BOOK REVIEWS



Finite Elements: Theory and Algorithms. Sashikumaar Ganesan and Lutz Tobiska (eds). Cambridge–IISc Series. Cambridge University Press, University Printing House, Cambridge CB2 8BS and IISc Press, Bengaluru. 2017. viii + 208 pages. Price: Rs 495.

Finite element methods (FEMs) is an important area in numerical analysis of partial differential equations (PDEs). In modern scientific computing, FEMs has gained popularity in approximating the numerical solutions of PDEs. Historically, though this method originated in solid mechanics, subsequently it has enjoyed tremendous success in a variety of applications, including fluids, plates, time-dependent problems, etc.

This book is written in a readerfriendly manner and can serve as a reference book for the first course in FEMs. It contains nine chapters covering weak formulation of elliptic PDEs, finite element construction and the interpolation theory, approximation of biharmonic problem, parabolic problem, linear elasticity, Mindlin–Reissner plate, Stokes and Navier–Stokes problems, and some algorithmic aspects of implementation.

Chapter 1 of the book begins with an introduction to Sobolev spaces and mentions some notions from functional analysis and proving the Banach contraction principle. The definition of weak derivative and Sobolev spaces are introduced; the statements of Sobolev imbedding theorem and the trace theorems are included with appropriate references for the proofs.

In chapter 2, weak formulation is introduced and the well-posedness of the elliptic scalar problem is proved. This chapter motivates the weak formulation for setting up an abstract framework for the Lax–Milgram lemma, which guarantees the existence and uniqueness of a weak solution to the elliptic problem. The lemma is proved and its use is illustrated with examples of Laplace equation and a convection-diffusion-reaction equation. It closes by introducing the standard Galerkin method and proving the fundamental Cea's lemma yielding the quasi best approximation for the Galerkin method.

Chapters 3 and 4 are devoted to the finite element construction and the interpolation theory respectively. The popular Lagrange finite elements on simplicial and cubical meshes are discussed in detail and the construction of global finite element spaces is described by following a necessary result to glue the local shape functions. The concept of mapped elements and a brief discussion on isoparametric finite elements are included to make the reader familiar with the forthcoming difficulties in implementation. Serendipity element, Argyris triangle and Bells triangles are some of the more popular examples included in the discussion. The interpolation theory is an important subject to understand the approximation properties of finite elements. The fundamental tools in this are Bramble-Hilbert lemma, change of variables with affine transformations and appropriate interpolation operators. The discussion and technical details are clearly presented with appropriate references, whenever required. Aubin-Nitsche duality argument to improve the L^2 -norm error estimate is explained in detail. Some discussion on Scott-Zhang interpolation for less smooth functions is added.

Chapter 5 discusses the approximation of Biharmonic equation by classical C^1 conforming and nonconforming FEMs that include rectangular Adini element, triangular Morley element and a nonconforming tetrahedral element. It is well known that as the implementation of C^1 finite elements is complicated, classical nonconforming FEMs became attractive. It is necessary to understand these methods from both theoretical and practical points of view. Readers interested in fourth-order problems and nonconforming methods will surely benefit from the discussion in this chapter.

Chapter 6 deals with the numerical analysis of parabolic problems using FEM in space variables and various discretizations in time variable. The discussion on A-stable and L-stable methods is precise. The time discretization includes the backward Euler method, Crank-Nicolson method, fractional step θ -method and Galerkin method. The chapter features a well-structured discussion on operator splitting techniques for high-dimensional problems. Chapter 7 presents a brief discussion on finite element approximation of linear elasticity problem and Mindlin-Reissner plate. The finite element approximation of Stokes and Navier-Stokes problems is discussed in chapter 8. The weak formulation of Stokes problem and its well-posedness through Ladyzhenskaya-Babuska-Brezzi condition are outlined. Conforming and nonconforming discretizations that are inf-sup stable have also been discussed. A brief section on equalorder stable approximation is added to facilitate the state-of-the-art discussion in the literature.

Without implementation of FEMs, there is no true sense of numerical analysis, be it in industrial or academic research. In chapter 9, the authors provide from their practical experience, description of algorithms that are useful to write the codes for implementing FEMs. This includes various discussions on meshhandling, assembly, boundary conditions and solvers, etc.

The bibliography covers most of the classical monographs and research articles.

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When I scanned the contents of this edition of the Annual Review of Public Health, my attention was immediately drawn to the article by Peter Craig et al. entitled 'Natural experiments: an overview of methods, approaches, and contributions to public health intervention