left(t_1^3-t_\mu^3\right)\left(at_1+\frac{bt_1^{\,2}}{2}\right)+\frac{b\theta}{40}\left(t_1^5-t_\mu^5\right)+\left(t_1-t_\mu\right)$$

$$\left(at_1+\frac{bt_1^{\,2}}{2}+\frac{a\theta}{6}t_1^3+\frac{b\theta}{8}t_1^4\right)\Bigg)+CI_c\Bigg(\Bigg(at_1+\frac{bt_1^{\,2}}{2}+\frac{a\theta}{6}t_1^3+\frac{b\theta}{8}$$

$$t_1^4\Bigg)(t_1-M)-\frac{a}{2}\left(t_1^2-M^2\right)-\frac{b}{6}\left(t_1^3-M^3\right)-\frac{a\theta}{24}$$

$$(t_1^4-M^4)-\frac{b\theta}{40}\left(t_1^5-M^5\right)-\frac{\theta}{6}\left(t_1^3-M^3\right)\left(at_1+\frac{bt_1^{\,2}}{2}\right)+\frac{a\theta}{8}$$

$$(t_1^4-M^4)+\frac{b\theta}{20}\left(t_1^5-M^5\right)\Bigg)-\frac{pI_eM^2}{6}(3a+2bM)\Bigg] \tag{28}$$

Thus necessary conditions for minimizing the cost are

$$\frac{\partial TC_2(t_1,T)}{\partial t_1}=0,\frac{\partial TC_2(t_1,T)}{\partial T}=0 \tag{29}$$

And the sufficient conditions for minimizing the $TC_2(t_1,T)$ are

$$\frac{\partial^2 TC_2(t_1,T)}{\partial t_1^{\,2}}>0,\frac{\partial^2 TC_2(t_1,T)}{\partial T^2}>0\ \&$$

$$\left[\frac{\partial^2 TC_2(t_1,T)}{\partial t_1^{\,2}}\right]\left[\frac{\partial^2 TC_2(t_1,T)}{\partial T^2}\right]-$$

$$\left[\frac{\partial^2 TC_2(t_1,T)}{\partial T\partial t_1}\right]^2>0 \tag{30}$$

Case 3: When $M\ge t_1$

In this case there is no interest charged since the inventory is fully consumed. But interest will be earned till the day of account settlement.
Interest Earned

$$IE_3=pI_e\left[\int_0^{t_1}t(a+bt)dt+(M-t_1)\int_0^{t_1}(a+bt)dt\right]$$

$$IE_3=pI_e\left[\frac{t_1^2}{6}(bt_1-3a)+Mt_1(a+\frac{bt_1}{2})\right] \tag{31}$$

Interest Charged $IC_3=0$ (32)

The total inventory cost per unit time is given by

$$TC_3(t_1,T)=\frac{1}{T}[DC+OC+HC+SC$$

$$+LS+IC_3-IE_3] \tag{33}$$

$$TC_3(t_1,T)=\frac{1}{T}\Bigg[C_1\Bigg((1-\frac{\theta t_\mu^2}{2})\Big\{a\left(t_1-t_\mu\right)+\frac{b}{2}\left(t_1^2-t_\mu^2\right)$$

$$+\frac{a\theta}{6}\left(t_1^3-t_\mu^3\right)+\frac{b\theta}{8}\left(t_1^4-t_\mu^4\right)\Big\}-a\left(t_1-t_\mu\right)$$

$$-\frac{b}{2}\left(t_1^2-t_\mu^2\right)\Bigg)+A+C_3\delta\Bigg(aT(T-t_1)+(bT-a)$$

$$\frac{(T^2-t_1^{\,2})}{2}-\frac{b}{3}(T^3-t_1^{\,3})\Bigg)-C_2\left(\frac{a\delta}{6}\left(t_1^3-T^3\right)\right.$$

$$+\frac{b\delta}{24}\left(t_1^4-T^4\right)+(T-t_1)\delta\left(\frac{at_1^{\,2}}{2}+\frac{bt_1^{\,3}}{6}\right)+(1-\delta T)$$

$$\left((T-t_1)\left(at_1+\frac{bt_1^{\,2}}{2}\right)-\frac{a}{2}(T^2-t_1^{\,2})-\frac{b}{6}(T^3-t_1^{\,3})\right)\Bigg)$$

$$+h\left(\left(\frac{at_\mu^{\,2}}{2}+\frac{bt_\mu^{\,3}}{3}\right)-t_d\left(a\left(t_1-t_\mu\right)+\frac{b}{2}\left(t_1^2-t_\mu^2\right)+\frac{a\theta}{6}\right.\right.$$

$$\left.\left(t_1^3-t_\mu^3\right)+\frac{b\theta}{8}\left(t_1^4-t_\mu^4\right)\right)-\frac{a}{2}\left(t_1^2-t_\mu^2\right)-\frac{b}{6}\left(t_1^3-t_\mu^3\right)$$

$$-\frac{\theta t_\mu^{\,3}}{2}\left(a\left(t_1-t_\mu\right)+\frac{b}{2}\left(t_1^2-t_\mu^2\right)\right)+\frac{a\theta}{12}(t_1^4-t_\mu^4)-\frac{\theta}{6}$$

$$\left(t_1^3-t_\mu^3\right)\left(at_1+\frac{bt_1^{\,2}}{2}\right)+\frac{b\theta}{40}\left(t_1^5-t_\mu^5\right)+\left(t_1-t_\mu\right)$$

$$\left(at_1+\frac{bt_1^{\,2}}{2}+\frac{a\theta}{6}t_1^3+\frac{b\theta}{8}t_1^4\right)\Bigg)-pI_e\left(\frac{t_1^2}{6}(bt_1-3a)\right.$$

$$\left.+Mt_1(a+\frac{bt_1}{2})\right)\Bigg] \tag{34}$$

Thus necessary conditions for minimizing the cost are

$$\frac{\partial TC_3(t_1,T)}{\partial t_1}=0,\frac{\partial TC_3(t_1,T)}{\partial T}=0$$

And the sufficient conditions for minimizing $TC_3(t_1,T)$ are

$$\frac{\partial^2 TC_3(t_1,T)}{\partial t_1^{\,2}}>0,\frac{\partial^2 TC_3(t_1,T)}{\partial T^2}>0\ \& \tag{35}$$

$$\left[\frac{\partial^2 TC_3(t_1,T)}{\partial t_1^{\,2}}\right]\left[\frac{\partial^2 TC_3(t_1,T)}{\partial T^2}\right]-$$

$$\left[\frac{\partial^2 TC_3(t_1,T)}{\partial T\partial t_1}\right]^2>0 \tag{36}$$

4. Numerical Experiment

Case-1: When $M \le t_\mu$

We have supposed the data

$A=100, \theta=0.1, p=15, C_1=1, a=900,$ $b=0.02, \delta=0.01, C_2=1.5, C_3=0.6, h=2,$ $C=12, I_e=0.10, t_\mu=0.12, I_c=0.20, M=0.118.$

We obtained $t_1=0.160941$, $T=0.444417$ and total cost $TC_1(t_1,T)$ =383.142.

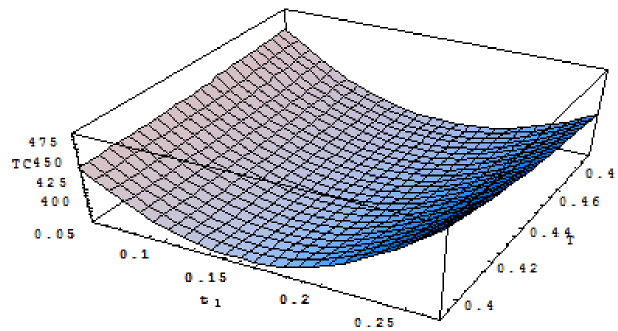

Figure 1. Graph of convexity between t_1 and T w.r.to total cost (Case-1)

Case-2: When $t_\mu \le M < t_1$

We have considered the following data

$A=100, \theta=0.1, p=15, C_1=1, a=900, b=0.02,$ 0.01,

$\delta=0.01, C_2=1.5, C_3=0.6, h=2,$

$C=12, I_e=0.10, t_\mu=0.12, I_c=0.20, M=0.17$.

We obtained $t_1=0.181704$, T = 0.443151 and total cost

$TC_2(t_1,T)$ = 353.46.

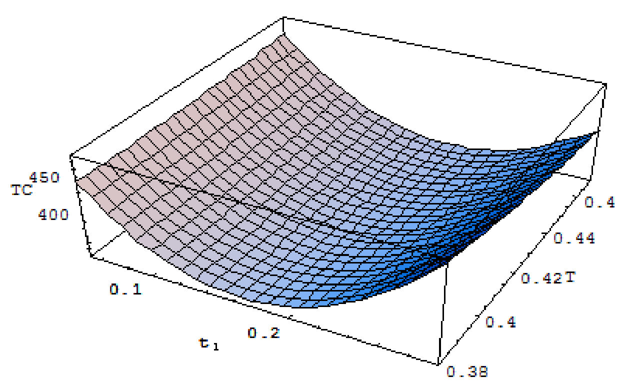

Figure 2. Graph of convexity between t_1 and T w.r.to total cost (Case-2)

Case-3: When $M \geq t_1$

We have supposed the data

$A = 100, \theta = 0.1, p = 15, C_1 = 1, a = 900, b = 0.02,$

$\delta = 0.01, C_2 = 1.5, C_3 = 0.6, h = 2,$

$C = 12, I_e = 0.10, t_\mu = 0.12, I_c = 0.20, M = 0.32$.

We obtained $t_1 = 0.218353$, T = 0.408938 and total cost

$TC_3(t_1, T)$ = 257.83

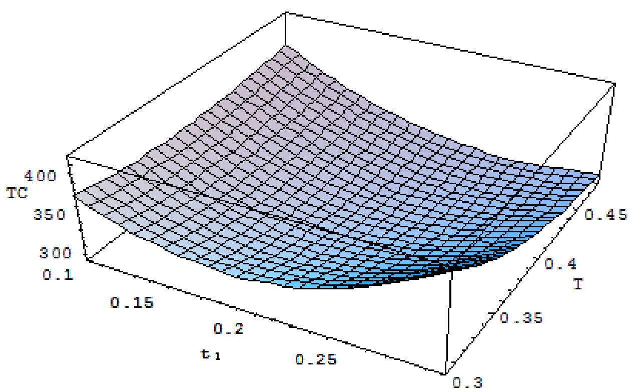

Figure 3. Graph of convexity between t_1 and T w.r.to total cost (Case-3)

5. Sensitivity Analysis

Sensitivity analysis with respect to various parameters

Case-1: **Table-(1)** Variation in lost sale cost and total cost
Table-(2) Variation in holding cost and total cost

Table 1. Variation in lost sale cost and total cost

C_3	t_1	T	$TC_1(t_1, T)$
0.7	0.160961	0.444309	383.223
0.8	0.160982	0.444201	383.304
0.9	0.161002	0.444092	383.385
1.0	0.161023	0.443984	383.467

Table 2. Variation in holding cost and total cost

h	t_1	T	$TC_1(t_1, T)$
2.1	0.158014	0.443404	385.72
2.2	0.155193	0.442432	388.212
2.3	0.152472	0.441449	390.621
2.4	0.149846	0.440602	392.952

Case-2: **Table-(3)** Variation in lost sale cost and total cost
Table-(4) Variation in holding cost and total cost

Table 3. Variation in lost sale cost and total cost

C_3	t_1	T	$TC_2(t_1, T)$
0.7	0.181721	0.443046	353.515
0.8	0.181739	0.442941	353.584
0.9	0.181756	0.442835	353.654
1.0	0.181774	0.44273	353.723

Table 4. Variation in holding cost and total cost

h	t_1	T	$TC_2(t_1, T)$
2.1	0.178495	0.442388	356.743
2.2	0.175399	0.441656	359.931
2.3	0.172409	0.440954	363.015
2.4	0.16952	0.44028	363.063

Case-3: **Table-(5)** Variation in lost sale cost and total cost
Table-(6) Variation in holding cost and total cost

Table 5. Variation in lost sale cost and total cost

C_3	t_1	T	$TC_3(t_1, T)$
0.7	0.218366	0.408853	257.87
0.8	0.218378	0.408768	257.909
0.9	0.218391	0.408684	257.949
1.0	0.218404	0.408599	257.989

Table 6. Variation in holding cost and total cost

h	t_1		$TC_3(t_1, T)$
2.1	0.213907	0.408303	262.976
2.2	0.20964	0.407701	267.924
2.3	0.205542	0.40713	272.685
2.4	0.201603	0.406587	277.87

6. Observations

Case-1: When $M \leq t_\mu$

- If we increase the lost sale cost parameter 'C_3' then $TC_1(t_1, T)$ increases. (Table-1)
- If we increase the holding cost parameter 'h' then

$TC_1(t_1, T)$ increases. (Table-2)

Case-2: When $t_\mu \le M < t_1$

- If we increase the lost sale cost parameter 'C_3' then

  $TC_2(t_1, T)$ increases. (Table-3)

- If we increase the holding cost parameter 'h' then $TC_2(t_1, T)$ increases. (Table-4)

Case-3: When $M \ge t_1$

- If we increase the lost sale cost parameter 'C_3' then

  $TC_3(t_1, T)$ increases. (Table-5)

- If we increase the holding cost parameter 'h' then $TC_3(t_1, T)$ increases. (Table-6)

7. Conclusion

Here we have developed an optimal inventory model for non-instantaneous decaying items with shortages of goods. Shortages are allowed as partially backlogging rate that is some customers may stay for the products until a particular time or event. We also have considered demand rate is time dependent. To make inventory model more realistic, we also have considered trade credit facility and partial backlogging since all shortages cannot be fully backlogged. At the end numerical example and sensitivity analysis is elaborated. We also have used Mathematica software and we got a three dimensional graph of convexity of the total cost that shows a total minimization cost of the function. For the future research we can incorporate some other parameters of inventory control system.

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