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Estimation of Aerodynamic Derivatives in Pitch of A Wedge in Hypersonic Flow

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Abstract

In the present paper a theory for 2D slender bodies at high angle of attack in hypersonic flow has been developed to determine the aerodynamic derivatives in pitch for different Mach numbers. The present theory has been applied to a sharp thick wedge with attached shock case. Using the theory, a relation for a piston moving in a cylinder at any velocity and relations for stiffness and damping derivatives are obtained for zero incidence of the wedge and it is found to be dependent on flight Mach number and wedge semi vertex angle. The present method includes the thin wedge case as well, which was covered by Lighthill piston theory, and applies only for small amplitude and low reduced frequency case. Effect of viscosity and secondary wave reflections have been neglected. The results are obtained for wedges of different semi vertex angles and Mach numbers. It is observed that the stiffness and Damping derivatives increase with increase in the semi vertex angle and decrease with an increase in Mach number. At very high Mach number the Mach number independence principle holds good.

Keywords: Aerodynamic Derivative, Hypersonic Flow, Mach Number, Semi Vertex Angle, Stability Derivative

1. Introduction

The idea of hypersonic similitude is due to in[1], who investigated the two-dimensional and axi -symmetric irrotational equations of motion. Equivalence of a steady hypersonic flow on a slender body with an unsteady flow in one fewer space dimensions was pointed out by in [2]. Unsteady supersonic flow has been well studied by using the potential theory. Unsteady hypersonic flow has been studied by many researchers using unsteady analogues of either the shock expansion theory or the tangent wedge approximation[2,3]. For an oscillating wedge, more theoretical studies have been made, such a theory of oscillating aero foils at high Mach number is developed in[4]. Here the pitching oscillation is taken into consideration. In results that to a good approximation, any plane slab of fluid, initially perpendicular to the undisturbed flow, remains so as it is swept downstream and moves in its own plane under the laws of one dimensional unsteady motion. A parameter τ is introduced, whose purpose will be to serve as a measure of the maximum inclination angle of Mach waves in the flow field. Here it is assumed that $M_1\tau \leq 1$ and τ is of the order of maximum deflection of a stream line. So in the flow past an aerofoil at high Mach number, the perturbations and gradients are much larger in the lateral direction than those in the axial direction. Sychev's law of plane sections for 3-D slender bodies at high angles of attack in hypersonic flow[5]has been discussed. Based on this a new piston analogy has been given for a 2-D slender body at high angle of attack in hypersonic flow. The new piston analogy has been first applied to get the stiffness and damping coefficients of an oscillating flat plate at high angle of incidence for which the bow shock is attached. Next assuming that two flat plates have formed a sharp wedge, closed form formulae for stiffness and damping coefficients have been obtained for it. In have obtained hypersonic[6] and supersonic[7] similitude's for planar wedges using Ghosh's theory. In have obtained

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hypersonic similitude for planar wedges for hypersonic flow in Newtonian limit also[8]. Results are obtained for hypersonic flow of a perfect gas over the wedges of different semi-vertex angles. Viscous effect and wave reflections are not taken into account in this development.

2. Analysis

Let the flat plate aerofoil be of length L at mean angle of incidence θ and oscillating in pitch with small amplitude about a pivot point 0_1 ,at a distance x_0 from its apex. The angle of attack at any instant is α. So the velocity of the infinitesimal piston (an infinitesimal part of the aerofoil) at point x, can be written as

$$U_P = U_1 \sin\alpha + \dot{\alpha}\,(x - x_0) \tag{1}$$

To relate the piston velocity with pressure on the face of piston, in suggested the use of 3 terms in the isentropic expression for the pressure on a piston as a power series in its velocity. So a condition that the piston velocity should be less than or equal to free stream sound velocity is imposed to satisfy isentropic assumption. This is consistent with hypersonic small disturbance theory on which Lighthill's piston analogy is based. Then the Pressure ratio can be written as

$$\frac{P_2}{P_1} = 1 + A\left(\frac{U_P}{a_1}\right)^2 + A\left(\frac{U_P}{a_1}\right)\sqrt{B + \left(\frac{U_P}{a_1}\right)^2} \tag{2}$$

where, $A = \left(\frac{\gamma(\gamma+1)}{4}\right) B = \left(\frac{4}{\gamma+1}\right)^2$

The nose down moment,

$$-m = \int_0^L (x - x_0) P_2\, dx \tag{3}$$

Of the load distribution about the axis $= x_0 \,.\, P_2$ is the pressure on the windward surface. The pressure on the leeward surface is taken to be zero.

Thus the aerodynamic stiffness derivative is,

$$-C_{m_\alpha} = (\gamma+1)\tan\theta \left[2 + \frac{\sqrt{M_1^2 \sin^2\theta + \left(\frac{4}{\gamma+1}\right)^2}}{M_1 \sin\theta} + \frac{M_1 \sin\theta}{\sqrt{M_1^2 \sin^2\theta + \left(\frac{4}{\gamma+1}\right)^2}} \right]$$

$$\left[\frac{1}{2} - h_0 \cos^2\theta \right] \tag{4}$$

And the aerodynamic damping derivative is,

$$-C_{m_q} = \frac{(\gamma+1)\tan\theta}{\cos^2\theta}$$

$$\left[2 + \frac{\sqrt{M_1^2 \sin^2\theta + \left(\frac{4}{\gamma+1}\right)^2}}{M_1 \sin\theta} + \frac{M_1 \sin\theta}{\sqrt{M_1^2 \sin^2\theta + \left(\frac{4}{\gamma+1}\right)^2}} \right]$$

$$\left[\frac{1}{3} - h_0 \cos^2\theta + h_0^2 \cos^4\theta \right] \tag{5}$$

3. Results and Discussions

Figures 1 and 2 show the variation of Stiffness and damping derivatives for Mach number 5 at different semi-vertex angle of from 5 degrees to 30 degrees. From the Figure 1 and Figure 2 it is seen that as the semi vertex angle increases the stiffness and damping derivative values increases in magnitude due to the increase in the platform area of the wedge. It is also seen that due to the increase in the semi vertex angle of the wedge there is substantial increase in the plan form area of the wedge which results in 54 % increase in the stiffness derivative and 44 % increase in the damping derivative. From Figures 3 and 4 it is seen that when semi vertex angle is

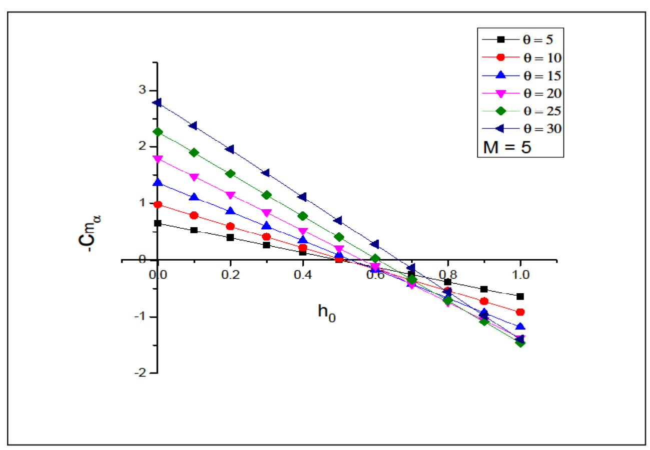

Figure 1. Variation of Stiffness derivative for M=5 with pivot position.

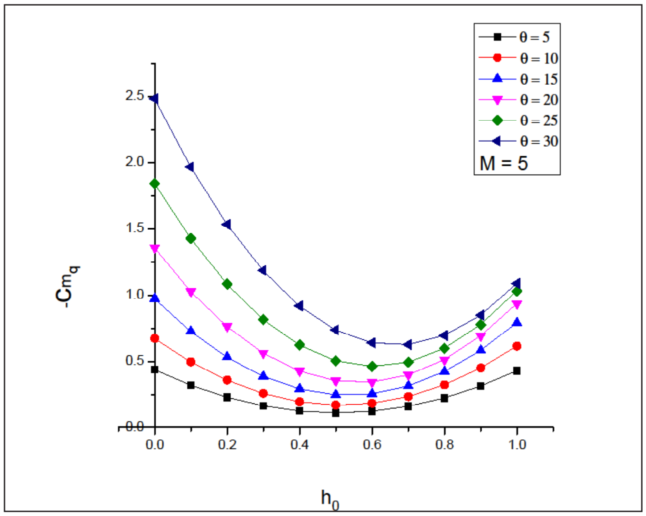

Figure 2. Variation of damping derivative for M=5, with pivot position.

increased to 15 and 20 degrees respectively results in 29 % and 40 % increment in stiffness and damping derivatives at Mach 10. Similarly, when the semi vertex angle is further increased to 25 and 30 degrees results in 22 % and 39 % increase in the stability derivatives in pitch. The stiffness derivative is the maximum at the h = 0, that is at the nose of the wedge, since at this point the moment arm will be the maximum and then it decreases linearly with the pivot position. Even in case of damping derivative we can see that magnitude increases with increase in semi vertex angle and the minima shifts towards the trailing edge of the wedge. From the Figures it is also observed that with the increase in the semi vertex angle of the wedge the center of pressure also shifts towards the trailing edge, the reason for this shift is due to the increase in the plan form area of the wedge and more so the major portion of plan form area is transferred towards the trailing edge leading to maximum plan form area available for the case at h = 1. Figure 3 and Figure4 show the variation of Stiffness and Damping derivative with pivot position for Mach number 10. It is observed that the magnitude in stiffness and Damping derivative decreases as the Mach number increases from 5 to 10. Further the same trend is observed for Mach number 12 and 15 as shown in Figures 5 to 8 and Figures 9, 10 shows the variation of Stiffness and damping derivative for Mach number 20 with pivot position. There is no much variation in the Magnitude of Stiffness and Damping derivative with pivot position for Mach number 15 to 20 thus adhering to Mach number independence Principle.

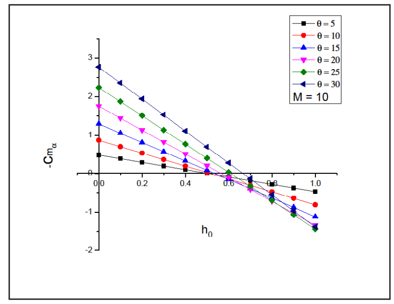

Figure 3. Variation of stiffness derivative M=10 with **pivot position.**

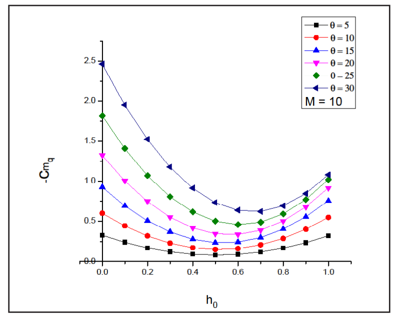

Figure 4. Variation of damping derivative for M=10 with pivot position.

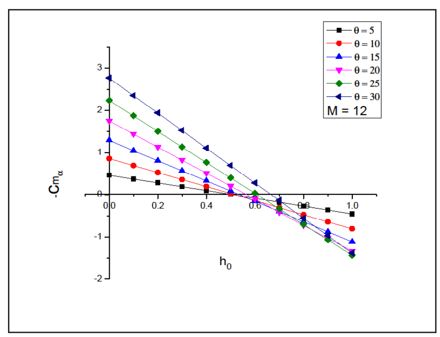

Figure 5. Variation of stiffness derivative for M=12 with pivot position

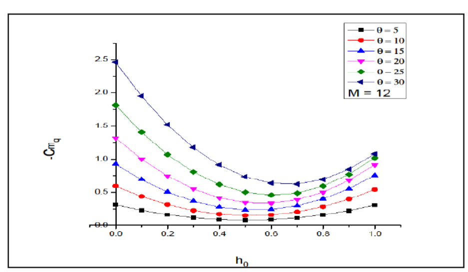

Figure 6. Variation of damping derivative for M=12 with pivot position.

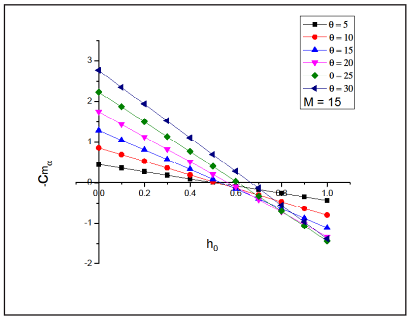

Figure 7. Variation of stiffness derivative for M=15with pivot position.

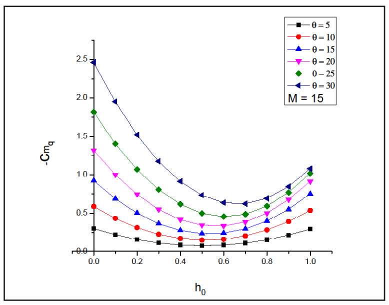

Figure 8. Variation of damping derivative for M=15with pivot position.

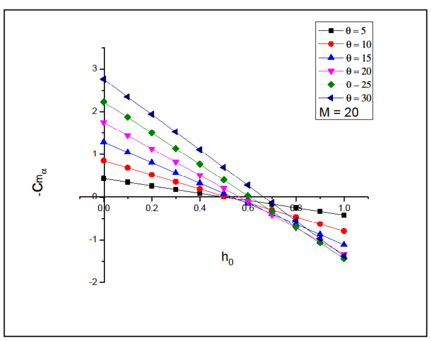

Figure 9. Variation of stiffness derivative for M=20 with pivot position.

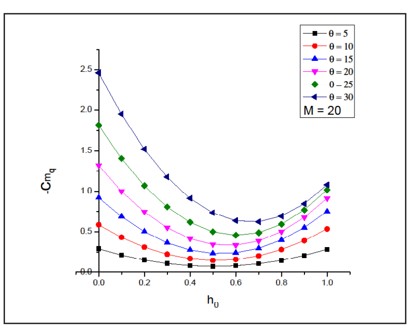

Figure 10. Variation of damping derivative for M=20 with pivot position.

4. Conclusion

All estimations are based on in viscid flow past sharp wedges. It is seen that the Stiffness and Damping derivative decreases with the increase in the Mach number. Further it increases with the increase in semi vertex angle. The Stiffness derivative decreases with pivot position whereas, in case of damping derivative first it decreases then attains a minima which is position of centre of pressure then increases. Secondary wave reflections generated from the body and reflected from the bow shock are neglected. Hui's has mentioned another set of waves generated due to the unsteady motion of the bow shock. These waves are also not considered in the present case. Finally, perturbation, tangential to the wedge surface is also neglected as per sychev law of plane sections.

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